OCR
RECOGNISING ACHIEVEMENT

Monday 13 May 2013 – Afternoon

AS GCE MATHEMATICS

4721/01 Core Mathematics 1

QUESTION PAPER

4733240613

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer book 4721/01
- List of Formulae (MF1)

Other materials required:
None

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

No calculator can be used for this paper

1 Express each of the following in the form $a\sqrt{5}$, where a is an integer.

(i) $4\sqrt{15}\times\sqrt{3}$ **[2]**

(ii) $\dfrac{20}{\sqrt{5}}$ **[1]**

(iii) $5^{\frac{3}{2}}$ **[1]**

2 Solve the equation $8x^6 + 7x^3 - 1 = 0$. **[5]**

3 It is given that $\mathrm{f}(x) = \dfrac{6}{x^2} + 2x$.

(i) Find $\mathrm{f}'(x)$. **[3]**

(ii) Find $\mathrm{f}''(x)$. **[2]**

4 **(i)** Express $3x^2 + 9x + 10$ in the form $3(x+p)^2 + q$. **[3]**

(ii) State the coordinates of the minimum point of the curve $y = 3x^2 + 9x + 10$. **[2]**

(iii) Calculate the discriminant of $3x^2 + 9x + 10$. **[2]**

5 **(i)** Sketch the curve $y = \dfrac{2}{x^2}$. **[2]**

(ii) The curve $y = \dfrac{2}{x^2}$ is translated by 5 units in the negative x-direction. Find the equation of the curve after it has been translated. **[2]**

(iii) Describe a transformation that transforms the curve $y = \dfrac{2}{x^2}$ to the curve $y = \dfrac{1}{x^2}$. **[2]**

6 A circle C has equation $x^2 + y^2 + 8y - 24 = 0$.

(i) Find the centre and radius of the circle. **[3]**

(ii) The point A (2, 2) lies on the circumference of C. Given that AB is a diameter of the circle, find the coordinates of B. **[2]**

7 Solve the inequalities

(i) $3 - 8x > 4$, **[2]**

(ii) $(2x-4)(x-3) \leqslant 12$. **[5]**

8 A is the point (–2, 6) and B is the point (3, –8). The line l is perpendicular to the line $x - 3y + 15 = 0$ and passes through the mid-point of AB. Find the equation of l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

9 **(i)** Sketch the curve $y = 2x^2 - x - 6$, giving the coordinates of all points of intersection with the axes. **[5]**

(ii) Find the set of values of x for which $2x^2 - x - 6$ is a decreasing function. **[3]**

(iii) The line $y = 4$ meets the curve $y = 2x^2 - x - 6$ at the points P and Q. Calculate the distance PQ. **[4]**

10 The curve $y = (1 - x)(x^2 + 4x + k)$ has a stationary point when $x = -3$.

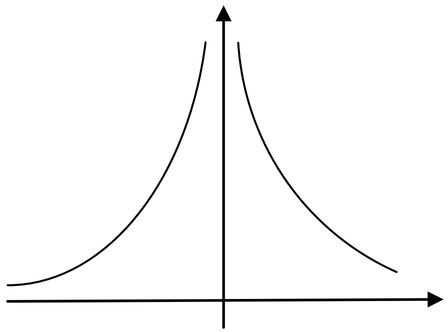
(i) Find the value of the constant k. **[7]**

(ii) Determine whether the stationary point is a maximum or minimum point. **[2]**

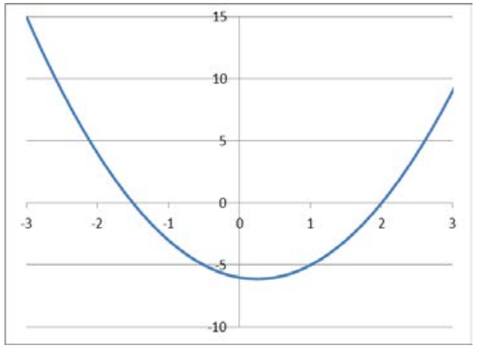
(iii) Given that $y = 9x - 9$ is the equation of the tangent to the curve at the point A, find the coordinates of A. **[5]**

