OCR
Oxford Cambridge and RSA

Monday 25 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 marks)

1 A rocket is launched vertically upwards from rest. The initial total mass of the rocket and its fuel is 1500 kg. The propulsion system of the rocket burns fuel at a constant rate of $20\,\text{kg}\,\text{s}^{-1}$ and the fuel is ejected vertically downwards with a speed of $1800\,\text{m}\,\text{s}^{-1}$ relative to the rocket. The only other force acting on the rocket is its weight. The acceleration due to gravity is constant throughout the motion. At time t s after launch, where $t \leqslant 60$, the speed of the rocket is $v\,\text{m}\,\text{s}^{-1}$. The rocket stops burning fuel 60 seconds after the launch.

(i) Show that, while fuel is being burnt,

$$\frac{dv}{dt} - \frac{1800}{75-t} = -g.$$

[6]

(ii) Solve this differential equation to find an expression for v in terms of t. Calculate, correct to 3 significant figures, the speed of the rocket when $t = 30$. **[6]**

2 Fig. 2 shows a uniform rigid rod AB of mass m and length a. The rod is freely hinged at A so that it can rotate in a vertical plane. The end B of the rod is attached to one end of a light elastic string BC of modulus λ and natural length a. The other end of the string, C, is fixed at a point vertically above A, where the distance AC is a. The rod makes an angle 2θ with the downward vertical, where $0 < \theta \leqslant \frac{\pi}{4}$.

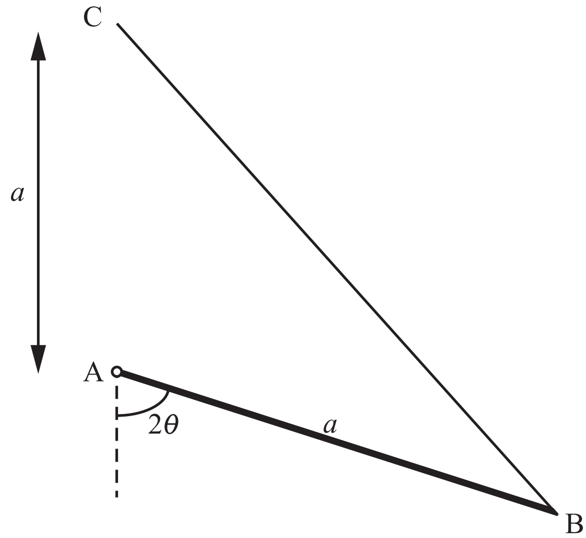

Fig. 2

(i) Find the potential energy, V, of the system relative to a situation in which the rod AB is horizontal, and hence show that

$$\frac{dV}{d\theta} = 2a\sin\theta(\lambda + mg\cos\theta - 2\lambda\cos\theta).$$

[5]

(ii) Show that if there is a position of equilibrium then $mg < \lambda \leqslant mg\left(1 + \frac{\sqrt{2}}{2}\right)$. Deduce that any such position of equilibrium is stable. **[7]**

Section B (48 marks)

3 A particle P of mass m kg is held at rest at a point O on a fixed plane inclined at an angle of 30° to the horizontal. P is released and moves down a line of greatest slope. The total resistance acting on P is kv^2 N, where k is a positive constant and where $v\,\mathrm{m\,s^{-1}}$ is the velocity of P when P has travelled a distance x m from O.

(i) Write down an equation of motion for P and show that

$$v^2 = \frac{mg}{2k}\left(1 - e^{-\frac{2kx}{m}}\right).$$

[7]

It is given that $k = 0.2$, $m = 3$ and P travels a distance of 1.5 m before reaching the foot of the plane.

(ii) Show, by integration, that the work done against the resistance in the first 1.5 m of the motion is

$$\frac{9}{4}g\left(5e^{-0.2} - 4\right)\,\mathrm{J},$$

and verify that this is equal to the loss in mechanical energy of P. **[6]**

At the bottom of the slope the particle P moves onto a smooth horizontal plane without loss of speed; a force then acts on P. This force, which acts in the direction of motion of P, has a magnitude of $\ln(2t + 1)$ N where t s is the time from the moment that P begins to move horizontally. When travelling horizontally there are no resistances to motion acting on P.

