OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **2639**

Mechanics 3

Thursday **23 MAY 2002** Afternoon 1 hour 20 minutes

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\text{m}\,\text{s}^{-2}$.
- You are permitted to use a graphic calculator in this paper.

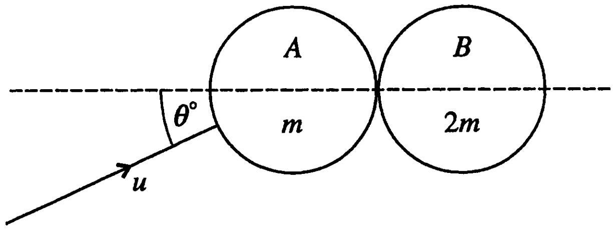
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 A particle is moving with simple harmonic motion in a straight line. The period is 0.2 s and the amplitude of the motion is 0.3 m. Find the maximum speed of the particle. [3]

2



A sphere A of mass m, moving on a horizontal surface, collides with another sphere B of mass $2m$, which is at rest on the surface. The spheres are smooth and uniform, and have equal radius. Immediately before the collision, A has velocity u at an angle $\theta°$ to the line of centres of the spheres (see diagram). Immediately after the collision, the spheres move in directions that are perpendicular to each other.

(i) Find the coefficient of restitution between the spheres. [4]

(ii) Given that the spheres have equal speeds after the collision, find θ. [3]

3 An aircraft of mass 80 000 kg travelling at $90\,\text{m s}^{-1}$ touches down on a straight horizontal runway. It is brought to rest by braking and resistive forces which together are modelled by a horizontal force of magnitude $(27\,000 + 50v^2)$ newtons, where $v\,\text{m s}^{-1}$ is the speed of the aircraft. Find the distance travelled by the aircraft between touching down and coming to rest. [8]

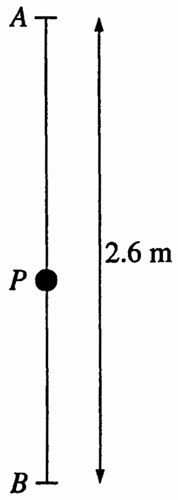
4 For a bungee jump, a girl is joined to a fixed point O of a bridge by an elastic rope of natural length 25 m and modulus of elasticity 1320 N. The girl starts from rest at O and falls vertically. The lowest point reached by the girl is 60 m vertically below O. The girl is modelled as a particle, the rope is assumed to be light, and air resistance is neglected.

(i) Use energy considerations to find the mass of the girl. [4]

(ii) Find the tension in the rope when the girl is at the lowest point. [2]

(iii) Find the acceleration of the girl when she is at the lowest point. [3]

5



Two points A and B lie on a vertical line with A at a distance 2.6 m above B. A particle P of mass 10 kg is joined to A by an elastic string and to B by another elastic string (see diagram). Each string has natural length 0.8 m and modulus of elasticity 196 N. The strings are light and air resistance may be neglected.

(i) Verify that P is in equilibrium when P is vertically below A and the length of the string PA is 1.5 m. [3]

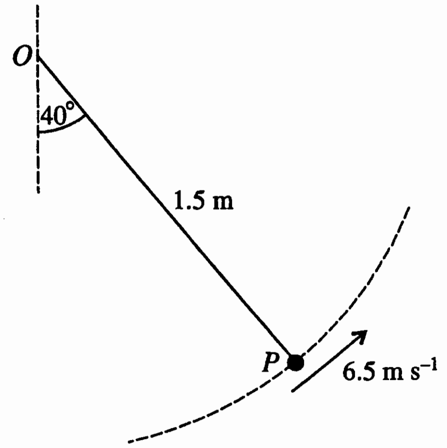
The particle is set in motion along the line AB with both strings remaining taut. The displacement of P below the equilibrium position is denoted by x metres.

(ii) Show that the tension in the string PA is $245(0.7 + x)$ newtons, and the tension in the string PB is $245(0.3 - x)$ newtons. [2]

(iii) Show that the motion of P is simple harmonic, and find the period. [5]

[Questions 6 and 7 are printed overleaf.]

6



A particle P of mass 0.3 kg is moving in a vertical circle. It is attached to the fixed point O at the centre of the circle by a light inextensible string of length 1.5 m. When the string makes an angle of 40° with the downward vertical, the speed of P is $6.5\,\mathrm{m\,s^{-1}}$ (see diagram). Air resistance may be neglected.

