OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2638

Mechanics 2

Wednesday **21 JANUARY 2004** Afternoon 1 hour 20 minutes

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\mathrm{m\,s^{-2}}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

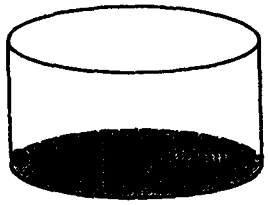
- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

 Registered Charity Number: 1066969

1 A child pulls a sledge at constant speed in a straight line along horizontal snow-covered ground. The rope attached to the sledge makes an angle of 25° with the horizontal and the tension is 30 N. Calculate the work done in moving the sledge 50 m. [3]

2



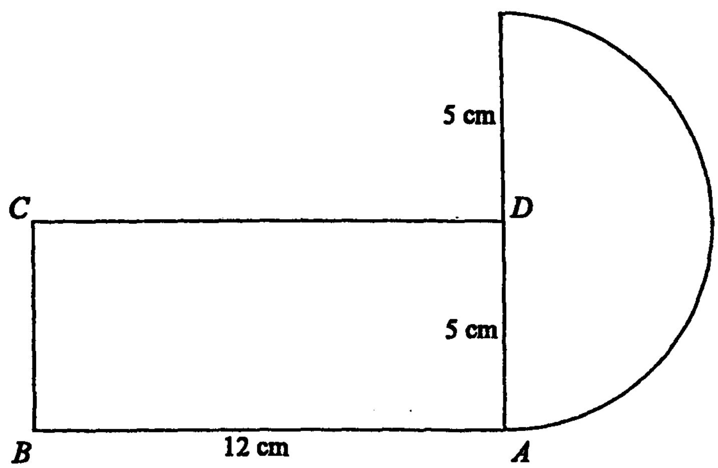
A small ball moves in a horizontal circle on the inside of a smooth hollow cylinder, in such a way that it remains in contact with both the curved surface and the base of the cylinder (see diagram). The mass of the ball is 0.1 kg and the radius of the base of the cylinder is 0.2 m. The ball moves with constant angular speed 3 rad s^{-1}.

(i) Find the magnitude of the force which the curved surface of the cylinder exerts on the ball. [2]

(ii) Find the kinetic energy of the ball. [3]

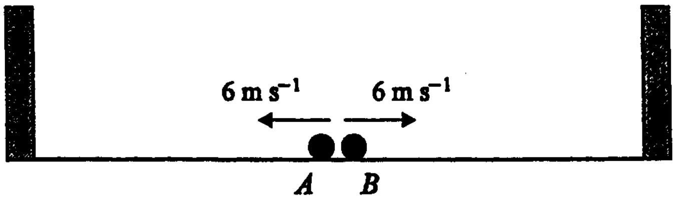
3 (i) A uniform semicircular lamina has radius 5 cm. Show that the distance from its centre to its centre of mass is 2.12 cm, correct to 3 significant figures. [2]

A uniform rectangular lamina *ABCD* has mass 2 kg and dimensions *AB* = 12 cm and *AD* = 5 cm. A uniform semicircular lamina has mass 3 kg and radius 5 cm. A single plane object is formed by attaching the rectangular lamina to the semicircular lamina, with the end *AD* coinciding with a radius of the semicircle (see diagram).



(ii) Calculate the distance of the centre of mass of the combined object from the point *B*. [6]

4



Two small spheres A and B, with masses 0.3 kg and 0.2 kg respectively, lie adjacent to each other on a smooth horizontal surface. The spheres are midway between two identical vertical walls. The spheres are projected directly towards the walls with speeds 6 m s^{-1} (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$. The coefficient of restitution between each sphere and each wall is also $\frac{2}{3}$.

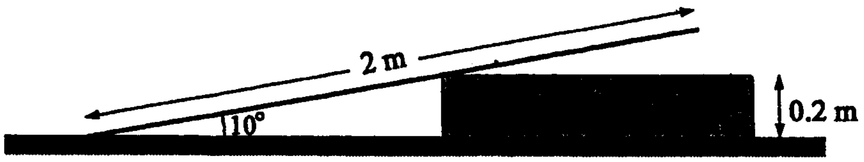
Calculate

(i) the speed of A after its first impact with a wall, [1]

(ii) the speeds of A and B after their first impact with each other, [6]

(iii) the magnitude of the impulse which sphere A exerts on sphere B at their first impact. [2]

5



A uniform plank of weight 200 N has length 2 m. The plank rests against a smooth step of height 0.2 m. One end of the plank lies on rough horizontal ground. The plank is in equilibrium and makes an angle of 10° with the horizontal (see diagram).

