

**ADVANCED GCE
MATHEMATICS (MEI)**

Mechanics 2

MONDAY 16 JUNE 2008

4762/01

Afternoon
Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):

- Answer Booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

- 1 (a) Disc A of mass 6 kg and disc B of mass 0.5 kg are moving in the same straight line. The relative positions of the discs and the \mathbf{i} direction are shown in Fig. 1.1.

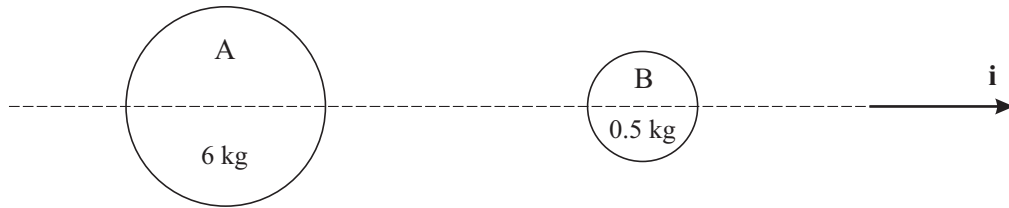


Fig. 1.1

The discs collide directly. The impulse on A in the collision is $-12\mathbf{i}$ N s and after the collision A has velocity $3\mathbf{i}$ m s⁻¹ and B has velocity $11\mathbf{i}$ m s⁻¹.

- (i) Show that the velocity of A just before the collision is $5\mathbf{i}$ m s⁻¹ and find the velocity of B at this time. [5]
- (ii) Calculate the coefficient of restitution in the collision. [3]
- (iii) After the collision, a force of $-2\mathbf{i}$ N acts on B for 7 seconds. Find the velocity of B after this time. [4]
- (b) A ball bounces off a smooth plane. The angles its path makes with the plane before and after the impact are α and β , as shown in Fig. 1.2.

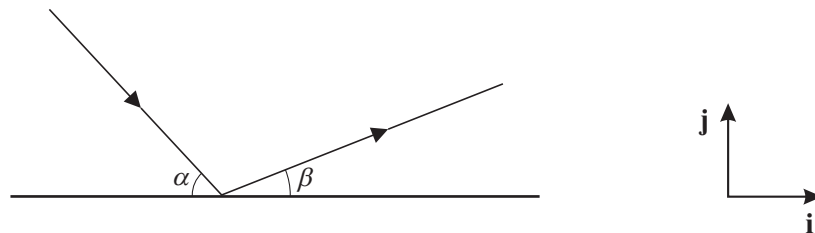


Fig. 1.2

The velocity of the ball before the impact is $u\mathbf{i} - v\mathbf{j}$ and the coefficient of restitution in the impact is e .

Write down an expression in terms of u , v , e , \mathbf{i} and \mathbf{j} for the velocity of the ball immediately after the impact. Hence show that $\tan \beta = e \tan \alpha$. [5]

2

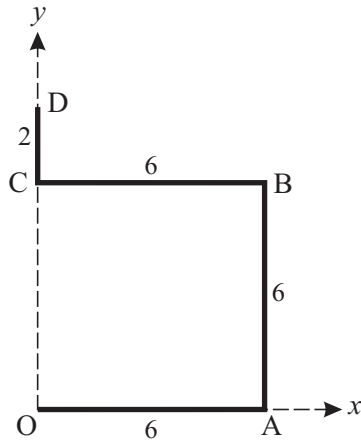


Fig. 2.1

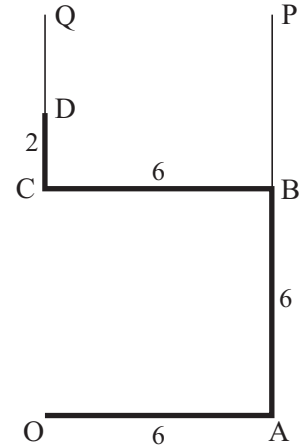


Fig. 2.2

A uniform wire is bent to form a bracket OABCD. The sections OA, AB and BC lie on three sides of a square and CD is parallel to AB. This is shown in Fig. 2.1 where the dimensions, in centimetres, are also given.