(i) Find the radial and transverse components of the acceleration of P at this instant. [2]

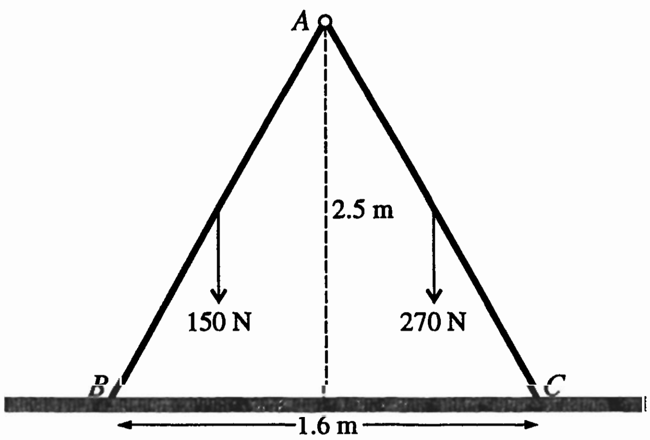
In the subsequent motion, with the string still taut and making an angle $\theta°$ with the downward vertical, the speed of P is $v\,\mathrm{m\,s^{-1}}$.

(ii) Use conservation of energy to show that $v^2 \approx 19.7 + 29.4\cos\theta°$. [4]

(iii) Find the tension in the string in terms of θ. [3]

(iv) Find the value of θ at the instant when the string becomes slack. [2]

7



A step-ladder is modelled as two uniform rods AB and AC, freely jointed at A. The rods are in equilibrium in a vertical plane with B and C in contact with a rough horizontal surface. The rods have equal lengths; AB has weight 150 N and AC has weight 270 N. The point A is 2.5 m vertically above the surface, and $BC = 1.6$ m (see diagram).

(i) Find the horizontal and vertical components of the force acting on AC at A. [7]

(ii) The coefficient of friction has the same value μ at B and at C, and the step-ladder is on the point of slipping. Giving a reason, state whether the equilibrium is limiting at B or at C, and find μ. [5]

1 $\omega = {}^{2\pi}\!/_{T} = {}^{2\pi}\!/_{0\cdot 2} = 10\pi$

maximum speed $= a\omega = 0\cdot 3 \times 10\pi = 3\pi = 9\cdot 42477... =$ $\mathbf{9\cdot 42\,ms^{-1}}$ (3 s.f.)

[3]

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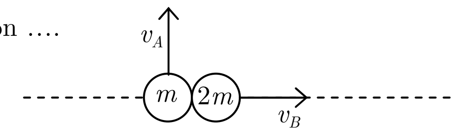
2 after collision ….



since the only impulse acts along the line of the centres, and it's given that the particles move away at right angles.

Conservation of Momentum $(\rightarrow)$ $\quad m(u\cos\theta) = (2m)\,v_B \quad \Rightarrow \quad v_B = \tfrac{1}{2}u\cos\theta$

Newton's Law of Impact $\quad \tfrac{1}{2}u\cos\theta - 0 = {}^{-}e(0 - u\cos\theta) \quad \Rightarrow \quad \boldsymbol{e = \tfrac{1}{2}}$ [4]

$v_A = u\sin\theta$ and given speeds are equal after collision $\quad v_B = v_A$

$$\tfrac{1}{2}u\cos\theta = u\sin\theta$$

$$\tan\theta = \tfrac{1}{2} \qquad \theta = \mathbf{26\cdot 6}^\circ \text{ (3 s.f.)}$$ [3]

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3 $F = ma$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = {}^{-}\tfrac{1}{80\,000}\left(27\,000 + 50v^2\right)$$

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = {}^{-}\tfrac{1}{1600}\left(540 + v^2\right)$$

$$\int \frac{v}{540 + v^2}\,\mathrm{d}v = {}^{-}\tfrac{1}{1600}\int \mathrm{d}x$$

$$\tfrac{1}{2}\ln\left(540 + v^2\right) = {}^{-}\tfrac{1}{1600}x + c$$

$$x = A - 800\ln\left(540 + v^2\right)$$

$x = 0,\ v = 90$

$$x = 800\ln(8640) - 800\ln\left(540 + v^2\right) = 800\ln\left(\frac{8640}{540 + v^2}\right)$$

distance till 'plane comes to rest $= 800\ln\left(\dfrac{8640}{540}\right)$

$= 800\ln(16)$

$= 3\,200\ln(2) = 2218\cdot 070... =$ **2220 m** (3 s.f.)

[8]

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4 at lowest point …

loss of G.P.E for girl = gain in E.P.E. of rope

$$mg \times 60 = \tfrac{1}{2} \cdot \tfrac{1320}{25} (60 - 25)^2$$

$$m = \mathbf{55\ kg}$$ [4]

tension in rope $= \tfrac{1320}{25} \cdot 35 = \mathbf{1848\ N}$ [2]

acceleration $= \dfrac{1848 - mg}{m} = \mathbf{23 \cdot 8\ \ ms^{-2}}$ **vertically upwards** [3]

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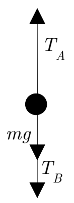
5 

when $PA = 1.5$ ….

