OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2638

Mechanics 2

Thursday **14 JUNE 2001** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\mathrm{m\,s^{-2}}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

 Registered Charity Number: 1066969 **[Turn over**

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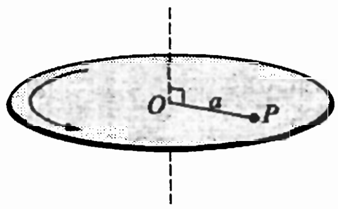
1 The engine of a car of mass 1000 kg is capable of a maximum power of 20 kW.

(i) With the engine working at maximum power the car can travel at a constant speed of $40\,\mathrm{m\,s^{-1}}$ on a horizontal straight road. Find the magnitude of the resistance to motion of the car. [3]

(ii) Assuming the resistance to motion is unchanged, find the acceleration of the car at an instant when it is travelling at $32\,\mathrm{m\,s^{-1}}$ on the same road with its engine working at maximum power. [2]

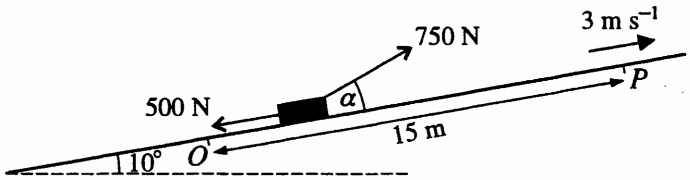
2



(i) The diagram shows a horizontal turntable, with centre O, which rotates with constant angular speed $\frac{1}{2}\sqrt{\left(\frac{g}{a}\right)}$. A particle P is situated on the turntable at a point whose distance from O is a. The coefficient of friction between the particle and the turntable is μ. The particle moves with the turntable and no sliding takes place. Show that $\mu \geqslant \frac{1}{4}$. [3]

(ii) Another particle Q is also situated on the turntable; it moves with the turntable and no sliding takes place. The speed of Q is $\frac{1}{4}\sqrt{(ag)}$. Find, in terms of a, the greatest and least possible values of the distance PQ. [3]

3



A crate of mass 100 kg is dragged up a slope, which is inclined at 10° to the horizontal, by a constant force. The force has magnitude 750 N and acts at an angle α upwards from the slope. The total resistance to motion of the crate has a constant magnitude of 500 N. The crate starts from rest at the point O and passes the point P, 15 m from O, with a speed of $3\,\mathrm{m\,s^{-1}}$ (see diagram).

(i) For the crate's motion from O to P, find

(a) the increase in kinetic energy, [1]

(b) the increase in gravitational potential energy, [2]

(c) the work done against the resistance to motion of the crate. [1]

(ii) Hence find the angle α. [3]

4 Two uniform rectangular blocks X and X' are made of the same material. X has length 24 cm and height 15 cm, and X' has length 30 cm and height 20 cm. X and X' have the same thickness. X' is placed on top of X with their front faces $ABCD$ and $A'B'C'D'$ in the same vertical plane.

(a)

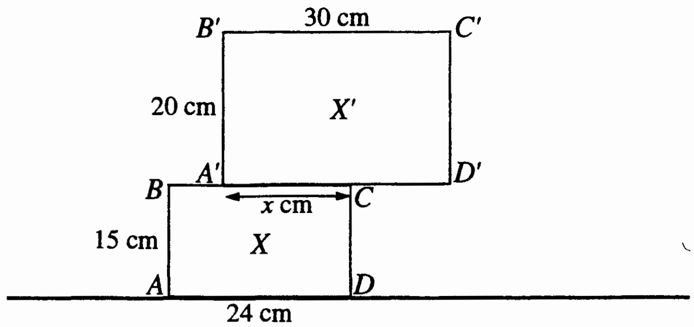


Fig. 1

X stands on a horizontal table, and the blocks rest in equilibrium with $A'C = x$ cm (see Fig. 1).

(i) State the least possible value of x when X and X' are not attached to each other. [1]

(ii) Find the least possible value of x when X and X' are glued together. [4]

(b)

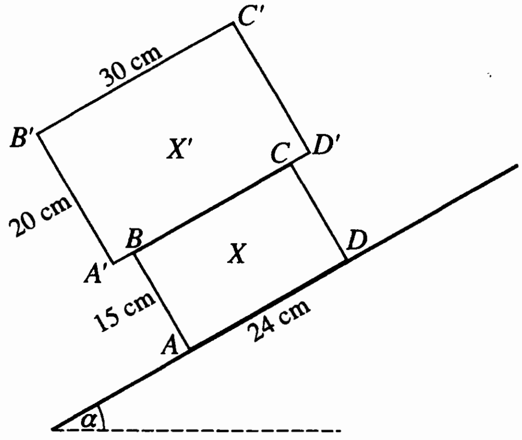


Fig. 2

X and X' are glued together with the mid-points of BC and $A'D'$ coinciding. The combined blocks are placed on a plane inclined at an angle α to the horizontal (see Fig. 2). The plane is sufficiently rough to prevent any sliding. Given that the blocks remain in equilibrium, find the greatest possible value of α. [4]

