OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **2642**

Probability & Statistics 2

Tuesday **17 JUNE 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 The rainfall, x cm, is recorded at a weather station on each of 6 randomly chosen August days between 1990 and 1999. The data can be summarised by $\Sigma x = 6.3$, $\Sigma x^2 = 11.95$. Calculate unbiased estimates for the mean and variance of the daily rainfall in August at the weather station in this period. [4]

2 The average number of weeds per square metre of a large lawn, measured each May over many years, is known to be 4.0. One May a randomly chosen square metre of lawn is found to contain 7 weeds. Using the Poisson tables, test, at the 5% significance level, whether this is evidence of an increase in the mean number of weeds per square metre of the lawn, stating your hypotheses clearly. [6]

3 The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that

$$P(X > 51) = 0.1841 \qquad \text{and} \qquad P(X > 60) = 0.0082.$$

(i) Show that $\sigma = 6.00$, and find the value of μ. [5]

(ii) The mean of 81 randomly chosen observations of X is denoted by $\bar{X}$. Find $P(\bar{X} > \mu - 1)$. [2]

4 Calls received by a car rescue service occur independently and at a constant average rate of 3 per minute.

(i) Find the probability that, in a randomly chosen period of 4 minutes, the number of calls received by the service is exactly 14. [3]

(ii) Find the longest period of time, in seconds to the nearest 0.1 s, for which the probability that no calls are received by the service is greater than 0.2. [4]

5 A company manufactures CDs.

(a) The percentage X of sulphur-based compounds in the ink used for CD labels is modelled by the distribution $N(\mu, 0.4^2)$. If $\mu > 1.5$ the CDs will be liable to a defect known as 'bronzing'. A random sample of 10 CDs is analysed and it is found that the sample mean is 1.70. Carry out a test, at the 5% significance level, of the null hypothesis $H_0 : \mu = 1.5$ against the alternative hypothesis $H_1 : \mu > 1.5$, stating your conclusion. [5]

(b) The probability that a randomly chosen CD manufactured by the company is defective is denoted by p. When the manufacturing process is working as intended, it is known that $p = 0.02$. Quality control is achieved by selecting a sample of 300 CDs and determining how many CDs in the sample are defective. A 5% significance test is then carried out of the null hypothesis $H_0 : p = 0.02$ against the alternative hypothesis $H_1 : p > 0.02$. Using a normal approximation, determine the critical region for the test. [6]

6 Archie is taking part in an archery competition. For each of his shots, the arrow strikes the target at a distance X metres from the centre, where the random variable X has probability density function given by

$$f(x) = \begin{cases} 12x^2(1-x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

The score for each arrow depends on its distance from the centre of the target, as given in the following table, where the probabilities are given correct to 4 decimal places.

Distance x metres	$0 \leqslant x < 0.25$	$0.25 \leqslant x < 0.5$	$0.5 \leqslant x < 0.75$	$0.75 \leqslant x < 1$
Score s	40	30	20	10
$P(S = s)$	0.0508	a	b	0.2617

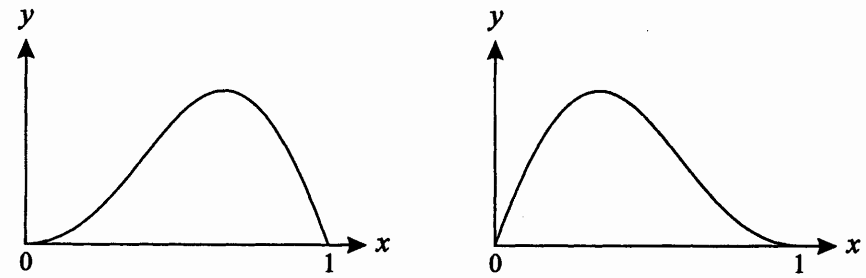
(i) Find the values of a and b, giving your answers correct to 4 decimal places. [5]

(ii) Find the expectation of

(a) X, [3]

(b) S. [2]

(iii) Bertie is also a competitor in the same competition. The graphs of the probability density functions for Archie and for Bertie are shown.



