OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4722

Core Mathematics 2

Monday **23 MAY 2005** Morning 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 A sequence S has terms $u_1, u_2, u_3, \ldots$ defined by

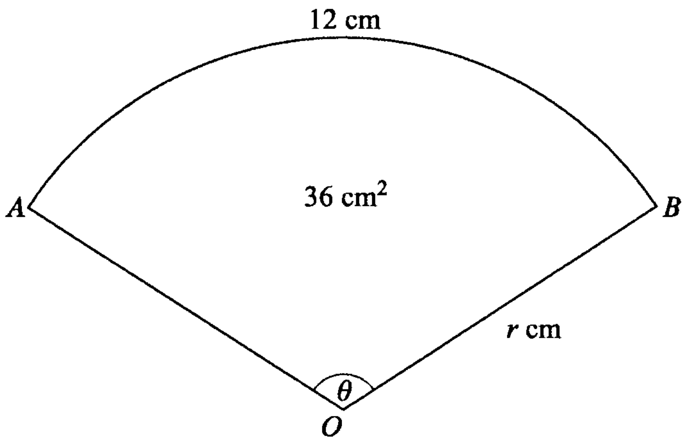
$$u_n = 3n - 1,$$

for $n \geqslant 1$.

(i) Write down the values of u_1, u_2 and u_3, and state what type of sequence S is. [3]

(ii) Evaluate $\sum_{n=1}^{100} u_n$. [3]

2



A sector OAB of a circle of radius r cm has angle θ radians. The length of the arc of the sector is 12 cm and the area of the sector is 36 cm^2 (see diagram).

(i) Write down two equations involving r and θ. [2]

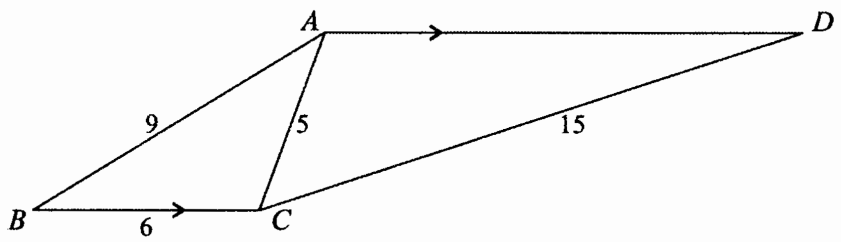
(ii) Hence show that $r = 6$, and state the value of θ. [2]

(iii) Find the area of the segment bounded by the arc AB and the chord AB. [3]

3 (i) Find $\int (2x + 1)(x + 3)\,dx$. [4]

(ii) Evaluate $\int_0^9 \frac{1}{\sqrt{x}}\,dx$. [3]

4



In the diagram, $ABCD$ is a quadrilateral in which AD is parallel to BC. It is given that $AB = 9$, $BC = 6$, $CA = 5$ and $CD = 15$.

(i) Show that $\cos BCA = -\frac{1}{3}$, and hence find the value of $\sin BCA$. [4]

(ii) Find the angle ADC correct to the nearest $0.1°$. [4]

5 The cubic polynomial $f(x)$ is given by

$$f(x) = x^3 + ax + b,$$

where a and b are constants. It is given that $(x + 1)$ is a factor of $f(x)$ and that the remainder when $f(x)$ is divided by $(x - 3)$ is 16.

(i) Find the values of a and b. [5]

(ii) Hence verify that $f(2) = 0$, and factorise $f(x)$ completely. [3]

6 **(i)** Find the binomial expansion of $\left(x^2 + \frac{1}{x}\right)^3$, simplifying the terms. [4]

(ii) Hence find $\int \left(x^2 + \frac{1}{x}\right)^3 \mathrm{d}x$. [4]

7 **(i)** Evaluate $\log_5 15 + \log_5 20 - \log_5 12$. [3]

(ii) Given that $y = 3 \times 10^{2x}$, show that $x = a\log_{10}(by)$, where the values of the constants a and b are to be found. [4]

[Questions 8 and 9 are printed overleaf.]

