OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2638

Mechanics 2

Tuesday **18 JUNE 2002** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

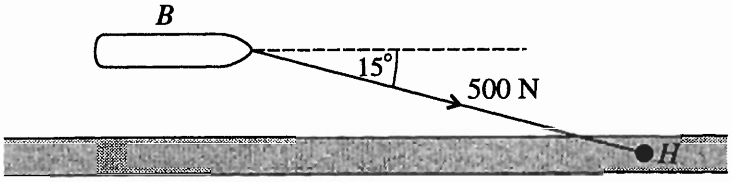
- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\ \mathrm{m\,s^{-2}}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1



A barge B is pulled along a canal by a horse H, which is on the tow-path. The barge and the horse move in parallel straight lines and the tow-rope makes a constant angle of 15° with the direction of motion (see diagram). The tow-rope remains taut and horizontal, and has a constant tension of 500 N.

(i) Find the work done on the barge by the tow-rope, as the barge travels a distance of 400 m. [3]

The barge moves at a constant speed and takes 10 minutes to travel the 400 m.

(ii) Find the power applied to the barge. [2]

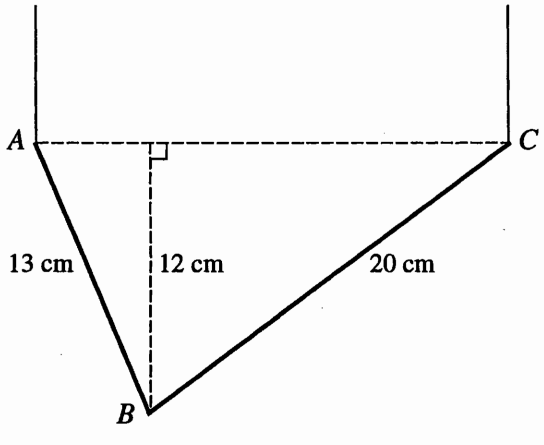
2 A uniform circular cylinder, of radius 6 cm and height 15 cm, is in equilibrium on a fixed inclined plane with one of its ends in contact with the plane.

(i) Given that the cylinder is on the point of toppling, find the angle the plane makes with the horizontal. [3]

The cylinder is now placed on a horizontal board with one of its ends in contact with the board. The board is then tilted so that the angle it makes with the horizontal gradually increases.

(ii) Given that the coefficient of friction between the cylinder and the board is $\frac{3}{4}$, determine whether or not the cylinder will slide before it topples, justifying your answer. [4]

3



A rigid body ABC consists of two uniform rods AB and BC, rigidly joined at B. The lengths of AB and BC are 13 cm and 20 cm respectively, and their weights are 13 N and 20 N respectively. The distance of B from AC is 12 cm. The body hangs in equilibrium, with AC horizontal, from two vertical strings attached at A and C. Find the tension in each string. [7]

4 A cyclist and his machine have a combined mass of 80 kg. The cyclist ascends a straight hill AB of constant slope, starting from rest at A and reaching a speed of 5 m s^{-1} at B. The level of B is 4 m above the level of A.

(i) Find the gain in kinetic energy and the gain in gravitational potential energy of the cyclist and his machine. [3]

During the ascent the resistance to motion is constant and has magnitude 70 N.

(ii) Given that the work done by the cyclist in ascending the hill is 8000 J, find the distance AB. [3]

At B the cyclist is working at 720 watts and starts to move in a straight line along horizontal ground. The resistance to motion has the same magnitude of 70 N as before.

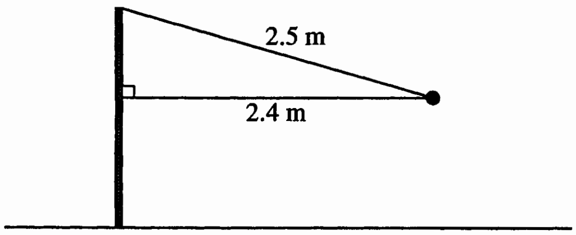
(iii) Find the acceleration with which the cyclist starts to move horizontally. [3]

5 An athlete 'puts the shot' with an initial speed of 19 m s^{-1} at an angle of $11°$ above the horizontal. At the instant of release the shot is 1.53 m above the horizontal ground. By treating the shot as a particle and ignoring air resistance, find

(i) the maximum height, above the ground, reached by the shot, [3]

(ii) the horizontal distance the shot has travelled when it hits the ground. [6]

6



A ball of mass 0.08 kg is attached by two strings to a fixed vertical post. The strings have lengths 2.5 m and 2.4 m, as shown in the diagram. The ball moves in a horizontal circle, of radius 2.4 m, with constant speed $v\text{ m s}^{-1}$. Each string is taut and the lower string is horizontal. The modelling assumptions made are that both strings are light and inextensible, and that there is no air resistance.

(i) Find the tension in each string when $v = 10.5$. [7]

(ii) Find the least value of v for which the lower string is taut. [3]

[Question 7 is printed overleaf.]

7 Two uniform smooth spheres, A and B, have the same radius. The mass of A is 0.24 kg and the mass of B is m kg. Sphere A is travelling in a straight line on a horizontal table, with speed 8 m s^{-1}, when it collides directly with sphere B, which is at rest. As a result of the collision, sphere A continues in the same direction with a speed of 6 m s^{-1}.

(i) Find the magnitude of the impulse exerted by A on B. [3]

(ii) Show that $m \leqslant 0.08$. [3]

It is given that $m = 0.06$.

(iii) Find the coefficient of restitution between A and B. [3]

On another occasion A and B are travelling towards each other, each with speed 4 m s^{-1}, when they collide directly.