- (i) Show that, referred to the axes shown in Fig. 2.1, the x -coordinate of the centre of mass of the bracket is 3.6. Find also the y -coordinate of its centre of mass. [6]

- (ii) The bracket is now freely suspended from D and hangs in equilibrium.

Draw a diagram showing the position of the centre of mass and calculate the angle of CD to the vertical. [5]

- (iii) The bracket is now hung by means of vertical, light strings BP and DQ attached to B and to D, as shown in Fig. 2.2. The bracket has weight 5 N and is in equilibrium with OA horizontal.

Calculate the tensions in the strings BP and DQ. [4]

The original bracket shown in Fig. 2.1 is now changed by adding the section OE, where AOE is a straight line. This section is made of the same type of wire and has length L cm, as shown in Fig. 2.3.

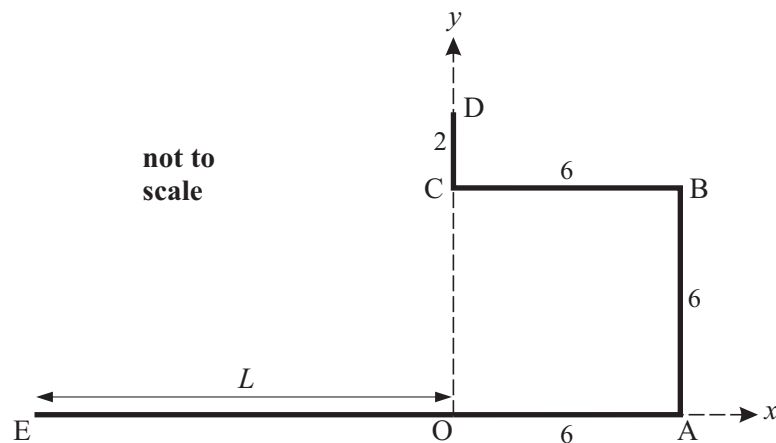


Fig. 2.3

The value of L is chosen so that the centre of mass is now on the y -axis.

- (iv) Calculate L . [4]

- 3 (a) Fig. 3.1 shows a framework in a vertical plane constructed of light, rigid rods AB, BC, AD and BD. The rods are freely pin-jointed to each other at A, B and D and to a vertical wall at C and D. There are vertical loads of $L\text{ N}$ at A and $3L\text{ N}$ at B. Angle DAB is 30° , angle DBC is 60° and ABC is a straight, horizontal line.

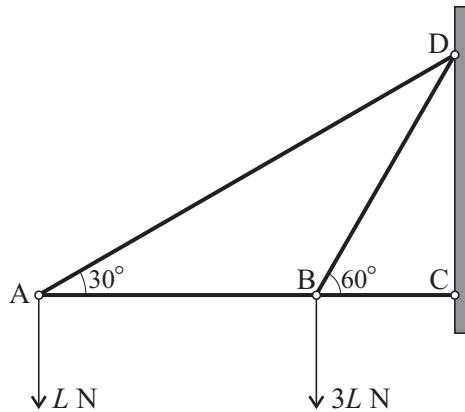


Fig. 3.1

- (i) Draw a diagram showing the loads and the internal forces in the four rods. [2]
- (ii) Find the internal forces in the rods in terms of L , stating whether each rod is in tension or in thrust (compression). [You may leave answers in surd form. Note that you are **not** required to find the external forces acting at C and at D.] [9]
- (b) Fig. 3.2 shows uniform beams PQ and QR, each of length $2l\text{ m}$ and of weight $W\text{ N}$. The beams are freely hinged at Q and are in equilibrium on a rough horizontal surface when inclined at 60° to the horizontal. You are given that the total force acting at Q on QR due to the hinge is horizontal. This force, $U\text{ N}$, is shown in Fig. 3.3.