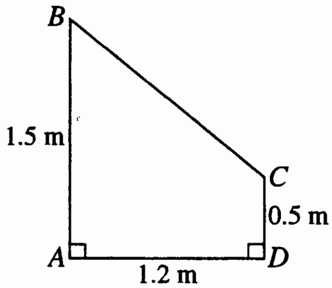
5 Two smooth spheres A and B, of equal radius and of masses 0.3 kg and 0.2 kg respectively, are free to move on a smooth horizontal table. A is moving with speed $5\,\mathrm{m\,s^{-1}}$ when it collides directly with B, which is stationary. As a result of the collision B starts to move with speed $4.5\,\mathrm{m\,s^{-1}}$.

(i) Find the coefficient of restitution between the spheres. [4]

(ii) The sphere B subsequently strikes a fixed barrier at right angles. The barrier exerts an impulse of magnitude 1.7 N s on B. Find the speed with which B rebounds from the barrier. [3]

(iii) Find also the speed with which B moves towards the barrier following its second collision with A. [4]

6 (i)



A uniform lamina $ABCD$ has the shape of a trapezium which is right-angled at A and D, and $AB = 1.5$ m, $AD = 1.2$ m and $CD = 0.5$ m, as shown in the diagram. Show that the distance of the centre of mass of the lamina from AB is 0.5 m. [4]

(ii) A cupboard door, of weight 165 N, is modelled by the uniform lamina $ABCD$. The door has smooth hinges at the points P and Q of the edge AB, which is vertical. $AP = BQ = 0.2$ m. The door is open and is in equilibrium. Find the magnitude and direction of the horizontal component of the force on the door at each of P and Q. [4]

(iii) A wedge is now placed between the door and the floor at D, exerting a vertically upward force on the door of magnitude F newtons. The horizontal components of the forces on the door at P and Q are now both zero and the door is in equilibrium. Calculate F, and state the magnitude of the resultant force on the door due to the hinges. [3]

7 A ball A is moving under gravity. A man throws another ball B, from a point O, at an instant when A is vertically above O and moving horizontally. B is thrown with initial speed $12\,\mathrm{m\,s^{-1}}$ in a direction making an angle of 40° above the horizontal. A and B collide at a point C which is at the same horizontal level as O.

(i) Find the time taken for B to travel from O to C. [2]

(ii) Find the distance OC. [2]

(iii) Show that A passes over O at a height of about 12.1 m. [3]

(iv) Find the angle between the direction of motion of A and the horizontal, immediately before A and B collide. [4]

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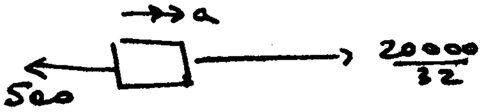
(1) Driving force $= \frac{20000}{40} = 500$. [3]

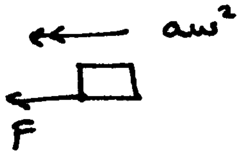
$a = 0$, so Resistance $=$ **500 N**



N2($\rightarrow$): $625 - 500 = 1000a$ [2]

$\therefore$ $a = 0.125\ ms^{-2}$

(2) (i)



N2 ($\leftarrow$) $F = ma \cdot \frac{1}{4}\frac{g}{a}$

$= \frac{mg}{4}$

also $N = mg$

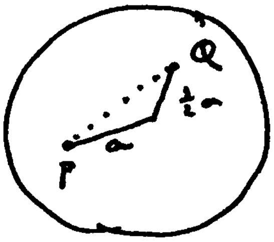
$F \leq \mu N$, so $\mu \geqslant \frac{\frac{mg}{4}}{mg}$ [3]

$\therefore \mu \geqslant \frac{1}{4}$

(ii) using $v = r\omega$,

$\frac{1}{4}\sqrt{ag} = r \cdot \frac{1}{2}\sqrt{\frac{g}{a}}$

$\Rightarrow r = \frac{1}{2}a$.



clearly, $\frac{1}{2}a \leq PQ \leq \frac{3}{2}a$ [3]

(3) (i) $\Delta KE = \frac{1}{2} \times 100 \times 3^2 = 450\ J$ [1]

$\Delta GPE = 100 \times 9.8 \times 15 \sin 10^\circ = 2553\ J$ [2]

$\Delta Rs = 500 \times 15 = 7500\ J$ [1]

So $750 \cos\alpha\ 15 = 450 + 2553 + 7500$, so $\alpha = 21.0^\circ$ (3sf)

[3]

(4)

$x_{max} = \underline{15}$ [1]