(↓) $\sum F = 10 \times 9 \cdot 8 + \tfrac{196}{0 \cdot 8} \times 0 \cdot 3 - \tfrac{196}{0 \cdot 8} \times 0 \cdot 7 = 98 + 73 \cdot 5 - 171 \cdot 5 = 0$

so the system is in equilibrium. [3]

at displacement x …

$T_{PA} = \tfrac{196}{0 \cdot 8}(1 \cdot 5 + x - 0 \cdot 8) = 245(0 \cdot 7 + x)$ $\qquad T_{PB} = \tfrac{196}{0 \cdot 8}(2 \cdot 6 - 1 \cdot 5 - x - 0 \cdot 8) = 245(0 \cdot 3 - x)$ [2]

N2(↓) $\qquad 10\ddot{x} = 245(0 \cdot 3 - x) - 245(0 \cdot 7 + x) + 98$

$\ddot{x} = {}^{-}49x$ $\qquad$ so SHM with period $T = {}^{2\pi}\!/_{7} = \mathbf{0 \cdot 898\ s}$ [5]

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6 acceleration ….

radial $= {}^{v^2}\!/_{r} = {}^{6 \cdot 5^2}\!/_{1 \cdot 5} = \mathbf{28\tfrac{1}{6}}$ $\qquad$ transverse $= {}^{-}g \sin 40^\circ = 6 \cdot 29931... = \mathbf{6 \cdot 30\ ms^{-2}}$ [2]

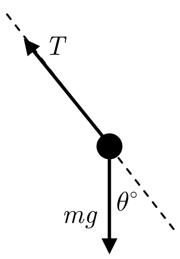
conservation of energy (relative to $\theta = 40^\circ$)

gain in G.P.E = loss in K.E.

$$0 \cdot 3 \times 9 \cdot 8 (1 \cdot 5 \cos 40^\circ - 1 \cdot 5 \cos \theta^\circ) = \tfrac{1}{2} \times 0 \cdot 3 (6 \cdot 5^2 - v^2)$$

$$29 \cdot 4 (\cos 40^\circ - \cos \theta^\circ) = 42 \cdot 25 - v^2$$

$$v^2 \approx 19 \cdot 7 + 29 \cdot 4 \cos \theta^\circ$$ (show) [4]



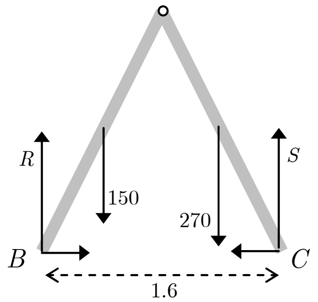
N2(radially) $\qquad T - mg \cos \theta^\circ = {}^{mv^2}\!/_{r}$

$$T \approx 2 \cdot 94 \cos \theta^\circ + {}^{0 \cdot 3 (19 \cdot 7 + 29 \cdot 4 \cos \theta^\circ)}\!/_{1 \cdot 5}$$

$$T \approx 8 \cdot 82 \cos \theta^\circ + 3 \cdot 94$$ [3]

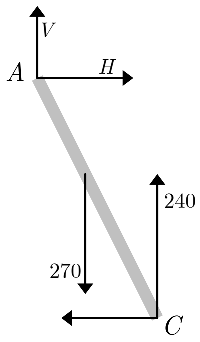
string becomes slack when … $\cos \theta^\circ = {}^{-3 \cdot 94}\!/_{8 \cdot 82}$ $\qquad \theta = 116 \cdot 5329... = \mathbf{117^\circ}$ (3 s.f.) [2]

7



Considering the step-ladder as a whole ….

M(B)	$1\cdot6S - 270\times1\cdot2 - 150\times0\cdot4 = 0$	$S = 240$
N2(↑)	$R + S - 150 - 270 = 0$	$R = 180$



Now considering AC alone …

N2(↑)	$V - 270 + 240 = 0$	$\boldsymbol{V = 30}$	
M(C)	$270\times0\cdot4 - H\times2\cdot5 - 30\times0\cdot8 = 0$	$\boldsymbol{H = 33\cdot6}$	[7]

The magnitudes of the frictional forces at A and C must both be 33.6.

Since the normal contact force at B is less than that at C, friction must be limiting at B and

$$\mu = F/R = 33\cdot6/180 = \mathbf{14/75} \qquad \left(= 0\cdot18666\dot{6}\right)$$

[5]