Question			Answer	Marks	Guidance	
1	(i)		$4\sqrt{45}$ $=12\sqrt{5}$	M1 A1 **[2]**	or $4\sqrt{5}\sqrt{3}\times\sqrt{3}$ (not just $4\sqrt{5\times3}\times\sqrt{3}$) or $\sqrt{720}$ or $\sqrt{240}\times\sqrt{3}$ or better Correctly simplified answer	For method mark, makes a correct start to manipulate the expression i.e. at least combines two parts correctly or splits one part correctly
1	(ii)		$\frac{20\sqrt{5}}{5}=4\sqrt{5}$	B1 **[1]**	**cao**, do not allow unsimplified, do not allow if clearly from wrong working	
1	(iii)		$5\sqrt{5}$	B1 **[1]**	**cao www**, do not allow unsimplified, do not allow if clearly from wrong working	
2			$k=x^3$ $8k^2+7k-1=0$ $(8k-1)(k+1)=0$ $k=\frac{1}{8},\ k=-1$ $x=\frac{1}{2},\ x=-1$	M1* DM1 * A1 M1 A1 **[5]**	Use a substitution to obtain a quadratic or factorise into 2 brackets each containing x^3 Correct method to solve a quadratic Both values of k correct Attempt to cube root at least one value to obtain x Both values of x correct and no other values	**No marks** if whole equation cube rooted etc. **No marks** if straight to formula with no evidence of substitution at start and no cube rooting/cubing at end. **Spotted solutions:** If **M0 DMO** or **M1 DM0** **SC B1** $x=-1$ **www** **SC B1** $x=\frac{1}{2}$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)

Question			Answer	Marks	Guidance	
3	(i)		$f(x) = 6x^{-2} + 2x$ $f'(x) = -12x^{-3} + 2$	M1 A1 B1 **[3]**	kx^{-3} obtained by differentiation $-12x^{-3}$ $2x$ correctly differentiated to 2	**ISW** incorrect simplification after correct expression
3	(ii)		$f''(x) = 36x^{-4}$	M1 A1 **[2]**	Attempt to differentiate **their (i)** i.e. at least one term "correct" Fully correct **cao** No follow through for **A** mark	Allow constant differentiated to zero **ISW** incorrect simplification after correct expression
4	(i)		$3\left(x^2+3x\right)+10$ $=3\left(x+\frac{3}{2}\right)^2-\frac{27}{4}+10$ $=3\left(x+\frac{3}{2}\right)^2+\frac{13}{4}$	B1 M1 A1 **[3]**	$\left(x+\frac{3}{2}\right)^2$ $10-3p^2$ or $\frac{10}{3}-p^2$ Allow $p=\frac{3}{2}$, $q=\frac{13}{4}$ **A1 www**	If p, q found correctly, then **ISW** slips in format. $3(x + 1.5)^2 - 3.25$ **B1 M0 A0** $3(x + 1.5) + 3.25$ **B1 M1 A1 (BOD)** $3(x + 1.5x)^2 + 3.25$ **B0 M1 A0** $3(x^2 + 1.5)^2 + 3.25$ **B0 M1 A0** $3(x - 1.5)^2 + 3.25$ **B0 M1 A1 (BOD)** $3x(x + 1.5)^2 + 3.25$ **B0M1A0**
4	(ii)		$\left(-\frac{3}{2},\frac{13}{4}\right)$	B1 B1 **[2]**	FT i.e. – their p FT i.e. their q	If restarted e.g. by differentiation: Correct method to find x value of minimum point **M1** Fully correct answer **www A1**
4	(iii)		$9^2 - (4 \times 3 \times 10)$ $= -39$	M1 A1 **[2]**	Uses b^2-4ac Ignore >0, <0 etc. **ISW** comments about number of roots	Use of $\sqrt{b^2-4ac}$ is **M0** unless recovered

Question			Answer	Marks	Guidance	
5	**(i)**			B1 B1 **[2]**	Excellent curve for $y=\frac{2}{x^2}$ in either quadrant Excellent curve for $y=\frac{2}{x^2}$ in other quadrant and no more. **SC B1** Reasonably correct curves in 1st and 2nd quadrants and no more	**N.B. Ignore 'feathering' now that answers are scanned.** For Excellent: Correct shape, not touching axes, asymptotes clearly the axes. Allow slight movement away from asymptote at one end but not more. Not finite. For **SC B1**, graph must not touch axes more than twice.
5	**(ii)**		$y=\frac{2}{(x+5)^2}$	M1 A1 **[2]**	$\frac{2}{(x+5)^2}$ or $\frac{2}{(x-5)^2}$ seen Fully correct, must include "y =" or "f(x) ="	
5	**(iii)**		Stretch scale factor $\frac{1}{2}$ parallel to y-axis	B1 B1 **[2]**	Or "stretched" etc; **do not accept** squashed, compressed etc. **oe** e.g. scale factor $1/\sqrt{2}$ parallel to x-axis	**0/2** if more than one type of transformation mentioned **ISW** non-contradictory statements For "parallel to the y-axis" allow "vertically", "up", "in the (positive) y direction". **Do not accept** "in/on/across/up/along/to/towards the y-axis"
6	**(i)**		Centre (0, −4) $x^2+(y+4)^2-16-24=0$ Radius = $\sqrt{40}$	B1 M1 A1 **[3]**	 $(y\pm4)^2-4^2$ seen (or implied by correct answer) Do not allow **A** mark from $(y-4)^2$	 Or attempt at $r^2=f^2+g^2-c$ **A0** for $\pm\sqrt{40}$
6	**(ii)**		(−2, −10)	B1FT B1FT **[2]**	FT through centre given in **(i)** FT through centre given in **(i)**	i.e. (their $2x-2$, their $2y-2$) Apply same scheme if equation of diameter found and attempt to solve simultaneously; no marks until a correct value of x/y found.