(iii) Given that the impulse of the force over the first T seconds is 20 N s show that T satisfies

$$T = \frac{40 + 2T - \ln(2T+1)}{2\ln(2T+1)}.$$

[7]

(iv) Use an iterative process based on the equation in part **(iii)**, with a suitable starting value, to find T correct to 3 decimal places. **[2]**

(v) Find the velocity of P after P has travelled horizontally for T seconds. **[2]**

Question 4 begins on page 4.

4 **(i)** Show, by integration, that the moment of inertia of a thin uniform rigid rod of length $3a$ and mass $2m$ about an axis through one end and perpendicular to the rod is $6ma^2$. **[4]**

A pendulum consists of a thin uniform rigid rod AB of length $3a$ and mass $2m$ and a uniform circular disc of radius a, mass m and centre C. The end B of the rod is rigidly attached to a point on the circumference of the disc in such a way that ABC is a straight line. The pendulum is initially at rest with B vertically below A. The pendulum is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through A where the axis is perpendicular to the plane of the disc (see Fig. 4). At time $t = 0$ the pendulum is set in motion with initial angular velocity ω.

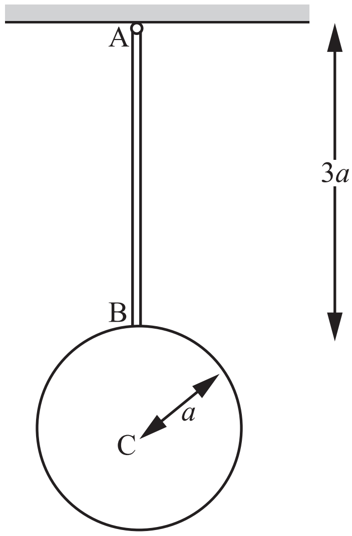

Fig. 4

(ii) Show that the angular velocity $\dot{\theta}$ when the pendulum makes an angle θ with the downward vertical is given by

$$\dot{\theta}^2 = \omega^2 + k(\cos\theta - 1),$$

where k is a constant to be determined in terms of a and g. **[8]**

(iii) Find, in terms a, g and θ, the angular acceleration of the pendulum. **[2]**

The pendulum is making small oscillations about the equilibrium position.

(iv) Show that the motion is approximately simple harmonic, and find the approximate period of oscillations in terms of a and g. **[2]**

(v) Now suppose θ is such that θ^3 and higher powers can be neglected. Show that

$$\frac{\mathrm{d}t}{\mathrm{d}\theta} \approx \left(\omega^2 - \frac{1}{2}k\theta^2\right)^{-\frac{1}{2}},$$

and hence, by integration, express θ in terms of k, ω and t. **[8]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

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4764/01 Mechanics 4

PRINTED ANSWER BOOK

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OCR supplied materials:
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Candidate forename		Candidate surname	

Centre number						Candidate number				

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1 (i)	

1 (ii)

2 (i)

2 (ii)

3 (i)

3 (ii)	

3 (iii)	

3 (iv)	
3 (v)	

4 (i)

4 (ii)	

4 (iii)	
4 (iv)	

4 (v)

(answer space continued on next page)

4 (v)	(continued)

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4764**: Mechanics 4

Advanced GCE

Mark Scheme for June 2018

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given.

Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate’s script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

‘Fresh starts’ will not affect an earlier decision about a misread.

Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

Question			Answer	Marks	Guidance
1	(i)		$(m+\delta m)(v+\delta v)-mv-\delta m(v-1800)=-mg\delta t$	M1	Impulse = change in momentum (allow F for mg)
			$m\frac{\delta v}{\delta t}+v\frac{\delta m}{\delta t}+\frac{\delta m}{\delta t}\frac{\delta v}{\delta t}\delta t-v\frac{\delta m}{\delta t}+1800\frac{\delta m}{\delta t}=-mg$	M1	Form differential equation (must replace F for this mark)
			$m\frac{\mathrm{d}v}{\mathrm{d}t}+1800\frac{\mathrm{d}m}{\mathrm{d}t}=-mg$	A1	Stating correct differential equation can imply previous M marks
			$m=1500-20t\Rightarrow\frac{\mathrm{d}m}{\mathrm{d}t}=-20$	B1	
			$(1500-20t)\frac{\mathrm{d}v}{\mathrm{d}t}+1800(-20)=-(1500-20t)g$	M1	Substitute m and $\frac{\mathrm{d}m}{\mathrm{d}t}$ into their differential equation - dependent on both previous M marks
			$\frac{\mathrm{d}v}{\mathrm{d}t}-\frac{1800}{75-t}=-g$	E1	
				[6]	
1	(ii)		$\int\mathrm{d}v=\int\left(\frac{1800}{75-t}-g\right)\mathrm{d}t$	M1	Separate variables
				M1	Integrate (must involve logs)
			$v=-1800\ln(75-t)-gt+c$	A1	Correct + constant of integration
			$t=0, v=0\Rightarrow c=1800\ln 75$	M1	Use condition to obtain c
			$v=1800\ln\left(\frac{75}{75-t}\right)-gt$	A1	
			$t=30, v=625\,\mathrm{m\,s^{-1}}$	A1	625.4861228
				[6]	

