OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2638

Mechanics 2

Wednesday **23 JUNE 2004** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

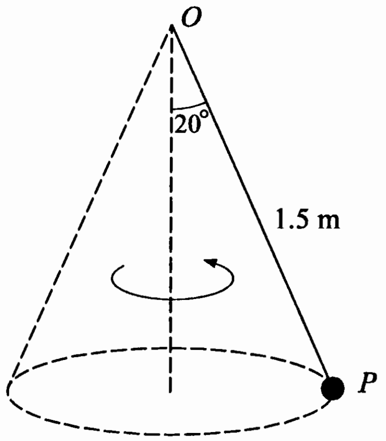
- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\text{m}\,\text{s}^{-2}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1



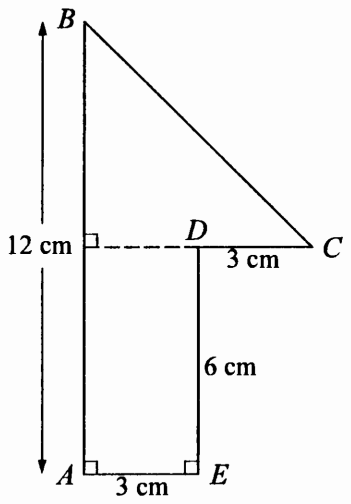
A particle P, of mass 0.1 kg, is attached to one end of a light inextensible string of length 1.5 m. The other end of the string is attached to a fixed point O. The particle moves in a horizontal circle with constant angular speed ω rad s^{-1}, with the string taut and inclined at a constant angle of 20° to the vertical (see diagram).

(i) Find the tension in the string. [2]

(ii) Find ω. [3]

2 A small sphere of mass 0.2 kg is dropped from rest at a height of 0.5 m above horizontal ground. It falls vertically, hits the ground and rebounds vertically upwards, coming to instantaneous rest at a height of 0.3 m above the ground. Calculate the magnitude of the impulse which the ground exerts on the sphere when it rebounds. Air resistance is ignored. [5]

3



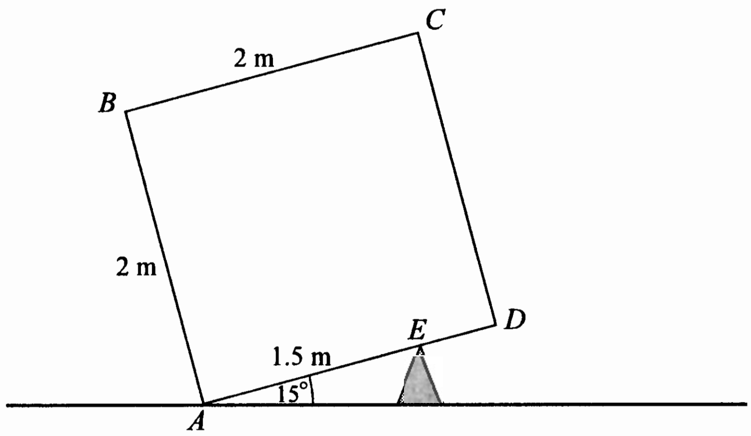
A uniform lamina *ABCDE* consists of a rectangle and a right-angled isosceles triangle, as shown in the diagram. $AB = 12$ cm, $CD = 3$ cm, $DE = 6$ cm and $AE = 3$ cm.

(i) Show that the distance of the centre of mass of the lamina from the line *AB* is 1.75 cm. [3]

(ii) Find the distance of the centre of mass of the lamina from the line *AE*. [3]

(iii) The lamina is freely suspended from *B*. Calculate the angle that *BA* makes with the vertical. [2]

4



A uniform square board of mass 10 kg and side 2 m is modelled as a lamina *ABCD*. The board is in equilibrium in a vertical plane with the point *A* on rough horizontal ground. The edge *AD* rests on a fixed wedge whose point of contact, *E*, is smooth. The distance *AE* is 1.5 m and the edge *AD* makes an angle of 15° with the horizontal (see diagram).

Calculate

(i) the magnitude of the force which the board exerts on the wedge at *E*, [4]

(ii) the magnitude of the frictional force acting at *A*. [2]

A small object of mass m kg is now fixed to the board at *B*.

