OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4762

Mechanics 2

Monday **19 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 6 printed pages and 2 blank pages.

1 **(a)** Two small spheres, P of mass 2 kg and Q of mass 6 kg, are moving in the same straight line along a smooth, horizontal plane with the velocities shown in Fig. 1.1.

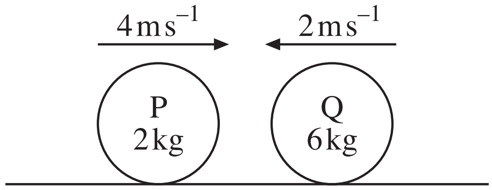

Fig. 1.1

Consider the direct collision of P and Q in the following two cases.

(i) The spheres coalesce on collision.

(A) Calculate the common velocity of the spheres after the collision. [3]

(B) Calculate the energy lost in the collision. [2]

(ii) The spheres rebound with a coefficient of restitution of $\frac{2}{3}$ in the collision.

(A) Calculate the velocities of P and Q after the collision. [6]

(B) Calculate the impulse on P in the collision. [2]

(b) A small ball bounces off a smooth, horizontal plane. The ball hits the plane with a speed of $26\,\mathrm{m\,s^{-1}}$ at an angle of $\arcsin \frac{12}{13}$ to it. The ball rebounds at an angle of $\arcsin \frac{3}{5}$ to the plane, as shown in Fig. 1.2.

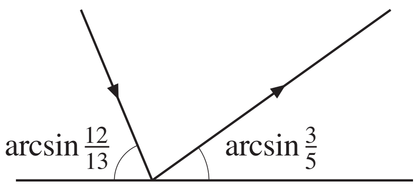

Fig. 1.2

Calculate the speed with which the ball rebounds from the plane.

Calculate also the coefficient of restitution in the impact. [6]

2 Two heavy rods AB and BC are freely jointed together at B and to a wall at A. AB has weight 90 N and centre of mass at P; BC has weight 75 N and centre of mass at Q. The lengths of the rods and the positions of P and Q are shown in Fig. 2.1, with the lengths in metres.

Initially, AB and BC are horizontal. There is a support at R, as shown in Fig. 2.1. The system is held in equilibrium by a vertical force acting at C.

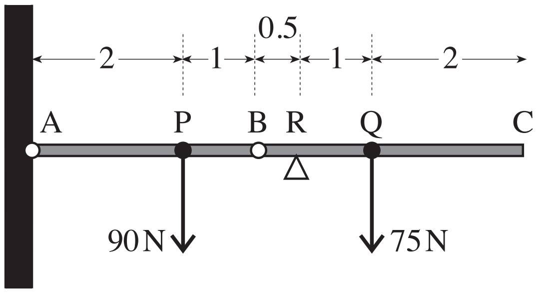

Fig. 2.1

(i) Draw diagrams showing all the forces acting on rod AB and on rod BC.

Calculate the force exerted on AB by the hinge at B and hence the force required at C. [6]

The rods are now set up as shown in Fig. 2.2. AB and BC are each inclined at 60° to the vertical and C rests on a rough horizontal table. Fig. 2.3 shows all the forces acting on AB, including the forces X N and Y N due to the hinge at A and the forces U N and V N in the hinge at B. The rods are in equilibrium.

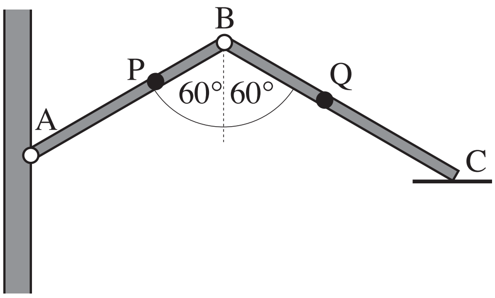

Fig. 2.2

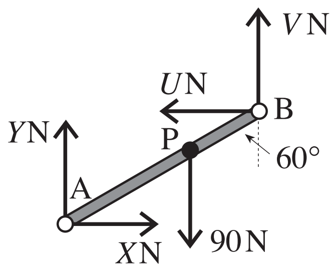

Fig. 2.3

(ii) By considering the equilibrium of rod AB, show that $60\sqrt{3} = U + V\sqrt{3}$. [3]

(iii) Draw a diagram showing all the forces acting on rod BC. [1]

(iv) Find a further equation connecting U and V and hence find their values. Find also the frictional force at C. [8]

3 **(a)** A car of mass 900 kg is travelling at a steady speed of $16\,\text{m s}^{-1}$ up a hill inclined at $\arcsin 0.1$ to the horizontal. The power required to do this is 20 kW.