Question			Answer	Marks	Guidance
2	(i)		$V=-\frac{1}{2}mga\cos 2\theta+\ldots \;\text{L}\; +\frac{\lambda a^2\left[(2\cos\theta-1)^2-(\sqrt{2}-1)^2\right]}{2a}$	B1 M1*	B1 first term, M1 genuine attempt at extension + substitution into $\frac{\lambda x^2}{2a}$ - allow omission of $-\frac{1}{2}\lambda a(\sqrt{2}-1)^2$ term for full marks
			$V=-\frac{1}{2}mga\cos 2\theta+\frac{1}{2}\lambda a(2\cos\theta-1)^2-\frac{1}{2}\lambda a(\sqrt{2}-1)^2$	A1	
			$\frac{\mathrm{d}V}{\mathrm{d}\theta}=mga\sin 2\theta+\lambda a(2\cos\theta-1)(-2\sin\theta)$	M1dep*	Differentiates their V of the correct form (all terms)
			or $mga\sin 2\theta-2\lambda a\sin\theta\cos\theta+2\lambda a\sin\theta$		
			$=2a\sin\theta(\lambda+mg\cos\theta-2\lambda\cos\theta)$	E1	
				[5]	
2	(ii)		$\frac{\mathrm{d}V}{\mathrm{d}\theta}=0\Rightarrow\lambda+mg\cos\theta-2\lambda\cos\theta=0\;\;(\text{Q}\sin\theta\neq 0)$	B1	
			$\cos\theta=\frac{\lambda}{2\lambda-mg}\;\Rightarrow\frac{\sqrt{2}}{2}\leq\frac{\lambda}{2\lambda-mg}<1$	M1	Make $\cos\theta$ the subject and use of $0<\theta\leq\frac{\pi}{4}$
			$mg<\lambda\leq mg\left(1+\frac{\sqrt{2}}{2}\right)$	E1	Complete argument
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}=2a\cos\theta(\lambda+mg\cos\theta-2\lambda\cos\theta)+2a\sin\theta(-mg\sin\theta+2\lambda\sin\theta)$	M1	Attempt to differentiate using the product rule
				A1	Correct differentiation
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}=2a\sin^2\theta(2\lambda-mg)$ and $\lambda>mg$	M1	Simplify $V^{//}$ and use of $\lambda>mg$ or mention that $2\lambda-mg>0$
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}>2mga\sin^2\theta$ so $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}>0\Rightarrow$ stable	E1	
				[7]	

Question			Answer	Marks	Guidance
3	(i)		$mv\frac{dv}{dx} = mg\sin 30 - kv^2$	B1	
			$x = 2m\int \frac{v\,dv}{mg - 2kv^2}$	M1	Separate and attempt to integrate (must get in terms of logs)
			$x = -\frac{m}{2k}\ln\left(mg - 2kv^2\right) + c$	A1	
			$x = 0, v = 0 \Rightarrow c = \frac{m}{2k}\ln mg$	M1	Use initial conditions (dependent on previous M mark)
			$x = \frac{m}{2k}\ln\left(\frac{mg}{mg - 2kv^2}\right)$	A1	
			$\frac{mg}{mg - 2kv^2} = e^{\frac{2kx}{m}} \Rightarrow mg - 2kv^2 = mge^{-\frac{2kx}{m}}$ leading to $v^2 = \ldots$	M1	Re-arrange to make v^2 the subject (dependent on previous M marks)
			$v^2 = \frac{mg}{2k}\left(1 - e^{-\frac{2kx}{m}}\right)$	E1	
				[7]	