Question			Answer	Marks	Guidance	
7	(i)		$8x < -1$	B1	soi, allow $-8x > 1$ but not just $8x + 1 < 0$	Allow $\le$ or $\ge$ for first mark Do not **ISW** if contradictory Do not allow $\le$ or $\ge$
			$x < -\frac{1}{8}$	B1	Correct working only, allow $-\frac{1}{8} > x$ Do not allow $\frac{1}{-8}$	
				[2]		
7	(ii)		$2x^2 - 10x \le 0$	M1*	Expand brackets and rearrange to collect all terms on one side	No more than one incorrect term
			$2x(x-5) \le 0$	DM1*	Correct method to find roots of resulting quadratic	Allow $(2x + 0)(x - 5)$ **Do not allow $(2x - 4)(x - 3)$,** this is the original expression.
				A1	0, 5 seen as roots – could be on sketch graph	
				DM1*	Chooses "inside region" for their roots of their **resulting** quadratic (not the original)	Dependent on first M1 only
			$0 \le x \le 5$	A1	Do **not** accept strict inequalities for final mark	Allow "$x \ge 0$, $x \le 5$", "$x \ge 0$ **and** $x \le 5$" but do not allow "$x \ge 0$ **or** $x \le 5$"
				[5]		
8			Midpoint of AB is $\left(\frac{-2+3}{2}, \frac{6+-8}{2}\right)$	M1	Correct method to find midpoint – can be implied by one correct value	**NB – "correct" answer can be found with wrong mid-pt. Check working thoroughly.**
			$\left(\frac{1}{2}, -1\right)$	A1		
			Gradient of given line = $\frac{1}{3}$	B1	Must be stated or used – just rearranging the equation is not sufficient	
			Gradient of $l = -3$	B1FT	Use of $m_1 m_2 = -1$ (may be implied), allow for any initial non-zero numerical gradient	
			$y + 1 = -3\left(x - \frac{1}{2}\right)$	M1	Correct equation for line, any non-zero numerical gradient, through their $\left(\frac{1}{2}, -1\right)$	
				A1	Correct equation in any three-term form	
			$6x + 2y - 1 = 0$	A1 **[7]**	$k(6x + 2y - 1) = 0$ for integer k **www**	Must include "= 0"

Question		Answer	Marks	Guidance	
9	(i)	$(2x+3)(x-2)=0$	M1	Correct method to find roots	
		$x=-\frac{3}{2},\ x=2$	A1	Correct roots	
			B1	Reasonably symmetrical positive quadratic curve, must cross x axis	
			B1	y intercept $(0, -6)$ only	Indicated on graph or clearly stated, but there must be a curve
			B1	Good curve, with correct roots indicated and min point in 4th quadrant (not on axis)	Only allow final B1 if curve is clearly intended to be a quadratic symmetrical about min point in 4th quadrant
			[5]		
9	(ii)	$\frac{dy}{dx}=4x-1=0$	M1	Attempt to find x coordinate of vertex by differentiating and equating/comparing to zero, completing the square, finding the mid-point of their roots oe	**SC** Award **B1 (FT)** for $x<0$ if clearly from their graph
		Vertex when $x=\frac{1}{4}$	A1	cao	**NB Look for solution to 9ii done in the space below 9i graph**
		$x<\frac{1}{4}$	A1 FT	$x <$ their vertex, allow $\leq$	
			[3]		
9	(iii)	$2x^2-x-6=4$	M1	Set quadratic expression equal to 4	
		$2x^2-x-10=0$ $(2x-5)(x+2)=0$	M1	Correct method to solve **resulting** three term quadratic	**Not** $2x^2-x-6=0$ with no use of 4
		$x=\frac{5}{2},\ x=-2$	A1	Must have both solutions – no mark for one spotted root	
		Distance $PQ=4½$	B1FT	FT from their x values found from their resulting quadratic, provided $y=4$	Allow $\frac{9}{2}$ oe, but do **not** accept unsimplified expressions like $\sqrt{\frac{81}{4}}$
			[4]		