(i) Show that the force which the plank exerts on the step is 171 N, correct to 3 significant figures. [3]

(ii) Find the least possible coefficient of friction between the plank and the ground. [6]

[Questions 6 and 7 are printed overleaf.]

6 The resistance to motion of a car, of mass 1500 kg, has magnitude $(Av + Bv^2)$ N, where A and B are constants and v m s^{-1} is the speed of the car. The maximum power of the car's engine is 30 kW.

(i) Given that the maximum speed of the car on a horizontal road is 50 m s^{-1}, show that $A + 50B = 12$. [4]

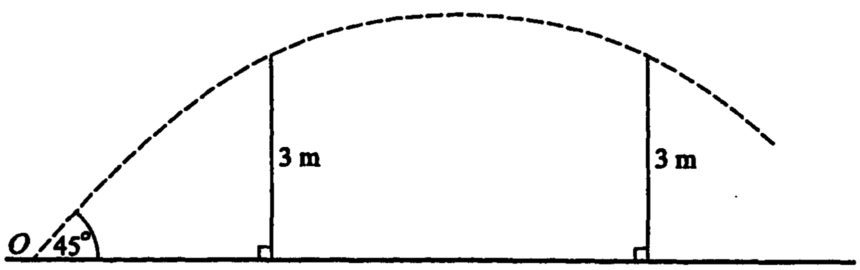
(ii) It is given also that the maximum speed of the car is 30 m s^{-1} when travelling uphill on a road inclined at $\alpha°$ to the horizontal, where $\sin \alpha° = \frac{1}{21}$. Find another equation connecting A and B, and hence show that $B = 0.1$. [4]

(iii) Find the maximum acceleration at an instant when the car is travelling on a horizontal road at a speed of 40 m s^{-1}. [5]

7 A particle is projected with speed 12 m s^{-1} at an angle of elevation θ from a point O on a horizontal plane, and it moves freely under gravity. The horizontal and upward vertical displacements of the particle from O at any subsequent time, t seconds, are x m and y m respectively.

(i) Express x and y in terms of θ and t, and hence show that

$$y = x \tan \theta - \frac{gx^2}{288 \cos^2 \theta}.$$ [4]



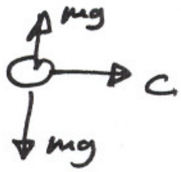
Two thin poles of height 3 m stand vertically in the plane of the path of the particle. When $\theta = 45°$, the particle just passes over the tops of both poles (see diagram).

(ii) Find the horizontal distance between the poles. [4]

(iii) Find the direction of motion of the particle as it passes over the second pole. [5]

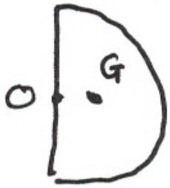
MECHANICS 2 JAN 2004

① Work done $= (30\cos 25^\circ) \times 50 = \underline{1.36\ \text{kJ}}$ (3 s.f.)

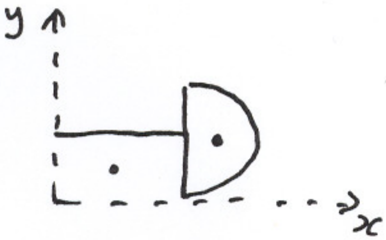
② 

N2 (→) $C = m(r\omega^2)$

$= 0.1\,(0.2 \times 3^2) = \underline{0.18\ \text{N}}$

③ 

$OG = \dfrac{2r\sin\alpha}{3\alpha} = \dfrac{10\sin\frac{\pi}{2}}{\left(\frac{3\pi}{2}\right)} = \dfrac{20}{3\pi} = \underline{2.12\ \text{cm}}$ (3sf)

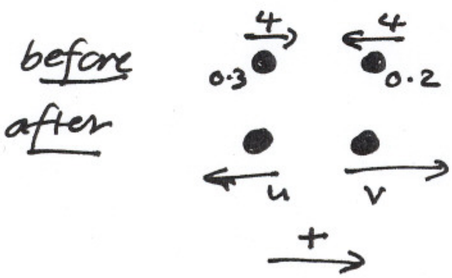


$5\begin{bmatrix}\bar{x}\\ \bar{y}\end{bmatrix} = 2\begin{bmatrix}6\\ 2.5\end{bmatrix} + 3\begin{bmatrix}12 + \frac{20}{3\pi}\\ 5\end{bmatrix}$

$\bar{x} = 10.873\ldots \qquad \bar{y} = 4$

$BG = \sqrt{(10.873\ldots^2 + 4^2)} = \underline{11.6\ \text{cm}}$ (3s.f.)