Calculate the resistance to the motion of the car. [4]

(b) A small box of mass 11 kg is placed on a uniform rough slope inclined at $\arccos \frac{12}{13}$ to the horizontal. The coefficient of friction between the box and the slope is μ.

(i) Show that if the box stays at rest then $\mu \geqslant \frac{5}{12}$. [3]

For the remainder of this question, the box moves on a part of the slope where $\mu = 0.2$.

The box is projected up the slope from a point P with an initial speed of $v\,\text{m s}^{-1}$. It travels a distance of 1.5 m along the slope before coming instantaneously to rest. During this motion, the work done against air resistance is 6 joules per metre.

(ii) Calculate the value of v. [5]

As the box slides back down the slope, it passes through its point of projection P and later reaches its initial speed at a point Q. During this motion, once again the work done against air resistance is 6 joules per metre.

(iii) Calculate the distance PQ. [6]

[Question 4 is printed overleaf.]

4 Fig. 4.1 shows four uniform rods, OA, AB, BE and CD, rigidly fixed together to form a frame. The rods have weights proportional to their lengths and these lengths, in centimetres, are shown in Fig. 4.1.

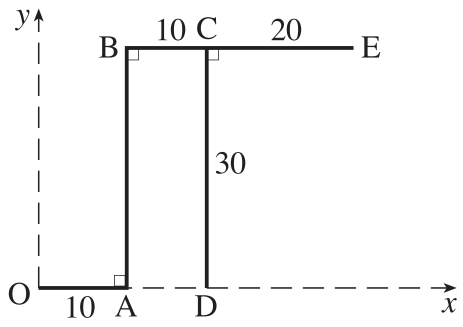

Fig. 4.1

(i) Calculate the coordinates of the centre of mass of the frame, referred to the axes shown in Fig. 4.1. [5]

The bracket shown in Fig. 4.2 is made of uniform sheet metal with cross-section the frame shown in Fig. 4.1. The bracket is 40 cm wide and its weight is 60 N. It stands on a horizontal plane containing Ox and Oz.

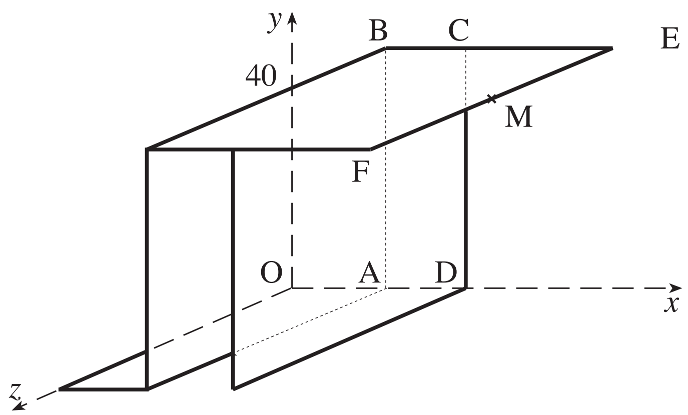

Fig. 4.2

(ii) Write down the coordinates of the centre of mass of the bracket, referred to the axes shown in Fig. 4.2. [2]

A force P N acts vertically downwards at the point M, shown in Fig. 4.2. M is the mid-point of EF. The bracket is on the point of tipping.

(iii) Calculate the value of P. [4]

In another situation, a horizontal force Q N acts through M parallel to EB and in the direction from E to B. The value of Q is increased from zero with the bracket in equilibrium at all times.

(iv) Draw a diagram showing the forces acting on the bracket when it is on the point of tipping. [1]

(v) If the limiting frictional force between the bracket and the plane is 30 N, does the bracket slide or tip first as Q is increased? [5]