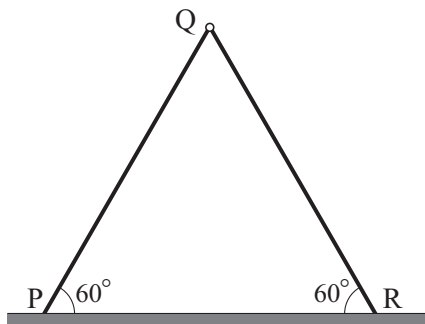


Fig. 3.2

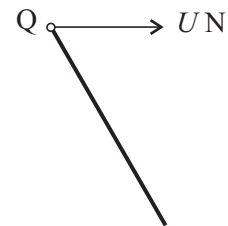


Fig. 3.3

Show that the frictional force between the floor and each beam is $\frac{\sqrt{3}}{6}W\text{ N}$. [7]

4 (a)

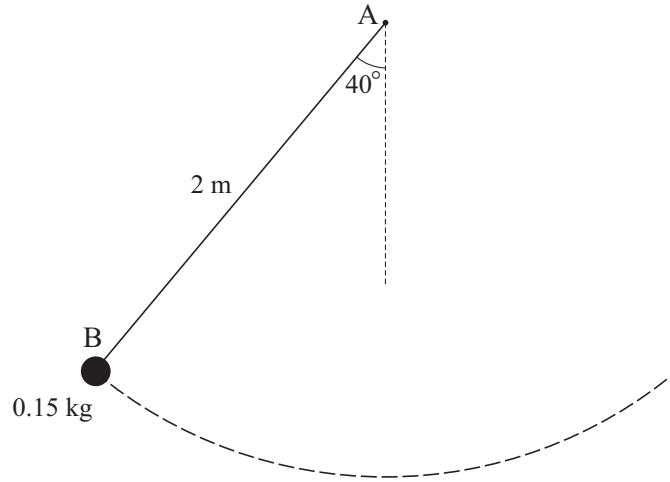


Fig. 4

A small sphere of mass 0.15 kg is attached to one end, B, of a light, inextensible piece of fishing line of length 2 m. The other end of the line, A, is fixed and the line can swing freely.

The sphere swings with the line taut from a point where the line is at an angle of 40° with the vertical, as shown in Fig. 4.

- (i) Explain why no work is done on the sphere by the tension in the line. [1]
- (ii) Show that the sphere has dropped a vertical distance of about 0.4679 m when it is at the lowest point of its swing and calculate the amount of gravitational potential energy lost when it is at this point. [4]
- (iii) Assuming that there is no air resistance and that the sphere swings from rest from the position shown in Fig. 4, calculate the speed of the sphere at the lowest point of its swing. [2]
- (iv) Now consider the case where
- there is a force opposing the motion that results in an energy loss of 0.6 J for every metre travelled by the sphere,
 - the sphere is given an initial speed of 2.5 m s^{-1} (and it is descending) with AB at 40° to the vertical.

Calculate the speed of the sphere at the lowest point of its swing. [5]

- (b) A block of mass 3 kg slides down a uniform, rough slope that is at an angle of 30° to the horizontal. The acceleration of the block is $\frac{1}{8}g$.

Show that the coefficient of friction between the block and the slope is $\frac{1}{4}\sqrt{3}$. [6]

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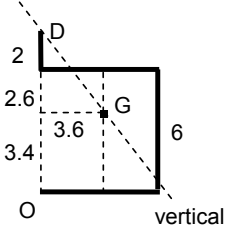
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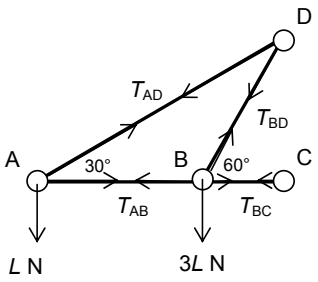
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4762 Mechanics 2

