Friday 16 May 2014 – Afternoon

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Use standard series formulae to find $\sum_{r=1}^{n} r(r-2)$, factorising your answer as far as possible. **[5]**

2 Fig. 2 shows the unit square, OABC, and its image, OA′B′C′, after undergoing a transformation.

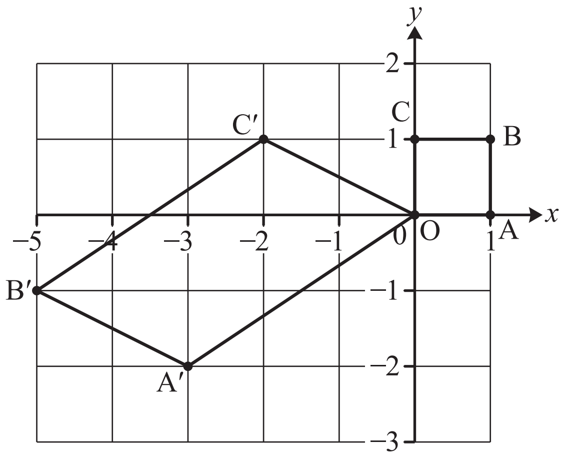

Fig. 2

(i) Write down the matrix **T** representing this transformation. **[2]**

The quadrilateral OA′B′C′ is reflected in the x-axis to give a new quadrilateral, OA″B″C″.

(ii) Write down the matrix representing reflection in the x-axis. **[1]**

(iii) Find the single matrix that will transform OABC onto OA″B″C″. **[2]**

3 You are given that $z = 2+3j$ is a root of the quartic equation $z^4 - 5z^3 + 15z^2 - 5z - 26 = 0$. Find the other roots. **[7]**

4 Use the identity $\frac{1}{2r+3} - \frac{1}{2r+5} \equiv \frac{2}{(2r+3)(2r+5)}$ and the method of differences to find $\sum_{r=1}^{n} \frac{1}{(2r+3)(2r+5)}$, expressing your answer as a single fraction. **[5]**

5 The roots of the cubic equation $3x^3 - 9x^2 + x - 1 = 0$ are α, β and γ. Find the cubic equation whose roots are $3\alpha - 1$, $3\beta - 1$ and $3\gamma - 1$, expressing your answer in a form with integer coefficients. **[7]**

6 Prove by induction that $\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$. **[7]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{x^2 - 5}{(x+3)(x-2)(ax-1)}$, where a is a constant.

(i) Find the coordinates of the points where the curve crosses the x-axis and the y-axis. **[2]**

(ii) You are given that the curve has a vertical asymptote at $x = \dfrac{1}{2}$. Write down the value of a and the equations of the other asymptotes. **[3]**

(iii) Sketch the curve. **[4]**

(iv) Find the set of values of x for which $y > 0$. **[3]**

8 You are given the complex number $w = 2 + 2\sqrt{3}\mathrm{j}$.

(i) Express w in modulus-argument form. **[3]**

(ii) Indicate on an Argand diagram the set of points, z, which satisfy both of the following inequalities.

$$-\frac{\pi}{2} \leqslant \arg z \leqslant \frac{\pi}{3} \text{ and } |z| \leqslant 4$$

Mark w on your Argand diagram and find the greatest value of $|z - w|$. **[9]**

9 You are given that $\mathbf{A} = \begin{pmatrix} 1 & 3 & -1 \\ -1 & \alpha & -1 \\ -2 & -1 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3\alpha - 1 & -8 & \alpha - 3 \\ 5 & 1 & 2 \\ 2\alpha + 1 & -5 & \alpha + 3 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} \gamma & 0 & 0 \\ \beta & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix}$.

(i) Show that $\beta = 0$. **[2]**

(ii) Find γ in terms of α. **[2]**

(iii) Write down $\mathbf{A}^{-1}$ for the case when $\alpha = 2$. State the value of α for which $\mathbf{A}^{-1}$ does not exist. **[3]**

(iv) Use your answer to part **(iii)** to solve the following simultaneous equations.

$$\begin{aligned} x + 3y - z &= 25 \\ -x + 2y - z &= 11 \\ -2x - y + 3z &= -23 \end{aligned}$$

[5]

END OF QUESTION PAPER

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Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Turn over

Section A (36 marks)

1

2 (i)	
2 (ii)	
2 (iii)	

3	
	(answer space continued on next page)

(answer space continued on next page)

3	**(continued)**
4	
	(answer space continued on next page)

4	**(continued)**
5	
	(answer space continued on next page)

5	(continued)

6	
	(answer space continued on next page)

(answer space continued on next page)

6	(continued)

Section B (36 marks)

7 (i)	
7 (ii)	
7 (iii)	

7 (iv)

8 (i)	
8 (ii)	
	(answer space continued on next page)

8 (ii)	**(continued)**

9 (i)	
9 (ii)	

9 (iii)	
9 (iv)	
	(answer space continued on next page)

9 (iv)	(continued)

Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4755**: Further Concepts for Advanced Mathematics