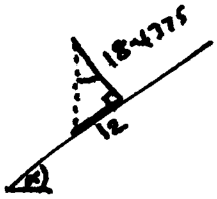
from A, $(24\times15)\times12 + (20\times30)\times(39-x) = (24\times15+20\times30)\bar{x}$

for non-toppling $\dfrac{360\times12+600(39-x)}{960} \leq 24$

$\therefore \quad \underline{\underline{x \geq 7.8\text{cm}}}$ [4]

$(24\times15)\times7\tfrac{1}{2} + (20\times30)\times25 = (24\times15+20\times30)\bar{y}$

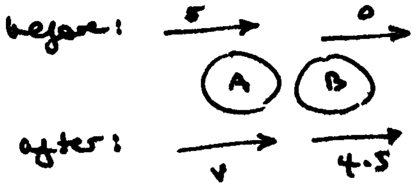
$\therefore \bar{y} = 18.4375$, and $\bar{x} = 12$.



$\therefore \tan\alpha \leq \dfrac{12}{18.4375}$

$\underline{\underline{\alpha \leq 33.057\ldots}}^\circ$ [4]

(5) Assuming 'it's not an oblique' impact, (?)



(i) Cons. of mom: $0.3\times5+0 = 0.3v + 0.2\times4.5$

$\therefore v = 2$.

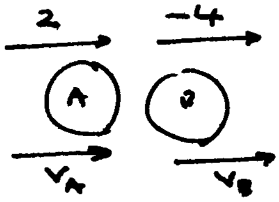
and so $e = \dfrac{2.5}{5} = \underline{\underline{0.5}}$ [4]

(ii)

$1.7 = 0.2v - -0.2\times4.5$

$\therefore \underline{\underline{v = 4}}\ ms^{-1}$ [3]

(iii)



C. of M.: $0.3\times2 + 0.2\times-4 = 0.3v_A + 0.2v_B$

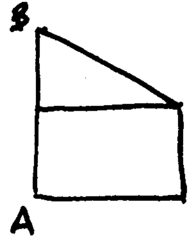
$\rightarrow 3v_A + 2v_B = -2$ —— (1)

N's law of impacts: $v_B - v_A = 0.5(2--4)$

$\rightarrow v_A - v_B = -3$ —— (2)

$\therefore \quad \underline{\underline{v_B = 1.4\,ms^{-1}}}$ [4]

(6)

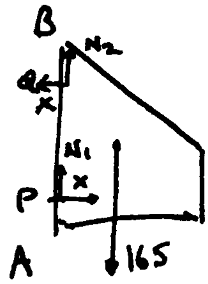


M_{AB}: $(\frac{1}{2}\times1.2\times1)\times0.4+(0.5\times1.2)\times0.6 = 1.2\bar{x}$

$\rightarrow \bar{x} = \frac{0.24+0.36}{1.2}$

$= \underline{\underline{0.5}}$ m [4]

(ii)

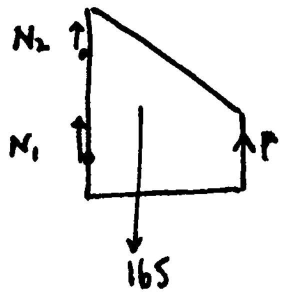


$M_Q(\circlearrowleft)$: $X\times1.1 - 165\times0.5 = 0$

So horiz forces are :

$\underline{\underline{P: 75\text{ N} \rightarrow \text{ and } Q: 75\text{ N} \leftarrow}}$ [4]

(iii)



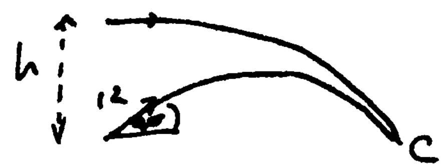
$M_{AB}(\circlearrowleft)$: $F\times1.2 - 165\times0.5 = 0$

So $F = \underline{\underline{68.75\text{ N}}}$

& hinge resultant $= 165 - 68.75$

$= \underline{\underline{96.25\text{ N}}}$ [3]

(7)



ball B : $\begin{cases} x = 12\cos40^\circ t \\ y = 12\sin40^\circ - 4.9t^2 \end{cases}$

At C, $y=0$ so $t = \frac{12\sin40^\circ}{4.9}$, $= \underline{\underline{1.57}}$ s [2]

and $OC = 12\cos40^\circ \times t_1$

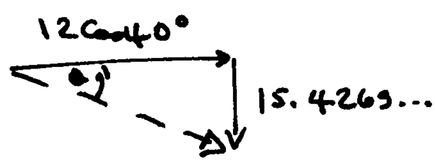
$= \underline{\underline{14.5}}$ m [2]

For A, $y = h - 4.9t^2$, so at t_1 with $y=0$,

$h = 12.14\ldots \approx \underline{\underline{12.1\text{m}}}$. [3]

A : $\dot{x} = 12\cos40^\circ$ and $\dot{y} = -9.8t$.

So at C :



So $\theta = \underline{\underline{59.2^\circ}}$ below horiz . [4]