Q 1	mark	comment	sub
(a) (i) In i direction: $6u - 12 = 18$ so $u = 5$ i.e. $5\mathbf{i} \text{ m s}^{-1}$ either In i direction: $0.5v + 12 = 0.5 \times 11$ $v = -13$ so $-13\mathbf{i} \text{ m s}^{-1}$ or $6 \times 5 + 0.5v = 6 \times 3 + 0.5 \times 11$ $v = -13$ so $-13\mathbf{i} \text{ m s}^{-1}$	M1 E1 M1 B1 A1 M1 A1 A1	Use of I-M Accept $6u - 12 = 18$ as total working. Accept 5 instead of $5\mathbf{i}$. Use of I-M Use of + $12\mathbf{i}$ or equivalent Accept direction indicated by any means PCLM Allow only sign errors Accept direction indicated by any means	5
(ii) Using NEL: $\frac{11-3}{-13-5} = -e$ $e = \frac{4}{9}$ (0.4)	M1 F1 F1	Use of NEL. Condone sign errors but not reciprocal expression FT only their -13 (even if +ve) FT only their -13 and only if $-ve$ (allow 1 s.f. accuracy)	3
(iii) In i direction: $-2 \times 7 = 0.5v - 0.5 \times 11$ $v = -17$ so $-17\mathbf{i} \text{ m s}^{-1}$ or $-2\mathbf{i} = 0.5\mathbf{a}$ so $\mathbf{a} = -4\mathbf{i} \text{ m s}^{-2}$ $v = 11\mathbf{i} - 4\mathbf{i} \times 7$ $v = -17$ so $-17\mathbf{i} \text{ m s}^{-1}$	M1 M1 A1 A1 M1 A1 M1 A1	Use of $\mathbf{I} = \mathbf{F}t$ Use of $\mathbf{I} = m(\mathbf{v} - \mathbf{u})$ For ± 17 cao. Direction (indicated by any means) Use of $\mathbf{F} = m\mathbf{a}$ For ± 4 Use of $\mathbf{u} = \mathbf{v}t$ cao. Direction (indicated by any means)	4
(b) $u\mathbf{i} + ev\mathbf{j}$ $\tan \alpha = \frac{v}{u}, \tan \beta = \frac{ev}{u}$ $\tan \beta = e \left(\frac{v}{u} \right) = e \tan \alpha$	B1 B1 M1 B1 E1	For u For ev Use of tan. Accept reciprocal argument. Accept use of their components Both correct. Ignore signs. Shown. Accept signs not clearly dealt with.	5
	17		

Q 2		mark	comment	sub
(i)	$(2+3\times 6)\left(\frac{\bar{x}}{\bar{y}}\right)=6\binom{3}{0}+6\binom{6}{3}+6\binom{3}{6}+2\binom{0}{7}$ $20\left(\frac{\bar{x}}{\bar{y}}\right)=\binom{18+36+18}{18+36+14}=\binom{72}{68}$ $\bar{x}=3.6$ $\bar{y}=3.4$	M1 B1 B1 B1 E1 A1	Method for c.m. Total mass correct For any of the 1 st 3 RHS terms For the 4 th RHS term cao [If separate cpts, award the 2 nd B1 for 2 x- terms correct and 3 rd B1 for 2×7 in y term]	6
(ii)	 $\arctan\left(\frac{3.6}{2+(6-3.4)}\right)=\arctan\left(\frac{3.6}{4.6}\right)$ <p>so 38.047... so 38.0° (3 s. f.)</p>	B1 B1 M1 B1 A1	Diagram showing G vertically below D 3.6 and their 3.4 correctly placed (may be implied) Use of arctan on their lengths. Allow reciprocal of argument. Some attempt to calculate correct lengths needed 2 + (6 – their 3.4) seen cao	5
(iii)	moments about D $5\times 3.6=6\times T_{BP}$ so tension in BP is 3 N Resolve vert: $3+T_{DQ}=5$ so tension in DQ is 2 N	M1 F1 M1 F1	moments about D. No extra forces FT their values if calc 2nd Resolve vertically or moments about B. FT their values if calc 2nd	4
(iv)	We require x-cpt of c.m. to be zero either $(20+L)\bar{x}=20\times 3.6-\frac{1}{2}L^2$ or $2\times 6\times(0.5\times 6)+6\times 6-0.5\times L^2=0$ $L=12$	M1 B1 A1 A1	A method to achieve this with all cpts For the $0.5\times L^2$ All correct	4
		19		