Mark Scheme 4762
June 2006

Q 1		mark		Sub
(a) (i) (A)	PCLM $\rightarrow$ +ve $2\times4-6\times2=8v$ $v = -0.5$ so 0.5 m s^{-1} in opposite direction to initial motion of P	 M1 A1 A1	 Use of PCLM and correct mass on RHS Any form Direction must be negative and consistent or clear. Accept use of a diagram.	3
(B)	$0.5\times2\times4^2+0.5\times6\times2^2-0.5\times8\times(-0.5)^2$ = 27 J	M1 A1	Use of KE. Must sum initial terms. Must have correct masses FT **their** (A) only	2
(ii) (A)	PCLM $\rightarrow$ +ve $2\times4-6\times2=2v_P+6v_Q$ $v_P+3v_Q=-2$ NEL $\rightarrow$ +ve $\frac{v_Q-v_P}{-2-4}=-\frac{2}{3}$ $v_Q-v_P=4$ $v_Q=0.5$ so 0.5 m s^{-1} in orig direction of P $v_P=-3.5$ so 3.5 m s^{-1} in opp to orig dir of P	 M1 A1 M1 A1 A1 A1	 Use of PCLM Any form NEL Any form cao. Direction need not be made clear. cao. Direction must be negative and consistent or clear (e.g diag)	6
(B)	$\rightarrow$ +ve $2\times-3.5-2\times4=-15$ N s so 15 N s in opp to orig direction	 M1 A1	 Use of change in momentum with correct mass. FT (A). Dir must be clear (e.g. diag)	2
(b)	Let $\alpha=\arcsin(12/13)$ and $\beta=\arcsin(3/5)$ Parallel: $26\cos\alpha=u\cos\beta$ so $26\times\frac{5}{13}=u\times\frac{4}{5}$ and u = 12.5 Perp: $e=\frac{u\sin\beta}{26\sin\alpha}$ $=\frac{12.5\times\frac{3}{5}}{26\times\frac{12}{13}}=\frac{5}{16}$	 M1 A1 A1 M1 F1 F1	 PCLM parallel to plane attempted. At least one resolution correct NEL on normal components attempted. FT **their** u FT **their** u	6
				19

Q 2		mark		Sub
(i)	Diagrams	B1	Internal force at B must be shown	
	cw moments about A			
	$2 \times 90 - 3R_{B} = 0$	M1	1st moments equation attempted for either force.	
	$R_{B} = 60$ so 60 N upwards	A1	Accept direction not specified	
	cw moments about R: $T \downarrow$			
	$75 \times 1 + 3T - 60 \times 0.5 = 0$	M1	2nd moments equation for other force. All forces present. No extra forces.	
		A1	Allow only sign errors	
	$T = -15$ so 15 N upwards	A1	Direction must be clear (accept diag)	6
(ii)	cw moments about A			
	$90 \times 2\cos 30 - V \times 3\cos 30 - U \times 3\cos 60 = 0$	M1	Moments equation with resolution. Accept terms missing	
		A1	All correct. Allow only sign errors.	
	giving $60\sqrt{3} = U + V\sqrt{3}$	E1	Clearly shown	3
(iii)	Diagram	B1	*U* and *V* correct with labels and arrows	1
(iv)	ac moments about C			
	$75 \times 2\cos 30 + 3.5V\cos 30 - 3.5U\cos 60 = 0$	M1	Moments equation with resolution. Accept term missing	
		B1	At least two terms correct (condone wrong signs)	
	$\frac{300}{7}\sqrt{3} = U - V\sqrt{3}$	A1	Accept any form	
	Solving for *U* and *V*	M1	Any method to eliminate one variable	
	$U = \frac{360\sqrt{3}}{7}$ (= 89.0768…)	A1	Accept any form and any reasonable accuracy	
	$V = \frac{60}{7}$ (= 8.571428…)	F1	Accept any form and any reasonable accuracy	
			[Either of *U* and *V* is cao. FT the other]	
	Resolve $\rightarrow$ on BC			
	F = *U*	M1		
	so frictional force is $\frac{360\sqrt{3}}{7}$ N	F1		
	(= 89.1 N (3 s. f.))			8
				18

Q 3		mark		Sub
(a)	$20000 = (R + 900g \times 0.1) \times 16$ R = 368 so 368 N	M1 B1 A1 A1	Use of $P = Fv$, may be implied. Correct weight term All correct	4
(b) (i)	$F_{max} = \mu mg \cos\alpha$ Force down slope is weight cpt $mg\sin\alpha$ Require $\mu mg\cos\alpha \geq mg\sin\alpha$ so $\mu \geq \tan\alpha = \frac{5}{12}$	B1 B1 E1	Correct expression for F_{max} **or** wt cpt down slope (may be implied and in any form) Identifying $\sin\alpha$ as $5/13$ or equivalent Proper use of $F \leq \mu R$ or equivalent. [$\mu = \tan\alpha$ used WW; SC1]	3
(ii)	**either** $0.5 \times 11 \times v^2$ $= 11g \times 1.5 \times \frac{5}{13} + 0.2 \times 11g \times 1.5 \times \frac{12}{13} + 9$ $v^2 = 18.3717...$ v = 4.2862… so 4.29 m s^{-1} (3 s. f.) **or** + ve up the slope $-11g \times \frac{5}{13} - 0.2 \times 11g \times \frac{12}{13} - 6 = 11a$ a = - 6.1239 m s^{-2} v^2 = - 3a v = 4.286 m s^{-1}	M1 B1 B1 A1 A1 M1 B1 A1 M1 A1	Use of work energy with at least three required terms attempted Any term RHS. Condone sign error. Another term RHS. Condone sign error. All correct . Allow if trig consistent but wrong cao Use of N2L Any correct term on LHS use of appropriate *uvast* c.a.o.	5
(iii)	**continued overleaf**			