④ speed of A (and B) after wall impact $= 6 \times \frac{2}{3} = \underline{4\ \text{ms}^{-1}}$



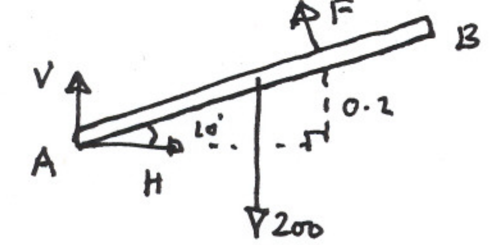
MOM $0.2v - 0.3u = {}^{-}0.8 + 1.2$

IMPACT $v + u = -\frac{2}{3}(-4-4)$

$\left.\begin{aligned} 2v - 3u &= 4 \\ v + u &= \tfrac{16}{3} \end{aligned}\right\}$ $\underline{u = 4/3}$, $\underline{v = 4}$

Impulse $= 4 \times 0.2 - {}^{-}4 \times 0.2 = \underline{1.6\ \text{Ns}}$

(5)



$M(A) \quad F\left(\frac{0.2}{\sin 10°}\right) - 2000\cos 10° = 0$

$F = 1000 \sin 10° \cos 10° = 171.010..$

$= \underline{171} \quad (3sf.)$

$N2(\rightarrow) \quad H - F\sin 10° = 0$
$N2(\uparrow) \quad V - 200 + F\cos 10° = 0$
$\Big\} \quad H = 29.6955... \quad V = 31.5879...$

$H \leq \mu V \quad \Rightarrow \quad \mu \geq \frac{H}{V} = 0.94009...$

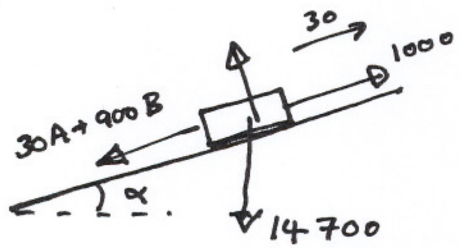
$\therefore$ least possible $\underline{\mu = 0.940} \quad (3sf.)$

(6)

at max. speed $\begin{cases} FV = 30\,000, \; F = \frac{30\,000}{50} = 600 \\ \text{Resistance} = 50A + 2500B \end{cases}$

$\therefore \; 50A + 2500B = 600 \quad \Rightarrow \quad \underline{A + 50B = 12}$

(ii)



$N2(\nearrow) \quad 1000 - 14700\sin\alpha - (30A + 900B) = 0$

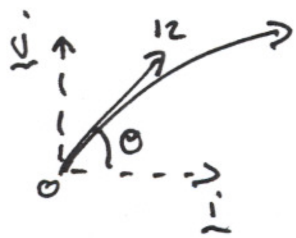
$30A + 900B = 300$

$\underline{A + 30B = 10}$

hence $\underline{A = 7} \qquad \underline{B = 0.1}$

At $40\,ms^{-1}$....

(7)



$$\underset{\sim}{a} = \begin{bmatrix} 0 \\ -g \end{bmatrix} \qquad \underset{\sim}{v} = \int \underset{\sim}{a}\, dt = \begin{bmatrix} 12\cos\theta \\ 12\sin\theta - gt \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \int \underset{\sim}{v}\, dt = \begin{bmatrix} 12\cos\theta\, t \\ 12\sin\theta\, t - \frac{1}{2}gt^2 \end{bmatrix}$$

$$\therefore \quad y = 12\sin\theta\, t - \frac{1}{2}gt^2$$

$$= 12\sin\theta\left(\frac{x}{12\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{12\cos\theta}\right)^2$$

$$y = x\tan\theta - \frac{gx^2}{288\cos^2\theta}$$

(AG)

Substitute $\theta = 45°$, $y = 3$

$$3 = x - \frac{9.8x^2}{144}$$

$$9.8x^2 - 144x + 432 = 0$$

$$x = \frac{144 \pm \sqrt{3801.6}}{19.6} = 10.4927\ldots$$

or $4.20116\ldots$

$\therefore$ horizontal separation of poles $= 10.49.. - 4.20.. = \underline{6.29\text{ m}}$

time at 2nd pole $= \dfrac{10.4927\ldots}{12\cos 45°} = 1.23657\ldots$