Q 3		mark	comment	sub
(a) (i)		B1 B1	Internal forces all present and labelled All forces correct with labels and arrows (Allow the internal forces set as tensions, thrusts or a mixture)	2
(ii)	<p>A \uparrow $T_{AD} \sin 30 - L = 0$ so $T_{AD} = 2L$ so $2L$ N (T)</p> <p>A \rightarrow $T_{AB} + T_{AD} \cos 30 = 0$ so $T_{AB} = -\sqrt{3}L$ so $\sqrt{3}L$ N (C)</p> <p>B \uparrow $T_{BD} \sin 60 - 3L = 0$ so $T_{BD} = 2\sqrt{3}L$ so $2\sqrt{3}L$ N (T)</p> <p>B \rightarrow $T_{BC} + T_{BD} \cos 60 - T_{AB} = 0$ so $T_{BC} = -2\sqrt{3}L$ so $2\sqrt{3}L$ N (C)</p>	M1 A1 M1 F1 M1 A1 M1 F1 E1	Equilibrium equation at a pin-joint attempted 1 st ans. Accept + or -. Second equation attempted 2 nd ans. FT any previous answer(s) used. Third equation attempted 3 rd ans. FT any previous answer(s) used. Fourth equation attempted 4 th ans. FT any previous answer(s) used. All T/C consistent [SC 1 all T/C correct WWW]	9
(b)	<p>Leg QR with frictional force $F \leftarrow$ moments c.w. about R $U \times 2l \sin 60 - Wl \cos 60 = 0$</p> <p>Horiz equilibrium for QR $F = U$</p> <p>Hence $\frac{1}{2}W = \sqrt{3}F$ and so $F = \frac{\sqrt{3}}{6}W$</p>	M1 A1 A1 M1 E1 M1 E1	Accept only 1 leg considered (and without comment) Suitable moments equation. Allow 1 force omitted a.c. moments c.w. moments A second correct equation for horizontal or vertical equilibrium to eliminate a force (U or reaction at foot) [Award if correct moments equation containing only W and F] * This second equation explicitly derived Correct use of 2 nd equation with the moments equation Shown. CWO but do not penalise * again.	7
		18		

Q 4	mark	comment	sub
(a) (i) Tension is perp to the motion of the sphere (so WD, $Fd \cos \theta = 0$)	E1		1
(ii) Distance dropped is $2 - 2 \cos 40 = 0.467911..$ GPE is mgh so $0.15 \times 9.8 \times 0.467911... = 0.687829... \text{ J}$	M1 E1 M1 B1	Attempt at distance with resolution used. Accept $\sin \leftrightarrow \cos$ Accept seeing $2 - 2 \cos 40$ Any reasonable accuracy	4
(iii) $0.5 \times 0.15 \times v^2 = 0.687829...$ so $v = 3.02837... \text{ so } 3.03 \text{ m s}^{-1} \text{ (3 s. f.)}$	M1 F1	Using KE + GPE constant FT their GPE	2
(iv) $\frac{1}{2} \times 0.15 (v^2 - 2.5^2)$ $= 0.687829... - 0.6 \times \frac{40}{360} \times 2\pi \times 2$ $v = 2.06178... \text{ so } 2.06 \text{ m s}^{-1} \text{ (3 s. f.)}$	M1 B1 M1 A1 A1	Use of W-E equation (allow 1 KE term or GPE term omitted) KE terms correct WD against friction WD against friction correct (allow sign error) cao	5
(b) N2L down slope: $3g \sin 30 - F = 3 \times \frac{1}{8}g$ so $F = \frac{9g}{8} \text{ (= 11.025)}$ $R = 3g \times \frac{\sqrt{3}}{2} \text{ (= 25.4611...)}$ $\mu = \frac{F}{R} = \frac{\sqrt{3}}{4} \text{ (= 0.43301...)}$	M1 A1 A1 B1 M1 E1	Must have attempt at weight component Allow sign errors. Use of $F = \mu R$ Must be worked precisely	6
	18		

4762 Mechanics 2

General Comments

Many good scripts were seen in response to this paper with the majority of candidates able to make some progress worthy of credit on every question. One disappointing aspect was the poor algebraic manipulation of equations and the inability of a large number of candidates to manipulate surds. The majority of candidates seemed to understand the principles and could use them efficiently. However, some did not clearly identify the principle or process being used; as has happened in previous sessions, those parts of the questions that were least well done were those that required an explanation or interpretation of results or that required the candidate to show a given answer. Those candidates who drew good diagrams were generally more successful than those who didn't. The majority of candidates appeared to have no problem in finishing the paper but a small number gave the impression of having had to rush the last part of question 4.