(iii) Assuming that the board does not slip, calculate the value of m for which the board is about to topple. [3]

5 Two particles of masses m kg and $2m$ kg are moving directly towards each other on a smooth horizontal plane, with speeds 2 m s^{-1} and 4 m s^{-1} respectively. When the particles collide, half of the total kinetic energy is lost. After the collision the particles move in the same direction as each other.

(i) Find the speeds of the particles after the collision. [8]

(ii) Find the coefficient of restitution. [2]

6 A car is moving along a straight horizontal road. The mass of the car is 1200 kg and the engine of the car is working at a constant rate of 50 kW. In a simple model the work done by the engine of the car is taken to be equal to the kinetic energy generated.

(i) Show that, in this model, the time taken to reach a speed of 30 m s^{-1} from rest is 10.8 s. [2]

The actual time to reach a speed of 30 m s^{-1} from rest is 13.6 s, and in this time the car moves a distance of 280 m. In a more refined model there is assumed to be a constant air resistance.

(ii) Show that the magnitude of the air resistance is 500 N. [3]

On another occasion the car moves up a straight hill, inclined at 4° to the horizontal, with its engine again working at 50 kW. Starting from rest, the car takes 26.8 s to reach a speed of 30 m s^{-1}.

(iii) Assuming that the air resistance remains 500 N, find the distance travelled in this time. [5]

7 The point O is 25 m above horizontal ground. A particle is projected from O with speed 40 m s^{-1} at an angle of elevation of 35° above the horizontal, and it moves freely under gravity. The particle hits the ground at the point A.

Calculate

(i) the greatest height above the ground reached by the particle, [3]

(ii) the time taken for the particle to travel from O to A, [4]

(iii) the speed and direction of motion of the particle immediately before it hits the ground at A. [6]

1	(i)	T cos20° = 0.1 x 9.8	M1		For resolving vertically	
		T = 1.04 N	A1	2		
	(ii)	r = 1.5sin20° (0.513 / 51.3cm)	B1		or ω=√(g/lcosθ) M1	
		Tsin 20° = 0.1rω^2 (res horiz)	M1		ω=√(9.8/1.5cos20°) A1	
		ω = 2.64	A1	3		**5**

2		v^2 = 2gh	M1		Energy or kinematic equ.	
		u = 3.13 m s^{-1} , √9.8, √g aef	A1		Speed of impact	
		v = 2.42 , √5.88, √0.6g aef	A1		Speed of rebound	
		Impulse = 0.2(2.42+3.13)	M1		±mass x sum of mag. of their vels.	
		1.11 Ns	A1√	5	√ above (must be positive)	**5**

3	(i)	Moments about axis *AB*	M1			
		(18+18)$\bar{x}$ = 18x1.5 + 18x2	A1		Equal areas can be implied	
		$\bar{x}$ = 1.75	A1	3	**AG** (answer must be validly obtained)	
	(ii)	Moments about *AE* or *B*	M1			
		36$\bar{y}$ = 18x3 + 18x8	A1		Or 36$\bar{z}$ = 18x4 +18 x 9 ($\bar{z}$ = 6.5)	
		$\bar{y}$ = 5.5	A1	3		
	(iii)	tanθ = 1.75/6.5	M1		must use 1.75 + their dist from B ($\bar{z}$)	
		θ = 15.1°	A1√	2	√ their $\bar{z}$	**8**

4	(i)	Dist. A to c.of m = ½√8, √2 (1.41)	B1		1x10x9.8cos15°&1x10x9.8sin15°	
		1.5R = 10x9.8x½√8xcos60° M(A) must recognise R is perp to AD n.b. (1-tan15°)cos15° = ½√2	M1 A1		1.5R+1x10x9.8sin15°= 1x10x9.8cos15°	
		R = 46.2 N	A1	4		
	(ii)	F = Rcos75°	M1		Resolving horizontally	
		F = 12.0	A1√	2	√ their R	
	(iii)	Moments about *A* (with R=0 at E)	M1		$\bar{x}$ =10/(10+m)& $\bar{y}$ =(10+2m)/(10+m)	
		m.9.8x2cos75°=10x9.8x½√8cos60°	A1		and tan15°= $\bar{x}$ / $\bar{y}$ (M1 + A1)	
		m = 13.7	A1	3		**9**