Question			Answer	Marks	Guidance
3	(ii)		Work done = $\int_0^{1.5} kv^2 \mathrm{d}x = \int_0^{1.5} \frac{mg}{2}\left(1-\mathrm{e}^{-\frac{2kx}{m}}\right)\mathrm{d}x = \left(\int_0^{1.5} \frac{3g}{2}\left(1-\mathrm{e}^{-\frac{2}{15}x}\right)\mathrm{d}x\right)$	B1	Limits not required for this mark
			$= \frac{3g}{2}\left[x+\frac{15}{2}\mathrm{e}^{-\frac{2}{15}x}\right]_0^{1.5}$	M1	Attempt to integrate (no limits required)
			$= \frac{3g}{2}\left(\frac{3}{2}+\frac{15}{2}\mathrm{e}^{-0.2}-\frac{15}{2}\right) = g\left(-9+\frac{45}{4}\mathrm{e}^{-0.2}\right) = \frac{9g}{4}\left(5\mathrm{e}^{-0.2}-4\right)$	E1	Establishes given result
			Initial mechanical energy = $3g\left(1.5\sin 30\right)\left(=\frac{9g}{4}\right)$	B1	
			Final mechanical energy = $\frac{1}{2}(3)\left(\frac{15g}{2}\left(1-\mathrm{e}^{-0.2}\right)\right)$	B1	
			Loss in ME = $g\left(\frac{9}{4}-\frac{45}{4}\left(1-\mathrm{e}^{-0.2}\right)\right) = \frac{9g}{4}\left(5\mathrm{e}^{-0.2}-4\right)$	E1	Convincingly shown
				[6]	

Question			Answer	Marks	Guidance
3	(iii)		$J = \int_0^T \ln(2t+1)\,\mathrm{d}t$	B1	
			$= \left[t\ln(2t+1)\right]_0^T - \int_0^T \frac{2t}{2t+1}\,\mathrm{d}t = \left[t\ln(2t+1)\right]_0^T - \int_0^T \left(1 - \frac{1}{2t+1}\right)\mathrm{d}t$	M1	Use integration by parts
				A1	Correctly applied to end of first stage
			$= \left[t\ln(2t+1) - \left(t - \frac{1}{2}\ln(2t+1)\right)\right]_0^T$	A1	
			or for $\int \ln(2t+1)\mathrm{d}t = \frac{1}{2}\left[(2t+1)\ln(2t+1) - (2t+1)\right]$		
			$T\ln(2T+1) - T + \frac{1}{2}\ln(2T+1) = 20$	M1	Equates their expression to 20
			$2T\ln(2T+1) - 2T + \ln(2T+1) = 40 \Rightarrow 2T\ln(2T+1) = 40 + 2T - \ln(2T+1)$	M1	Attempts to rearrange to make T the subject
			$T = \frac{40 + 2T - \ln(2T+1)}{2\ln(2T+1)}$	E1	**AG**
				[7]	
3	(iv)		$T_{n+1} = \frac{40 + 2T_n - \ln(2T_n+1)}{2\ln(2T_n+1)}$ with e.g. $T_1 = 1$	M1	Using given answer with suitable starting value to find at least one approximation to T
			$T = 9.347$	A1 **[2]**	9.3469489… (correct answer implies M mark)
3	(v)		$20 = 3(v - 3.6501...)$	M1	Use of Impulse = change in momentum (must equal 20 and correct number of terms)
			$v = 10.3\ \mathrm{m\,s^{-1}}$	A1 **[2]**	10.3167748…

Question			Answer	Marks	Guidance
4	(i)		Mass per unit length $=\dfrac{2m}{3a}$	B1	
			$I=\sum\dfrac{2m}{3a}x^2\delta x=\dfrac{2m}{3a}\int x^2\mathrm{d}x$	M1	M1 for $\int x^2\mathrm{d}x$ - limits not required for M mark
			$=\dfrac{2m}{3a}\int_0^{3a}x^2\mathrm{d}x=\dfrac{2m}{3a}\left[\dfrac{x^3}{3}\right]_0^{3a}$	A1	Correct integration with limits
			$=\dfrac{2m}{3a}\left(\dfrac{27a^3}{3}\right)=6ma^2$	E1 **[4]**	3 marks maximum if $\int_{-1.5a}^{1.5a}$ then parallel axis rule used
4	(ii)		M of I of disc about its axis: $\dfrac{1}{2}ma^2$	B1	
			$I_A=\left(\dfrac{1}{2}ma^2+m(4a)^2\right)(+6ma^2)$	M1	Use of parallel axis theorem
			$I_A=\dfrac{45}{2}ma^2$	A1	
			$\bar{x}=\dfrac{7}{3}a$	B1	Centre of mass from A
				M1	Conservation of energy (two terms for each)
			$\dfrac{1}{2}\left(\dfrac{45}{2}ma^2\right)(\theta^2-\omega^2)=7mga(\cos\theta-1)$	A1 A1	A1 for KE terms, A1 for PE terms **SC** If M0 then B1 for either term
			$\theta^2=\omega^2+k(\cos\theta-1)$ where $k=\dfrac{28g}{45a}$	E1	
				[8]	
4	(iii)		$2\theta\theta=-k\theta\sin\theta$	M1	Differentiating θ with respect to *t* or $C=I\alpha$
			$\theta=-\dfrac{14g}{45a}\sin\theta$	A1	$\alpha=-\dfrac{1}{\left(\frac{45ma^2}{2}\right)}(3mg)\left(\dfrac{7a}{3}\sin\theta\right)$
				[2]	