Archie

Bertie

By consideration of the shapes of the graphs, state with a reason which of Archie and Bertie is likely to score more points in the competition. [2]

[Question 7 is printed overleaf.]

7 A referendum is held, in which voters are required to answer Yes or No to a single question. The probability that a randomly chosen voter in the constituency of Coketown will answer Yes to the question is denoted by p.

(i) A random sample of 25 voters is chosen from Coketown. These voters are asked whether they intend to answer Yes or No.

(a) Find the critical region for a test, at the 5% significance level, of the null hypothesis $H_0 : p = 0.5$ against the alternative hypothesis $H_1 : p < 0.5$. You should state the values of any relevant probabilities obtained from tables. [3]

(b) State the conclusion of the test, in context, if 10 of the sample of 25 say they intend to answer Yes. [2]

It is known that, in two different constituencies, Abbotsea and Budmouth, the corresponding values of p are 0.4 and 0.5 respectively.

(ii) If the test described in part **(i)** is carried out in Abbotsea, find the probability that the result of the test is to accept the null hypothesis. [2]

One of the two constituencies, Abbotsea or Budmouth, is chosen at random (with equal probability), and from that constituency the test described in part **(i)** is carried out. Calculate the probability that

(iii) the result of the test is to accept the null hypothesis, [3]

(iv) the test leads to a Type II error. [3]

1 unbiased estimates …

$$\hat{\mu} = \overline{x} = \frac{\sum x}{n} = \frac{6.3}{6} = \mathbf{1 \cdot 05}$$ [1]

$$\hat{\sigma}^2 = \frac{n}{n-1}\left\{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2\right\} = \frac{6}{5}\left\{\frac{11 \cdot 95}{6} - 1 \cdot 05^2\right\} = \frac{6}{5} \times 0 \cdot 8892... = \mathbf{1 \cdot 067}$$ [3]

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2 $H_0 : \lambda = 4$ $H_1 : \lambda > 4$ [2]

Assuming H_0 ….. No. of weeds $X \sim \text{Po}(4)$

$$\begin{cases} p(X \geq 7) = 0 \cdot 1107 \\ p(X \geq 8) = 0 \cdot 0511 \\ p(X \geq 9) = 0 \cdot 0214 \end{cases}$$

Critical Region: Reject H_0 if $X \geq 9$ [2]

For given sample …..

$x = 7$ (<9) and so there is insufficient evidence of an increase in the mean no. of weeds. [2]

3

$p(X > 51) = 0 \cdot 1841$ $p(X > 60) = 0 \cdot 0082$

$1 - \Phi\left(\frac{51-\mu}{\sigma}\right) = 0 \cdot 1841$ $1 - \Phi\left(\frac{60-\mu}{\sigma}\right) = 0 \cdot 0082$

$\Phi\left(\frac{51-\mu}{\sigma}\right) = 0 \cdot 8159$ $\Phi\left(\frac{60-\mu}{\sigma}\right) = 0 \cdot 9918$

$\frac{51-\mu}{\sigma} = 0 \cdot 900$ $\frac{60-\mu}{\sigma} = 2 \cdot 400$

$\mathbf{51 - \mu = 0 \cdot 9\sigma}$ $\mathbf{60 - \mu = 2 \cdot 4\sigma}$

solving simultaneously gives …..

$\mu = \mathbf{45 \cdot 6}$

$\sigma = \mathbf{6 \cdot 00}$ [5]

$$\bar{X} \sim N\left(45 \cdot 6, \frac{36}{81}\right)$$

$$p\left(\bar{X} > \mu - 1\right) = 1 - \Phi\left(\frac{^-1}{\left(2/3\right)}\right) = \Phi(1 \cdot 5) = \mathbf{0 \cdot 933}(2)$$ [2]

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4 No. of calls in a period of 4 minutes … $X \sim \text{Po}(12)$

$$p(X = 14) = e^{-12}\frac{12^{14}}{14!} = 0 \cdot 090488... = \mathbf{0 \cdot 0905} \quad \text{(3 s.f.)}$$ [3]

For a period of T minutes, no. of calls $Y \sim \text{Po}(3T)$

$$p(0 \text{ calls}) > 0 \cdot 2 \Rightarrow e^{-3T} > 0 \cdot 2 \Rightarrow -3T > \ln 0 \cdot 2 \Rightarrow T < 0 \cdot 536479...$$

hence the longest period of time is 0.536479… minutes; that is **32.2 s** (1 d.p.) [4]

5 $H_0 : \mu = 1\cdot5$ $\qquad$ $H_1 : \mu > 1\cdot5$

For random samples of 10 CD's, if H_0 is true then …

$$Z = \frac{\bar{X} - 1\cdot5}{\sqrt{0\cdot4^2/10}} \sim N(0,\, 1) \quad \text{and so the } \textbf{rejection region will be} \quad \boldsymbol{Z > 1.645}$$

For this particular sample $\bar{x} = 1\cdot70$, giving $z = \dfrac{1\cdot7 - 1\cdot5}{\sqrt{0\cdot016}} = \mathbf{1\cdot581}$ $(< 1\cdot645)$ …. retain H_0

So there is insufficient evidence to conclude that the percentage of Sulphur compounds exceeds 1.5 [5]

$H_0 : p = 0\cdot02$ $\qquad$ $H_1 : p > 0\cdot02$

On the assumption of H_0,

in a random sample of 300 CD's, the no. of defectives $X \sim B(300,\, 0\cdot02) \approx N(6,\, 5\cdot88)$ [2]

Reject H_0 therefore, if $Z = \dfrac{(X - 0\cdot5) - 6}{\sqrt{5\cdot88}} > 1\cdot645$. Rearranged, this gives $X > 10\cdot489$

So the **critical region is** $\boldsymbol{X \geq 11}$ [4]

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6 $a = p(0\cdot25 \leq X < 0\cdot5) = \int_{0\cdot25}^{0\cdot5} (12x^2 - 12x^3)\,dx = \left[4x^3 - 3x^4\right]_{0\cdot25}^{0\cdot5} = \mathbf{0\cdot2617}$ (4 d.p.)

$b = 1 - 0\cdot0508 - 0\cdot2617 - 0\cdot2617 = \mathbf{0\cdot4258}$ [5]

$$E[X] = \int_0^1 (12x^3 - 12x^4)\,dx = \left[3x^4 - 2\cdot4\,x^5\right]_0^1 = \tfrac{3}{5}$$ [3]

$$E[S] = 40 \times 0\cdot0508 \; + \; 30 \times 0\cdot2617 \; + \; 20 \times 0\cdot4258 \; + \; 10 \times 0\cdot2617 = \mathbf{21\cdot016}$$ [2]

Bertie's probability distribution has positive skew so is more biased towards the lower distances that yield more points in the competition. [2]

7 $H_0 : p = 0\cdot5$ $H_1 : p < 0\cdot5$

In a random sample of 25 voters …… no. answering 'Yes' $Y \sim B(25,\ 0\cdot5)$

$p(Y \le 7) = 0\cdot0216$
$p(Y \le 8) = 0\cdot0539$ } so with a sig. level of 5%, the critical region is $\boldsymbol{Y \le 7}$ [3]

With $y = 10$ we can accept H_0 ; there being **insufficient evidence to conclude that p < 0.5** [2]

$p(\text{accept } H_0 \text{ in Abbotsea}) = p\left(Y > 7 \,\middle|\, Y \sim B(25, 0\cdot4)\right) = \mathbf{0\cdot846}(4)$ [2]

$$p(\text{accept } H_0) = \tfrac{1}{2} \times p\left(Y > 7 \,\middle|\, Y \sim B(25, 0\cdot4)\right) + \tfrac{1}{2} \times p\left(Y > 7 \,\middle|\, Y \sim B(25, 0\cdot5)\right)$$
$$= \tfrac{1}{2} \times 0\cdot8464 + \tfrac{1}{2} \times 0\cdot9784$$
$$= \mathbf{0\cdot912}(4)$$
[3]

$p(\text{Type II Error}) = p\left(\text{Abbotsea chosen } \textbf{and } Y > 7\right) = \tfrac{1}{2} \times 0\cdot8464 = \mathbf{0\cdot423}(2)$ [3]