Question			Answer	Marks	Guidance	
10	(i)		$y = -x^3 - 3x^2 + 4x - kx + k$	M1 A1	Attempt to multiply out brackets Can be unsimplified	Must have $\pm x^3$ and 5 or 6 terms
			$\frac{dy}{dx} = -3x^2 - 6x + 4 - k$	M1 A1	Attempt to differentiate **their** expansion (**M0** if signs have changed throughout)	**<u>If using product rule:</u>** Clear attempt at correct rule **M1*** Differentiates both parts correctly **A1** Expand brackets of both parts ***DM1**
			When $x = -3$, $\frac{dy}{dx} = 0$	M1* DM1*	Sets $\frac{dy}{dx} = 0$ Substitutes $x = -3$ into their $\frac{dy}{dx} = 0$	
			$-27 + 18 + 4 - k = 0$ $k = -5$	A1 **[7]**	**www**	Then as main scheme
10	(ii)		$\frac{d^2y}{dx^2} = -6x - 6$	M1	Evaluates second derivative at $x = -3$ or other fully correct method	**Alternate valid methods include:** 1) Evaluating gradient at either side of -3 2) Evaluating y at either side of -3 3) Finding other turning point and stating "negative cubic so min before max"
			When $x = -3$, $\frac{d^2y}{dx^2}$ is positive so min point	A1 **[2]**	No incorrect working seen in this part i.e. if second derivate is evaluated, it must be 12. (Ignore errors in k value)	
10	(iii)		$-3x^2 - 6x + 9 = 9$	M1	Sets their gradient function from (i) (or from a restart) to 9	Allow first **M** even if k not found but look out for correct answer from wrong working.
			$3x(x + 2) = 0$ $x = 0$ or $x = -2$	A1	Correct x-values	**<u>SEE NEXT PAGE FOR ALTERNATIVE METHODS</u>**
			When $x = 0$, $y = -9$ for line $y = -5$ for curve	M1	One of their x-values substituted into both curve and line/substituted into one and verified to be on the other	**Note:** Putting a value into $x^3 + 3x^2 - 4 = 0$ (where the line and curve meet) is equivalent
			When $x = -2$, $y = -27$ for line $y = -27$ for curve	M1	Conclusion that $x = -2$ is the correct value **<u>or</u>** Second x-value substituted into both curve and line/verified as above	<u>If curve equated to line before differentiating:</u>
			$x = -2$, $y = -27$	A1 **[5]**	$x = -2$, $y = -27$ **www *(Check k correct)***	**M0 A0**, can get **M1M1** but **A0 ww** Maximum mark **2/5**

Question			Answer	Marks	Guidance
10	(iii)		Alternative method Attempt to solve equations of curve and tangent simultaneously **and** uses valid method to establish at least one root of the resulting cubic ($x^3 + 3x^2 - 4 = 0$ oe) **M1** All roots found **A1** Either 1) States $x = -2$ is repeated root so tangent **M2** (If double root found but not explicitly stated that repeated root implies tangent then **M0** but **B1** if (−2, −27) found) Or 2) Substitutes one x value into their gradient function to determine if equal to gradient of the line **M1** Substitutes other x value into their gradient function to determine if equal to gradient of the line **or** conclusion that −2 is the correct one **M1** $x = -2$, $y = -27$ **A1 www** **SC** Trial and Improvement Finds at least one value at which the gradient of the curve is 9 **B1** Verifies on both line and curve **B1 2/5**		

APPENDIX 1

Allocation of method mark for solving a quadratic

e.g. $2x^2 - x - 6 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x - 3)(x + 2)$	**M1**	$2x^2$ and -6 obtained from expansion
$(2x - 3)(x + 1)$	**M1**	$2x^2$ and $-x$ obtained from expansion
$(2x + 3)(x + 2)$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$\dfrac{-1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -6}}{2 \times 2}$ earns **M1** (minus sign incorrect at start of formula)

$\dfrac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 6}}{2 \times 2}$ earns **M1** (6 for c instead of -6)

$\dfrac{-1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 6}}{2 \times 2}$ **M0** (2 sign errors: initial sign and c incorrect)

$\dfrac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -6}}{2 \times -6}$ **M0** ($2c$ on the denominator)

Notes – for equations such as $2x^2 - x - 6 = 0$, then $b^2 = 1^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for a in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - x - 6 = 0$$

$$2\left(x^2 - \frac{1}{2}x\right) - 6 = 0$$

$$2\left[\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right] - 6 = 0$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{49}{16}$$

$$x - \frac{1}{4} = \pm\sqrt{\frac{49}{16}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{1}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt