1 A student was investigating the relationship between the performance of two football teams, *A* and *B*, which had been in the same league for many years. She recorded the number of points gained by the two teams for each of 7 randomly chosen years. The results are given in the table below.

Year	1	2	3	4	5	6	7
Points gained by team *A*	41	82	62	58	50	60	80
Points gained by team *B*	65	79	81	66	83	36	70

(i) Find the value of Spearman's rank correlation coefficient between the points gained by team *A* and the points gained by team *B*. [4]

(ii) State what the value found in part (i) shows about the relationship between the performance of teams *A* and *B* over the selected 7 years. [1]

2 A chemist recorded the mass, y grams, of a substance which dissolved in 100 cm^3 of water at a temperature of x °C, for 10 different values of x. The results are summarised below.

$$n = 10,\ \Sigma x = 550,\ \Sigma y = 803,\ \Sigma x^2 = 38\,500,\ \Sigma y^2 = 65\,848.9,\ \Sigma xy = 47\,520.$$

(i) Calculate the value of the product moment correlation coefficient. [2]

(ii) Calculate the equation of the regression line of y on x, in the form $y = a + bx$, giving a and b correct to 3 significant figures. [4]

(iii) Assuming that $x = 45$ is a temperature within the range used by the chemist for his experiment, use the equation you found in part (ii) to estimate the mass, in grams, of the substance which will dissolve in 100 cm^3 of water at a temperature of 45 °C. [2]

3 A man is standing at a bus stop waiting for a 'number 37' bus. He makes the following modelling assumptions:

- at any time the probability that the next bus to arrive will be a number 37 bus is $\frac{1}{12}$,
- whether the next bus to arrive is a number 37 bus or not is independent of all other arrivals.

Starting from the time when the man first arrives at the bus stop, let X be the number of buses which arrive up to and including the first number 37 bus.

(i) State the distribution of X, giving the value(s) of any parameter(s). [2]

(ii) Calculate

(a) $P(X = 3)$, [2]

(b) $P(X > 5)$. [2]

(iii) State the value of $E(X)$. [1]

(iv) Give one reason why the modelling assumptions may not be correct. [1]

4 Mike is taking an examination module. He wants to achieve a top grade in this module and he can have a maximum of three attempts. If he gains the top grade at any particular attempt at the module, he will not take it again. At his first attempt he estimates that his probability of gaining a top grade is $\frac{7}{10}$. If he does not gain a top grade at the first attempt, he estimates that he has a probability of $\frac{16}{25}$ of gaining a top grade at the second attempt. If he does not gain a top grade in either of the first two attempts, he estimates that he has a probability of $\frac{29}{50}$ of gaining a top grade in the third (and last) attempt.

(i) Using Mike's estimates, show that the probability that he will gain a top grade is 0.955, correct to 3 significant figures. [4]

(ii) Sasha and Katie also take this examination. You may assume that their probabilities of gaining a top grade are the same as Mike's at each stage and the performances of the three students are independent of each other. Calculate the probability that exactly two of the three students gain a top grade. [4]

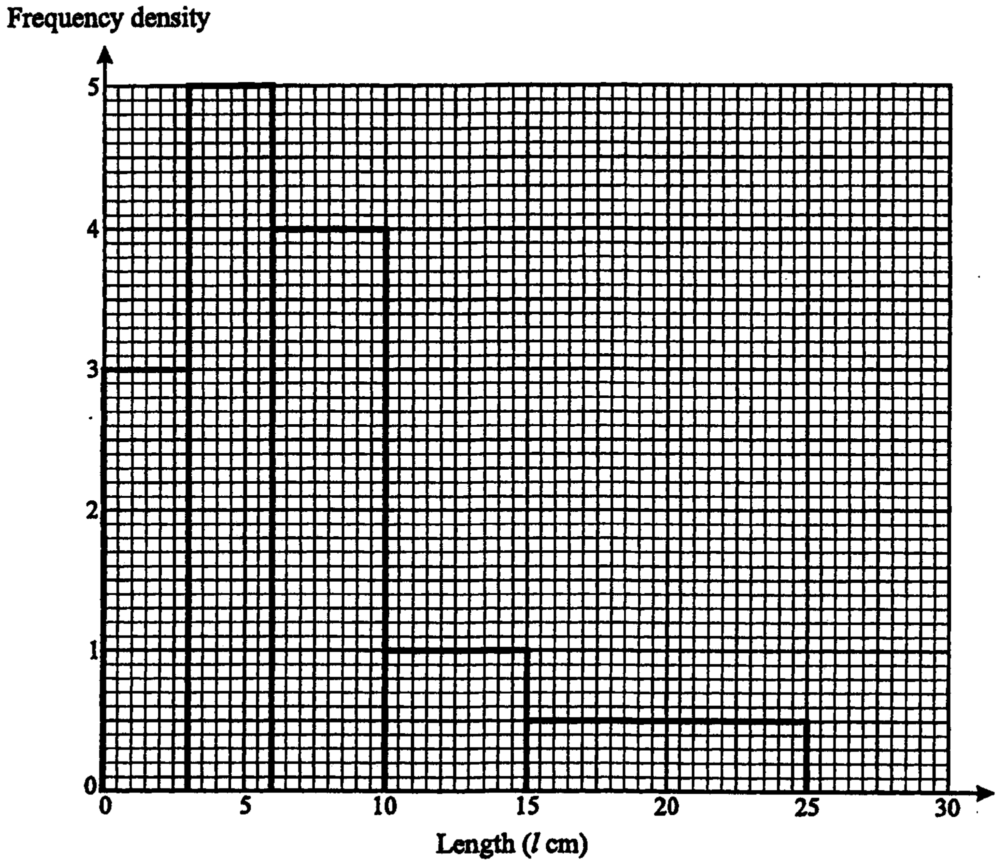
5 Each of 6 cards has a different single letter written on it. The letters on the cards are A, B, C, D, E and F. The cards are shuffled and then placed in a row.

(i) How many different possible arrangements of letters are there? [1]

(ii) In how many of these arrangements are the vowels (i.e. the letters A and E) next to each other? [4]

(iii) The cards are now shuffled and placed face down. Three of the cards are selected at random. Find the probability that at least one of the selected cards is a vowel. [4]

6 A student measured the length, l cm, for each of a sample of 50 pebbles. The results are summarised in the histogram drawn below.



You are given that the number of pebbles in the $0 \leqslant l < 3$ class is 9.

(i) Copy and complete the table below. [3]

Length (l cm)	Frequency
$0 \leqslant l < 3$	9
$3 \leqslant l < 6$	
$6 \leqslant l < 10$	
$10 \leqslant l < 15$	
$15 \leqslant l < 25$	

(ii) Use your completed table from part (i) to estimate the mean and standard deviation of the lengths of the pebbles in the sample. [6]

7 Jane and Dave each spin a coin 3 times. The coin which Jane uses is fair but Dave's coin is biased and for his coin the probability of turning up heads is $\frac{2}{3}$.

Let J be the number of heads that Jane obtains in 3 spins of her coin and let D be the number of heads that Dave obtains in 3 spins of his coin.

(i) Copy and complete the tables below to show the probability distributions of J and D. [5]

j	0	1	2	3
$P(J = j)$	$\frac{1}{8}$			

d	0	1	2	3
$P(D = d)$	$\frac{1}{27}$			

The random variable X is defined by the equation $X = J - D$.

(ii) Show that $P(X = 2) = \frac{1}{24}$. [4]

(iii) Calculate the probability that Jane and Dave obtain a total of at least 5 heads in their 6 spins. [4]

1

Team A ranks	7	1	3	5	6	4	2
Team B ranks	6	3	2	5	1	7	4
d	1	-2	1	0	5	-3	-2

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)} = 1 - \frac{6 \times 44}{7 \times 48} = \frac{3}{14} = \mathbf{0 \cdot 214} \qquad \text{(3 s.f.)}$$ [4]

There exists a weak level of agreement between the performance of the teams across years.

[1]

2

$$r = \frac{\frac{\sum xy}{n} - \bar{x}\,\bar{y}}{\sqrt{\frac{\sum x^2}{n} - \bar{x}^2}\sqrt{\frac{\sum y^2}{n} - \bar{y}^2}} = \frac{\frac{47\,520}{10} - 55.0 \times 80.3}{\sqrt{\frac{38\,500}{10} - 55.0^2}\sqrt{\frac{65\,848 \cdot 9}{10} - 80.3^2}} = \frac{335.5}{28 \cdot 722... \times 11 \cdot 696...} = \mathbf{0.999}$$ [2]

regression line of y on x ....

$$y - \bar{y} = \left(\frac{\frac{\sum xy}{n} - \bar{x}\,\bar{y}}{\frac{\sum x^2}{n} - \bar{x}^2}\right)(x - \bar{x})$$

$$y - 80 \cdot 3 = \left(\tfrac{335 \cdot 5}{825}\right)(x - 55)$$

$$y = 0 \cdot 40\dot{6}\,x + 57 \cdot 9\dot{3}$$

$$\boldsymbol{y = 57 \cdot 9 + 0 \cdot 407x} \qquad \text{(3 s.f.)}$$ [4]

$x = 45$ $\qquad$ $\hat{y} = 57 \cdot 9\dot{3} + 0 \cdot 40\dot{6} \times 45 = 76 \cdot 2\dot{3} = \mathbf{76 \cdot 2}$ [2]

3

$X \sim \text{Geo}\left(\frac{1}{12}\right)$

[2]

$\text{p}(X = 3) = \left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right) = \frac{121}{1728} = \mathbf{0 \cdot 0700}$ (3 s.f.)

[2]

$\text{p}(X > 5) = \left(\frac{11}{12}\right)^5 = \mathbf{0 \cdot 647}$ (3 s.f.)

[2]

$\text{E}[X] = \frac{1}{p} = \mathbf{12}$

[1]

The **assumption of independence** is likely to be unreasonable, since after one No. 37 has arrived it is probably less likely that the next bus is another No. 37.

[1]

4 $$\text{p}(\text{top grade}) = \tfrac{7}{10} + \tfrac{3}{10}\cdot\tfrac{16}{25} + \tfrac{3}{10}\cdot\tfrac{9}{25}\cdot\tfrac{29}{50} = 0\cdot 95464 = \mathbf{0\cdot 955} \quad (3\text{ s.f.}) \qquad (\text{show})$$

[4]

No. of students gaining a top grade … $X \sim \text{B}(3, 0\cdot 95464)$

$$\text{p}(X=2) = {}^3C_2\,(0\cdot 95464)^2\,(0\cdot 04536) = 0\cdot 124014... = \mathbf{0\cdot 124} \qquad (3\text{ s.f.})$$

[4]

---

5 No. of different arrangements $= 6! = \mathbf{720}$

[1]

No. with vowels adjacent $= 5! \times 2 = \mathbf{240}$

[4]

$$\text{p}(\text{at least one is a vowel}) = 1 - \text{p}(\text{none is a vowel}) = 1 - \frac{{}^5C_3}{{}^7C_3} = 1 - \frac{10}{35} = \mathbf{\frac{5}{7}}$$

[4]

6

length	f
$0 \le l < 3$	9
$3 \le l < 6$	**15**
$6 \le l < 10$	**16**
$10 \le l < 15$	**5**
$15 \le l < 25$	**5**

[3]

$$\text{estimate of mean} = \frac{1\cdot 5\times 9 + 4\cdot 5\times 15 + 8\times 16 + 12\cdot 5\times 5 + 20\times 5}{50} = \frac{371\cdot 5}{50} = \mathbf{7\cdot 43\ cm}$$

$$\text{s.d.} = \sqrt{\frac{1\cdot 5^2\times 9 + 4\cdot 5^2\times 15 + ...}{50} - 7\cdot 43^2} = \sqrt{\frac{4129\cdot 25}{50} - 7\cdot 43^2} = \sqrt{27\cdot 3801} = 5\cdot 232599... = \mathbf{5\cdot 23\ cm}$$

[6]

7

j	0	1	2	3
$\text{p}(J=j)$	$\mathbf{\tfrac{1}{8}}$	$\mathbf{\tfrac{3}{8}}$	$\mathbf{\tfrac{3}{8}}$	$\mathbf{\tfrac{1}{8}}$

d	0	1	2	3
$\text{p}(D=d)$	$\mathbf{\tfrac{1}{27}}$	$\mathbf{\tfrac{6}{27}}$	$\mathbf{\tfrac{12}{27}}$	$\mathbf{\tfrac{8}{27}}$

[5]

$$\text{p}(X=2) = \text{p}\big(J=3 \text{ and } D=1\big) + \text{p}\big(J=2 \text{ and } D=0\big) = \tfrac{1}{8}\cdot\tfrac{6}{27} + \tfrac{3}{8}\cdot\tfrac{1}{27} = \mathbf{\tfrac{1}{24}} \qquad (\text{show})$$

[4]

$$\text{p}\big(\text{total of 5 between } J \text{ and } D\big) = \text{p}\big(3,\,2\big) + \text{p}\big(2,\,3\big) = \tfrac{1}{8}\cdot\tfrac{12}{27} + \tfrac{3}{8}\cdot\tfrac{8}{27} = \mathbf{\tfrac{1}{6}}$$

[4]