8 The amounts of oil pumped from an oil well in each of the years 2001 to 2004 formed a geometric progression with common ratio 0.9. The amount pumped in 2001 was 100 000 barrels.

(i) Calculate the amount pumped in 2004. [2]

It is assumed that the amounts of oil pumped in future years will continue to follow the same geometric progression. Production from the well will stop at the end of the first year in which the amount pumped is less than 5000 barrels.

(ii) Calculate in which year the amount pumped will fall below 5000 barrels. [4]

(iii) Calculate the total amount of oil pumped from the well from the year 2001 up to and including the final year of production. [3]

9 **(a)** **(i)** Write down the exact values of $\cos \frac{1}{6}\pi$ and $\tan \frac{1}{3}\pi$ (where the angles are in radians). Hence verify that $x = \frac{1}{6}\pi$ is a solution of the equation

$$2\cos x = \tan 2x.$$ [3]

(ii) Sketch, on a single diagram, the graphs of $y = 2\cos x$ and $y = \tan 2x$, for x (radians) such that $0 \leqslant x \leqslant \pi$. Hence state, in terms of π, the other values of x between 0 and π satisfying the equation

$$2\cos x = \tan 2x.$$ [4]

(b) **(i)** Use the trapezium rule, with 3 strips, to find an approximate value for the area of the region bounded by the curve $y = \tan x$, the x-axis, and the lines $x = 0.1$ and $x = 0.4$. (Values of x are in radians.) [4]

(ii) State with a reason whether this approximation is an underestimate or an overestimate. [1]

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1 $u_1 = 3\times1-1=\mathbf{2}$ $u_2 = 3\times2-1=\mathbf{5}$ $u_3 = 3\times3-1=\mathbf{8}$ [3]

(**arithmetic progression**) $\sum_{n=1}^{100} u_n = \frac{1}{2}n\{2a+(n-1)d\} = 50\{4+99\times3\} = \mathbf{15\,050}$

[4]

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2 $\left.\begin{aligned} r\theta &= 12 \\ \tfrac{1}{2}r^2\theta &= 36 \end{aligned}\right\}$ $\Rightarrow$ $6r = 36$ $\Rightarrow$ $\boldsymbol{r=6}$ (show), $\boldsymbol{\theta=2}$ [2] [2]

area of segment $= 36 - \Delta OAB = 36 - \frac{1}{2}\times36\times\sin2 = 19\cdot6326... = \mathbf{19\cdot6}\ \text{cm}^2$ (3 s.f.)

[3]

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3 $\int(2x+1)(x+3)\,\mathrm{d}x = \int\left(2x^2+7x+3\right)\mathrm{d}x = \boldsymbol{\tfrac{2}{3}x^3+\tfrac{7}{2}x^2+3x+\mathrm{c}}$

[4]

$\int_0^9 \frac{1}{\sqrt{x}}\,\mathrm{d}x = \int_0^9 x^{-\frac{1}{2}}\,\mathrm{d}x = \left[2x^{\frac{1}{2}}\right]_0^9 = 6-0=\mathbf{6}$ [3]

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4 Cosine Rule in ΔABC …

$$9^2 = 5^2+6^2-2\times5\times6\times\cos B\hat{C}A$$
$$81 = 61-60\cos B\hat{C}A$$
$$\cos B\hat{C}A = {}^-\tfrac{20}{60} = {}^-\boldsymbol{\tfrac{1}{3}} \qquad \text{(show)}$$

hence $\sin B\hat{C}A = \sqrt{1-\cos^2 B\hat{C}A} = \sqrt{8/9} = \boldsymbol{\tfrac{2\sqrt{2}}{3}}$ [4]

$C\hat{A}D = B\hat{C}A$ (alternate angles)

Sine Rule in ΔACD …

$$\frac{\sin A\hat{D}C}{5} = \frac{\sin C\hat{A}D}{15}$$

$\sin A\hat{D}C = \frac{1}{3}\times\frac{2\sqrt{2}}{3} = \frac{2\sqrt{2}}{9}$ $A\hat{D}C = 18\cdot31673... = \mathbf{18\cdot3^\circ}$ (1 d.p.)