Question			Answer	Marks	Guidance
4	(iv)		For small $\theta, \sin\theta \approx \theta$ $\ddot{\theta} \approx -\frac{14g}{45a}\theta$ so approx. SHM	M1	Apply small angle approximation, $\ddot{\theta} = -\omega^2\theta$ and must state result is (approx.) simple harmonic
			$T = 2\pi\sqrt{\frac{45a}{14g}}$	A1	If M0 then B1 for correct T
				[2]	
4	(v)		$\dot{\theta}^2 = \omega^2 + k\left(\left(1 - \frac{1}{2}\theta^2 + \mathbf{n}\right) - 1\right)$	M1	Substitutes $1 - \frac{1}{2}\theta^2$ for $\cos\theta$ and attempt to make $\frac{\mathrm{d}t}{\mathrm{d}\theta}$ the subject
			$\frac{\mathrm{d}t}{\mathrm{d}\theta} \approx \left(\omega^2 - \frac{1}{2}k\theta^2\right)^{-\frac{1}{2}}$,	E1	**AG**
			$t = \int\left(\omega^2 - \frac{1}{2}k\theta^2\right)^{-\frac{1}{2}}\mathrm{d}\theta$ with $\theta = \omega\sqrt{\frac{2}{k}}\sin\phi \Rightarrow \mathrm{d}\theta = \omega\sqrt{\frac{2}{k}}\cos\phi\,\mathrm{d}\phi$	M1*A1	M1 for separate and attempt to integrate by substitution or by using formula – if using formula then A1 for either $\sqrt{\frac{2}{k}}\sin^{-1}(\ldots)$ or $\sin^{-1}\left(\frac{\theta}{\omega}\sqrt{\frac{k}{2}}\right)$
			$t = \sqrt{\frac{2}{k}}\sin^{-1}\left(\frac{\theta}{\omega}\sqrt{\frac{k}{2}}\right) + c$	A1	Ignore lack of $+c$ for this mark
			$t = 0, \theta = 0 \Rightarrow c = 0$	B1	Dependent on previous three marks
				M1dep*	Attempt to re-arrange to make θ the subject
			$\theta = \omega\sqrt{\frac{2}{k}}\sin\left(t\sqrt{\frac{k}{2}}\right)$	A1	Dependent on previous B mark
				[8]	

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Unit level raw mark and UMS grade boundaries June 2018 series

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AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

AS & Advanced GCE Mathematics				Max Mark	a	b	c	d	e	u
4721	01	C1 Core mathematics 1 (AS)	Raw	72	61	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4722	01	C2 Core mathematics 2 (AS)	Raw	72	55	49	43	37	31	0
			UMS	100	80	70	60	50	40	0
4723	01	C3 Core mathematics 3 (A2)	Raw	72	55	48	41	34	28	0
			UMS	100	80	70	60	50	40	0
4724	01	C4 Core mathematics 4 (A2)	Raw	72	54	47	40	34	28	0
			UMS	100	80	70	60	50	40	0
4725	01	FP1 Further pure mathematics 1 (AS)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4726	01	FP2 Further pure mathematics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4727	01	FP3 Further pure mathematics 3 (A2)	Raw	72	47	41	36	31	26	0
			UMS	100	80	70	60	50	40	0
4728	01	M1 Mechanics 1 (AS)	Raw	72	60	51	42	34	26	0
			UMS	100	80	70	60	50	40	0
4729	01	M2 Mechanics 2 (A2)	Raw	72	53	46	39	32	26	0
			UMS	100	80	70	60	50	40	0
4730	01	M3 Mechanics 3 (A2)	Raw	72	50	42	34	27	20	0
			UMS	100	80	70	60	50	40	0
4731	01	M4 Mechanics 4 (A2)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4732	01	S1 – Probability and statistics 1 (AS)	Raw	72	57	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4733	01	S2 – Probability and statistics 2 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4734	01	S3 – Probability and statistics 3 (A2)	Raw	72	59	50	41	32	24	0
			UMS	100	80	70	60	50	40	0
4735	01	S4 – Probability and statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4736	01	D1 – Decision mathematics 1 (AS)	Raw	72	55	48	42	36	30	0
			UMS	100	80	70	60	50	40	0
4737	01	D2 – Decision mathematics 2 (A2)	Raw	72	58	53	48	44	40	0
			UMS	100	80	70	60	50	40	0