5	(i)	8*m* – 2*m* = ±(*mx* + 2*my*)	M1		C.of Mom. Must be 8*m* – 2*m*	
		±6 = *x* + 2*y*	A1		a.e.f.	
		½mx^2+½2my^2 = ½(½m2^2+½2m4^2)	M1 A1		Loss of K.E. (½(ke)must be on correct side for M1 (18 = x^2 + 2y^2)	
		18 = $(6-2y)^2$ + 2y^2	M1		Or 18=x^2+2(½(6-x))^2 M0 for silly √	
		x=0 and/or *y*=3 registered	B1		Can simply disappear	
		x = 4	A1		S.R. B1 A0 for -4, -1	
		y = 1	A1	8		
	(ii)	*e* = 3 ÷ 6	M1		Correct use of (*x*-*y*) ÷ (4+2)	
		e = $\frac{1}{2}$	A1√	2	√ their (*x*-*y*) ÷ 6. no for e>1 or e<0	**10**

6	(i)	50 000t = ½x1200x30^2	M1		(540 000)	
		t = 10.8 s	A1	2	**AG** (answer must be validly obtained)	
	(ii)	W.D. = ½mv^2 + 280R (680 000 = 540 000 + 280R)	M1		Or 50000/v – R=1200a or WD against resistance = Power x Δt	
		50000x13.6=½x1200x30^2+280R	A1		50 000(13.6-10.8) = 280R	
		R = 500 N	A1	3	**AG** (answer must be validly obtained)	
	(iii)	mgh =1200g*d*sin4°	B1		Or 50000/v -500 -1200gsin4° =1200a	
		½x1200x30^2+1200x9.8xdsin4° = 50 000 x 26.8 – 500d	M1 A1		Must have 4 terms n.b. next M1 depends on this M1	
		1320.3d = 800 000	M1		Attempt at solving energy equation	
		d = 606 m	A1	5		**10**

7	(i)	0 = (40sin35°)^2 – 2x9.8xh	M1		or via 2 kinematic equations (t = 2.34)	
		h = 26.9	A1			
		51.9 m (26.9+25)	A1√	3	√ 25 +their h if sensible method	
	(ii) **P**	- 25 = 40sin35°.t – 4.9t^2	M1 A1		Next M1 depends on this being a 3 term quadratic	
		4.9t^2 – 40sin35°t – 25 = 0	M1		a.e.f. and attempt to solve quadratic	
		t = 5.59	A1	4		
	(iii)	v = 40sin35° - 9.8 x 5.59	M1		Use of v^2=u^2+2as (2x9.8x51.9) (top)	
		v = (-) 31.9 √ their 5.59 (t ii)	A1√		√ their 51.9 (i)	
		Speed=√(32.8^2+31.9^2)	M1		n.b. 40cos35° = 32.8	
		Speed = 45.7 m s^{-1}	A1√		√ their 31.9 (vertical component)	
		θ = tan^{-1}(31.9/32.8)	M1		Must be right way up	
		θ = 44.2° below horizontal	A1√	6	Or 45.8° to downward vertical	
		(above A1 √. √ their 31.9)			Direction can be clear from diagram	**13**
	(ii)	**Alternative Q**				
		O to Top t = 2.34	B1		40sin35°÷9.8	
		Top to A 51.9 = 4.9t^2	M1			
		t = 3.25	A1			
		Total t = 5.59	A1	(4)		
	(ii)	**Alternative R**				
		O to level t = 4.68	B1			
		Level to A 25=40sin35°t+4.9t^2	M1			
		t = 0.91	A1			
		Total t = 5.59	A1	(4)		
	(ii)	**Alternative S**			(ii) **Alternative T**	
		-25=xtan35°-9.8x^2/2.40^{2}cos^{2}35	M1		v^2 = 2x9.8x51.9	
		x = 183.4	A1		v = (-)31.9	
		183.4 = 40cos35°t	M1		-31.9 = 40sin35° - 9.8t	
		t = 5.59	A1		t = 5.59	