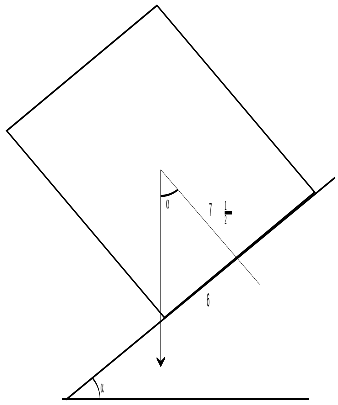
(iv) Find the speeds of A and B immediately after the collision. [4]

M2 June 2002

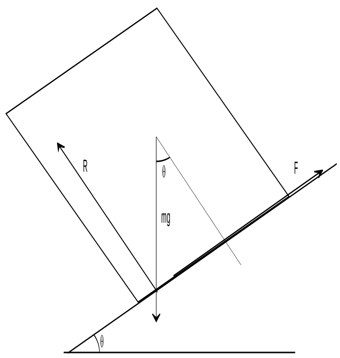
1) $W = (500\cos 15°) \times 400 = 193\text{kJ}$

$P = \frac{dW}{dt} = \frac{193185\ldots}{600} = 322\text{kW}$

2)



$$\alpha = \tan^{-1}\left(\frac{6}{7.5}\right) = \tan^{-1} 0.8 = 38.7°\ (3sf)$$



Assume equilibrium:

$N2(\nwarrow): R - mg\cos\theta = 0$

$N2(\nearrow): F - mg\sin\theta = 0$

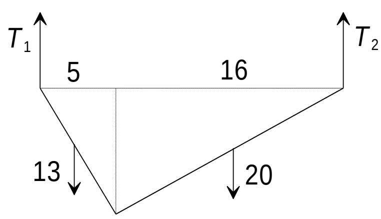
And as always $\frac{F}{R} \leq \mu$

So $\tan\theta \leq \frac{3}{4}$

And it starts to slip when $\tan\theta = \frac{3}{4}$.

This clearly happens before $\tan\theta = 0.8$, so it slips **before** it topples.

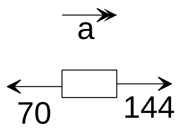
3)



$M_A(\circlearrowleft): T_2 \times 21 - 20 \times 13 - 13 \times 2.5 = 0$
$T_2 = 13.9$
$N2(\uparrow): T_1 + T_2 - 33 = 0$
So $T_1 = 19.1$

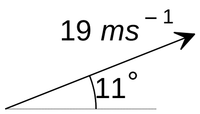
4) $\Delta GPE = 80 \times g \times 4 = 3136\text{J}$
$\Delta KE = \frac{1}{2} \times 80 \times 5^2 = 1000\text{J}$
Work done $= 8000 = 3136 + 1000 + 70s$
$\therefore s = 55.2\text{m}$

Using $P = Fv$, $F = 144$



And $a = \frac{74}{80} = 0.925\text{ms}^{-1}$

5)



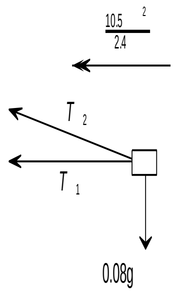
$$x = 19\cos 11°\, t$$
$$y = 19\sin 11°\, t - 4.9t^2 + 1.53$$

Max h when $\dot{y} = 0$, or simpler to use $v^2 = u^2 + 2as$ vertically,
So $0 = (19\sin 11°)^2 - 2 \times 9.8 \times h$
$\therefore h_{\max} = \frac{(19 \times \sin 11°)^2}{2 \times 9.8} + 1.53 = 2.20\text{m}$

$$y = 0 \Rightarrow 4.9t^2 - 19\sin 11°\, t - 1.53 = 0$$
$$\therefore t = 1.040083 \ldots$$

And so $x = 19.4\text{m}$

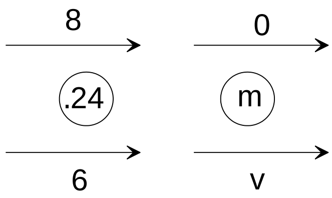
6)



$$N2(\uparrow): T_2 \sin\alpha - 0.08g = 0$$
$$\therefore T_2 = 2.8$$
$$N2(\leftarrow): T_1 + T_2 \times \frac{24}{25} = 0.08 \times \frac{10.5^2}{2.4}$$
$$\therefore T_1 = 0.987$$

Assume taut:
T_2 is still 2.8,
And $T_1 + 2.8 \times \frac{24}{25} = 0.08 \times \frac{v^2}{2.4}$
$T_1 \geq 0$ and so $v_{\min} = \sqrt{80.64} = 8.98$ms^{-1}

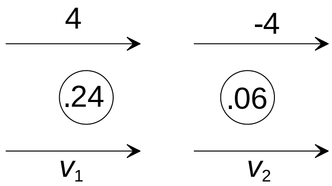
7)



For A, $I = 6 \times .24 - 8 \times .24 = -0.48$
So Impulse on B is $+0.48$Ns

For B, $0.48 = mv$.
Since $v \geq 6,\ m \leq \frac{0.48}{6} = 0.08$

Now given that $m = 0.06$, $v = \frac{0.48}{0.06} = 8$ms^{-1}
So $e = \frac{2}{8} = \frac{1}{4}$



cons. mom: $0.96 - 0.24 = 0.24v_1 + 0.06v_2$
N's law of impact: $v_2 - v_1 = \frac{1}{4} \times 8$
Giving

$$4v_1 + v_2 = 12$$
$$v_2 - v_1 = 2$$

$$\therefore v_1 = 2, v_2 = 4$$