[4]

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5 remainder theorem … $\left.\begin{aligned} \mathrm{f}({}^-1) &= 0 & {}^-1-a+b &= 0 \\ \mathrm{f}(3) &= 16 & 27+3a+b &= 16 \end{aligned}\right\}$ $\Rightarrow$ $28+4a=16$

$\boldsymbol{a = {}^-3, \quad b = {}^-2}$

[5]

$\therefore\ \mathrm{f}(2) = 8-6-2=0$ $\mathrm{f}(x) \equiv \boldsymbol{(x+1)^2(x-2)}$

[3]

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6 $\left(x^2+\frac{1}{x}\right)^3 \equiv \left(x^2\right)^3 + 3\left(x^2\right)^2\left(\frac{1}{x}\right) + 3\left(x^2\right)\left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3 = \boldsymbol{x^6 + 3x^3 + 3 + \frac{1}{x^3}}$ [4]

$\int\left(x^2+\frac{1}{x}\right)^3 \mathrm{d}x = \int\left(x^6 + 3x^3 + 3 + \frac{1}{x^3}\right)\mathrm{d}x = \boldsymbol{\tfrac{1}{7}x^7 + \tfrac{3}{4}x^4 + 3x - \frac{1}{2x^2} + \mathbf{c}}$ [4]

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7 $\log_5 15 + \log_5 20 - \log_5 12 = \left(\log_5 3 + \log_5 5\right) + \left(\log_5 4 + \log_5 5\right) - \left(\log_5 3 + \log_5 4\right) = 2\log_5 5 = \mathbf{2}$ [3]

$y = 3\times 10^{2x} \quad \Leftrightarrow \quad 10^{2x} = \tfrac{1}{3}y \quad \Leftrightarrow \quad 2x = \log_{10}\left(\tfrac{1}{3}y\right) \quad \Leftrightarrow \quad \boldsymbol{x = \tfrac{1}{2}\log_{10}\left(\tfrac{1}{3}y\right)}$ [4]

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8 amount pumped in 2004 $= 100\,000\times(0\cdot 9)^3 = \mathbf{72\,900}$ barrels [2]

amount pumped in the year $(2000+n) = 100\,000\times(0\cdot 9)^{n-1}$

$$100\,000\times(0\cdot 9)^{n-1} < 5000$$
$$(0\cdot 9)^{n-1} < 0\cdot 05$$
$$(n-1)\log 0\cdot 9 < \log 0\cdot 05$$
$$n > \frac{\log 0\cdot 05}{\log 0\cdot 9} + 1 = 29\cdot 433...$$

2030 will be the first year in which production falls below 5000 barrels. [4]

total amount $= \frac{a\left(1-r^n\right)}{1-r} = \frac{100\,000\left(1-0\cdot 9^{30}\right)}{1-0\cdot 9} = \mathbf{957\,609}$ barrels [3]

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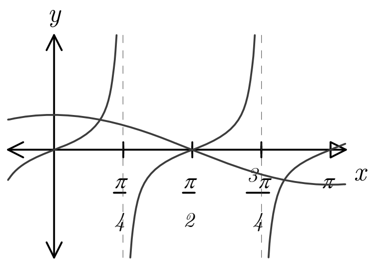
9 $\cos\tfrac{1}{6}\pi = \tfrac{1}{2}\sqrt{3}$ $\quad \tan\tfrac{1}{3}\pi = \sqrt{3}$ $\quad$ so for $x = \tfrac{1}{6}\pi$ .... $\quad 2\cos x = \sqrt{3} = \tan 2x$ [3]



other roots of $2\cos x = \tan 2x$ are $\boldsymbol{x = \tfrac{1}{2}\pi,\ \tfrac{5}{6}\pi}$ [4]

area of required region $\approx \tfrac{1}{2}\times 0\cdot 1[\tan 0\cdot 1 + 2\tan 0\cdot 2 + 2\tan 0\cdot 3 + \tan 0\cdot 4] = \mathbf{0\cdot 0774}$ (3 s.f.) [4]

the curve $y = \tan x$ is concave upwards so the trapezium rule gives an **overestimate**. [1]