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AS & Advanced GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – Introduction to advanced mathematics (AS)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – Concepts for advanced mathematics (AS)	Raw	72	53	47	41	36	31	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) Methods for Advanced Mathematics (A2): Written Paper	Raw	72	61	56	51	46	40	0
4753	02	(C3) Methods for Advanced Mathematics (A2): Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) Methods for Advanced Mathematics (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – Applications of advanced mathematics (A2)	Raw	90	63	56	49	43	37	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – Further concepts for advanced mathematics (AS)	Raw	72	55	51	47	43	40	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – Further methods for advanced mathematics (A2)	Raw	72	48	42	36	31	26	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – Further applications of advanced mathematics (A2)	Raw	72	63	56	49	42	35	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) Differential Equations (A2): Written Paper	Raw	72	61	54	48	42	35	0
4758	02	(DE) Differential Equations (A2): Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) Differential Equations (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – Mechanics 1 (AS)	Raw	72	51	44	37	31	25	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – Mechanics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – Mechanics 3 (A2)	Raw	72	61	54	48	42	36	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – Mechanics 4 (A2)	Raw	72	59	51	44	37	30	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – Statistics 1 (AS)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – Statistics 2 (A2)	Raw	72	54	47	41	35	29	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – Statistics 3 (A2)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – Decision mathematics 1 (AS)	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – Decision mathematics 2 (A2)	Raw	72	55	51	47	43	39	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) Numerical Methods (AS): Written Paper	Raw	72	57	52	48	44	39	0
4776	02	(NM) Numerical Methods (AS): Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) Numerical Methods (AS): Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

AS GCE Statistics (MEI)

				Max Mark	a	b	c	d	e	u
G241	01	Statistics 1 MEI	Raw	72		No entry in June 2018				
			UMS	100	80	70	60	50	40	0
G242	01	Statistics 2 MEI	Raw	72		No entry in June 2018				
			UMS	100	80	70	60	50	40	0
G243	01	Statistics 3 MEI	Raw	72		No entry in June 2018				
			UMS	100	80	70	60	50	40	0

AS GCE Quantitative Methods (MEI)

				Max Mark	a	b	c	d	e	u
G244	01	Introduction to Quantitative Methods (Written Paper)	Raw	72	58	50	43	36	28	0
G244	02	Introduction to Quantitative Methods (Coursework)	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
G245	01	Statistics 1	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
G246	01	Decision Mathematics 1	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0

Level 3 Certificate, Level 3 Extended Project and FSMQ raw mark grade boundaries June 2018 series

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Level 3 Certificate Mathematics - Quantitative Methods (MEI)

					Max Mark	a	b	c	d	e	u
G244	A	01	Introduction to Quantitative Methods with Coursework (Written Paper)	Raw	72	58	50	43	36	28	0
G244	A	02	Introduction to Quantitative Methods with Coursework (Coursework)	Raw	18	14	12	10	8	7	0
				UMS	100	80	70	60	50	40	0
				Overall	90	72	62	53	44	35	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI)

				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H866	02	Critical maths	Raw	60	44	39	34	29	24	0
		*To create the overall boundaries, component 02 is weighted to give marks out of 72	Overall	144	109	96	83	70	57	0

Level 3 Certificate Mathematics - Quantitative Problem Solving (MEI)

				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H867	02	Statistical problem solving	Raw	60	40	36	32	28	24	0
		*To create the overall boundaries, component 02 is weighted to give marks out of 72	Overall	144	104	92	80	69	57	0

Advanced Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	56	50	44	38	33	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0