3	**continued**			
(iii)	**either** Extra GPE balances WD against resistances	M1	Or equivalent	
	$mgx\sin\alpha$	B1		
	$= 6(x+3) + 0.2 \times 11g \times \cos\alpha(x+3)$	B1	One of 1st three terms on RHS correct	
		B1	Another of 1st 3 terms on RHS correct	
		A1	All correct. FT **their** *v* if used.	
	x = 4.99386… so 4.99 m (3 s. f.)	A1	cao.	6
	or	M1	Allow 1 term missing	
	$0.5 \times 11 \times 18.3717...$	B1	KE. FT **their** *v*	
	$= (1.5+x) \times 11g \times \frac{5}{13} - 6(1.5+x)$	B1	Use of 1.5 + *x* (may be below)	
	$-(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$	B1	WD against friction	
		A1	All correct	
	x = 4.99386… so 4.99 m (3 s. f.)	A1	cao.	
	or + ve down the slope			
	$11g \times {}^{5}/_{13} - 0.2 \times 11g \times {}^{12}/_{13} - 6 = 11a$	M1	N2L with all terms present	
		A1	all correct except condone sign errors	
	$a = 1.4145...$ m s^{-2}	**A1**		
	$4.286^2 = 2a(1.5+x)$	**M1**	use of appropriate *uvast*	
		B1	for (1.5 + *x*) (may be implied)	
	x = 4.99	A1	c.a.o.	
				18

Q 4		mark		Sub
(i)	$100\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=10\begin{pmatrix}5\\0\end{pmatrix}+30\begin{pmatrix}10\\15\end{pmatrix}+30\begin{pmatrix}20\\15\end{pmatrix}+30\begin{pmatrix}25\\30\end{pmatrix}$	M1	Correct method for c.m.	
		B1 B1	Total mass correct One c.m. on RHS correct [If separate components considered, B1 for 2 correct]	
	$100\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}1700\\1800\end{pmatrix}$ $\bar{x}=17$ $\bar{y}=18$	A1 A1	cao cao. [Allow SC 4/5 for $\bar{x}=18$ and $\bar{y}=17$]	5
(ii)	$(17,18,20)$	B1 B1	*x*- and *y*- coordinates. FT from (i). *z* coordinate	2
(iii)	cw moments about horizontal edge thro' D *x* component $P\times 20-60\times(20-17)=0$ *P* = 9	M1 B1 B1 A1	Or equivalent with all forces present One moment correct (accept use of mass or length) correct use of **their** $\bar{x}$ in a distance FT only **their** $\bar{x}$	4
(iv)	Diagram	B1	Normal reaction must be indicated acting vertically upwards at edge on O*z* and weight be in approximately the correct place.	1
(v)	On point of toppling ac moments about edge along O*z* $30\times Q-60\times 17=0$ *Q* = 34 Resolving horizontally *F* = *Q* As 34 > 30, slips first	M1 B1 F1 B1 B1	Or equivalent with all forces present Any moment correct (accept use of mass or length) FT only **their** $\bar{x}$ FT **their** Q correctly argued.	5
				17

4762 - Mechanics 2

General Comments

Many candidates could make some progress with at least some part of every question and gain some credit for their work. There were many candidates who had more difficulty with questions 2 and/or 3 than with questions 1 or 4. The standard of presentation of most scripts was pleasing but, as has happened in other sessions, in some cases diagrams were too poor to be useful. Many candidates do not appreciate the value of a diagram in finding a solution or in conveying to an examiner information relevant to the answer. Labelling of forces should be clear and unambiguous and the diagram large enough for the information to be clearly shown. Candidates who state in a solution which process or principle is being employed tend to be more successful than those who omit this information – stating the principle being employed seems to help the candidate to use it properly.
The standard of algebraic manipulation was, for some candidates, so low as to make it hard for the candidate to demonstrate knowledge of mechanics; this was seen particularly in the attempts to solve simultaneous equations or to rearrange formulae. Efforts were not helped by the use of some clumsy notation with regard to trigonometric functions. e.g. it was common to see expressions such as $\cos(\arcsin {}^{12}\!/_{13})$ with no attempt being made to simplify this.