Comments on Individual Questions

- 1 Many candidates obtained significant credit on this question, particularly in part a). However, those who failed to draw a diagram or failed to indicate the positive direction in some other way did not always include an adequate indication of direction in their answers.
- (a)(i) This part was well done by many candidates. However, only a minority remembered to indicate the direction of the velocity of B.
- (ii) This part posed few difficulties with the majority of candidates gaining full credit.
- (iii) Well done on the whole but the direction of B was not always clearly indicated.
- (b) Many candidates found this part difficult. Some failed to appreciate that they had been given the components of a velocity vector; tried resolving (which then led to complicated algebraic expressions that could not easily be simplified); others took u to be the initial velocity and v the final velocity and thus lost the thread of the argument because they were using the same notation to represent different things. It was common to see $\tan\alpha = \frac{v\mathbf{j}}{u\mathbf{i}}$ indicating a lack of understanding of vectors.
- 2) Many candidates scored highly on this question. They appeared to understand the principles and processes required and could structure their answers well.
- (i) The majority of candidates obtained full credit for this part of the question. Those who did not, but used column vector notation, generally made fewer errors than those who calculated the co-ordinates separately.
- (ii) It was pleasing to see many good responses to this part with almost all candidates able to draw a diagram showing the centre of mass vertically below D. However, some diagrams were too small to be helpful to the candidate; the lengths were not indicated clearly or correctly and so, some of the triangles used to find the required angle had little connection with either CD or the vertical.
- (iii) This part posed few difficulties.

Report on the Units taken in June 2008

- (iv) Most candidates approached this part well, appreciating that the x co-ordinate of the centre of mass had to be zero and then taking moments using their answer to part i). A small number of candidates went back to square one and attempted to find the moments of all of the individual elements again. Unfortunately, many who did this omitted one element.
- 3) This question was found hard by many candidates; some found the framework difficult, others had difficulty with part b). Few had difficulty with both parts.
- (a)(i) The quality of the diagrams offered for this part was poor. In many cases labelling was incomplete or omitted altogether and spurious reaction forces often appeared at A and B. Arrows were not always as clear as they could be as some candidates adjusted their diagram in the light of the answers they obtained for the next part of the question.
- (ii) The vast majority of candidates understood that they had to consider equilibrium at the pin joints to obtain the internal forces. However, there were many inconsistencies seen between the equations offered and the diagram drawn in (i). These inconsistencies were compounded by sign errors in solving the equations and, indeed, in following through. Algebraic manipulation of equations was poor, for instance many cases of $T \sin 30 = L$ leading to $T = \frac{1}{2} L$ were seen.
- (b) This part of the question required candidates to *show* a given answer. Those candidates who gained some credit for it usually drew a fully labelled and clear diagram showing all of the forces correctly, including the direction of the frictional force and the forces due to the hinge at Q. However, some candidates assumed that the weight of the structure (i.e. $2W$) acted at Q; others showed a weight W acting in the centre of the structure. In order to show the given result candidates had to consider one beam, take moments and resolve. Many chose to take moments about Q but forgot the normal reaction force at R. Full explanations of the processes and principles being employed were rare. On some scripts no consistent sign conventions were adopted and on others algebra was not used properly to establish the result.
- 4) Some excellent answers were seen to this question with few candidates attempting to use, inappropriately, the constant acceleration formulae and Newton's second law in part (a).
- (a)(i) Many candidates had difficulty with this part. Few could explain clearly that the tension was perpendicular to the motion of the sphere and hence did no work on it.
- (ii) This part posed few difficulties to the majority of candidates.
- (iii) Few had problems with this part.
- (iv) Most candidates could go some way to obtaining a solution to this part. The main errors were in the calculation of the work done, with some candidates not appreciating the circular nature of the motion. It was common to see 0.6×0.4769 used as the work done.
- (b) The majority of candidates could obtain some credit for this part. However, marks were lost in many cases because the candidates did not work precisely enough. They used decimals throughout but then stated the answer as a surd without any attempt to justify the agreement.