Comments on Individual Questions

1 Impulse and Momentum

Many candidates gained good marks on this question. The main cause of error was in failure to indicate clearly the direction of the vector answer.

(a) (i) (A) This part posed few problems for the majority of candidates.

(B) Many completely correct solutions were seen to this part. Errors were mainly arithmetic.

(ii) (A) It was pleasing to see many completely correct solutions to this part. Candidates who did not draw a fully labelled diagram were less successful than those who did. Without a diagram, the sign convention was not always as clear as it had to be and errors occurred, usually in the application of Newton's experimental law.

(B) Only the most able (identified by their success on the paper overall) gained full credit for this part. Many candidates failed to indicate direction and a few thought that impulse was equal to change in kinetic energy.

(b) A large number of candidates did well on this part of the question. Most appreciated that there would be no change in speed parallel to the plane and could hence calculate the speed of the rebound. Almost as many could find the coefficient of restitution either by analysing the motion perpendicular to the plane and applying Newton's experimental law or by using $\tan\beta = e \tan\alpha$.

2 Moments and Resolving

Unfortunately many candidates did not apply Newton's third law properly, hence they could not gain the credit for the diagrams required in part (i) and part (iii). However, the majority of candidates understood that moments were required along with some resolution of forces and could gain at least some credit in the other parts of the question.

(i) Many candidates could correctly calculate the force exerted on AB by the hinge at B and could go on to use this to attempt to evaluate the force at C. Of those candidates who failed to obtain the correct answer those who tackled the problem by looking at rods AB and BC separately tended to gain more credit than those who treated AC initially as a single rigid rod.

(ii) The majority of candidates encountered little difficulty with this part.

(iii) Many candidates applied Newton's third law correctly in the horizontal direction giving U in the opposite direction to the diagram for rod AB but failed to do so in the vertical direction and drew a diagram in which V acted vertically upwards as in the diagram for rod AB.

(iv) A large number of candidates made a good attempt at setting up a second equation in U and V and then went on to try to find a solution to the simultaneous equations. The majority of these then correctly deduced the size of the frictional force.

3 Work and Energy

It was very satisfying to see that the majority of candidates tried to use work-energy methods throughout this question and did so with some success. Correct solutions were not obtained in some cases because of the absence of a diagram or the omission of an indication about the sign convention adopted.

(a) Almost all of the candidates appreciated the need to use $P = Fv$ and very many of them obtained full credit for that part.

(b) (i) The amount of explanation required to establish the given result was not appreciated by the majority of candidates. It was common to see $\mu = 5/12$ hence $\mu \geq 5/12$ without any supporting reasoning. Persistent use of the notation $\sin(\arccos 12/13)$ without showing or stating that this was $5/13$ meant that many candidates presented an incomplete argument in support of their answers. Very few candidates mentioned the maximum value of the frictional force either implicitly or explicitly and some used rounded decimal fractions throughout and then claimed this was $5/12$.

(ii) Many good attempts were seen to this part of the question. Errors usually involved one of the terms in the work-energy equation. Those candidates who used Newton's second law and the constant acceleration formulae were not usually as successful as those using the work- energy principle.

(iii) Again, some attempts worthy of significant credit were seen for this part, with errors of a similar type to those in part (ii). Candidates who made their method clear usually gained more credit than those who attempted to write down single terms independently and then use these in an equation. Sign errors were common.

4 Centres of Mass

This question was attempted well by almost all candidates with a large number of candidates gaining significant credit. Some excellent solutions were seen.

(i) Most candidates had little or no difficulty with this part.

(ii) Few encountered problems with this part.

(iii) The majority of candidates could obtain some credit for this part with errors being mainly arithmetic. A few candidates attempted to take moments about the edge through O rather than the edge through D but forgot to include the term containing the reaction at the edge through D.

(iv) Many of the diagrams offered here were poor with forces either omitted or unlabelled. A large number of candidates did not appreciate that the reaction would act vertically upwards at some point on the edge through Oz but showed it acting along the edge through A. Others omitted it altogether.

(v) Errors in this part tended to be only arithmetic and it was pleasing to see that many candidates could produce a coherent argument to support their conclusion.