Advanced Subsidiary GCE

Mark Scheme for June 2014

1. Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

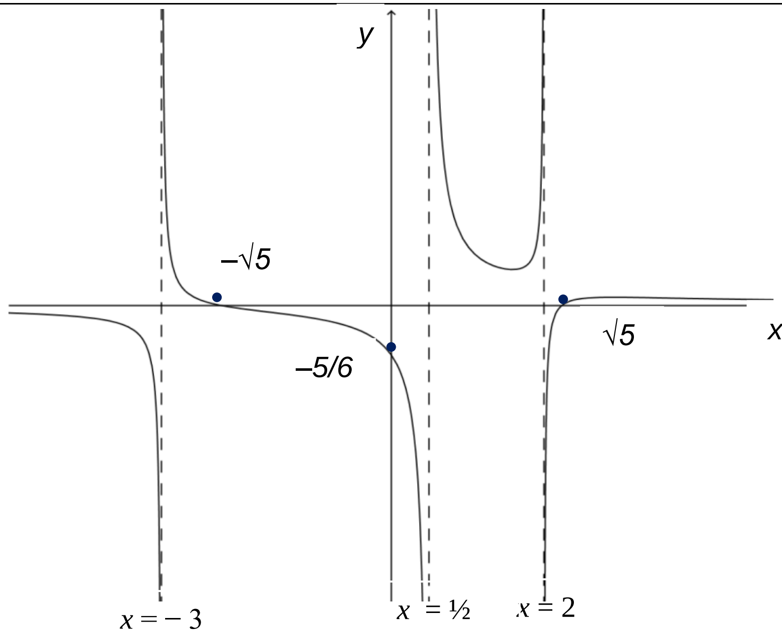
Question			Answer	Marks	Guidance
1			$\sum_{\gamma=1}^{n} r(r-2) = \sum_{\gamma=1}^{n} r^2 - 2\sum_{\gamma}^{n} r$	M1	Separate sum (may be implied)
			$= \frac{1}{6}n(n+1)(2n+1) - n(n+1)$	A1,A1	1 mark for each part oe
			$= \frac{1}{6}n(n+1)[(2n+1)-6]$	M1	$n(n+1)(\text{linear factor})$ seen
			$= \frac{1}{6}n(n+1)(2n-5)$	A1	Or $n(n+1)(2n-5)/6$ only, ie 1/6 must be a factor
				[5]	
2	(i)		$\begin{pmatrix} -3 & -2 \\ -2 & 1 \end{pmatrix}$	B1,B1	1 mark for each column. Must be a 2×2 matrix Condone lack of brackets throughout
				[2]	
2	(ii)		$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1	
				[1]	
2	(iii)		$\begin{pmatrix} -3 & -2 \\ 2 & -1 \end{pmatrix}$	B1,B1	1 mark for each column (no ft). Must be a 2×2 matrix
				[2]	

Question			Answer	Marks	Guidance
3			$z = 2 - 3\text{j}$ is also a root	B1	
			Either		
			$(z-(2+3\text{j}))(z-(2-3\text{j})) = ((z-2)+3\text{j})((z-2)-3\text{j})$	M1	Condone (z+ 2+ 3j)(z+ 2–3j)
			$= z^2 - 4z + 13$	A1	Correct quadratic
			$z^4 - 5z^3 + 15z^2 - 5z - 26 = (z^2 - 4z + 13)(z^2 - z - 2)$	M1 A1	Valid method to find the other quadratic factor. Correct quadratic
			$(z^2 - z - 2) = (z-2)(z+1)$		
			So the other roots are 2 and –1	A1,A1	1 mark for each root, cao
				[7]	
			Or		
			$2 + 3j + 2 - 3j + \gamma + \delta = 5$ oe	B1	Sum of roots with substitution of roots $2 \pm 3j$ for α and β
			$(2+3j)(2-3j)\gamma\delta = -26$ $\gamma\delta = -2$	M1	Attempt to obtain equation in $\gamma\delta$ using a root relation and $2 \pm 3j$
			$\Rightarrow 4 + \gamma + \delta = 5 \Rightarrow \gamma = 1 - \delta$		
			and $13\gamma\delta = -26 \Rightarrow \gamma\delta = -2$	M1	Eliminating γ or δ leading to a quadratic equation
			$\Rightarrow \delta(1-\delta) = -2 \Rightarrow \delta^2 - \delta - 2 = 0$	A1	Correct equation obtained
			$\Rightarrow (\delta+1)(\delta-2) = 0$		
			So the other roots are – 1 and 2.	A1,A1	1 mark for each, cao If 2, –1 guessed from $\gamma + \delta = 1$ and $\gamma\delta = -2$ give A1 A1 for these equations and A1A1 for the roots. **SC** factor theorem used. M1 for substitution of $z = -1$(or 2) or division by (z + 1) (or by z – 2), A1 if zero obtained, B1 for the root stated to be – 1(or 2). For the other root, similarly but M1A1A1 Max [**7/7**] Answers only get M0M0, max [**1/7**]
				[7]	

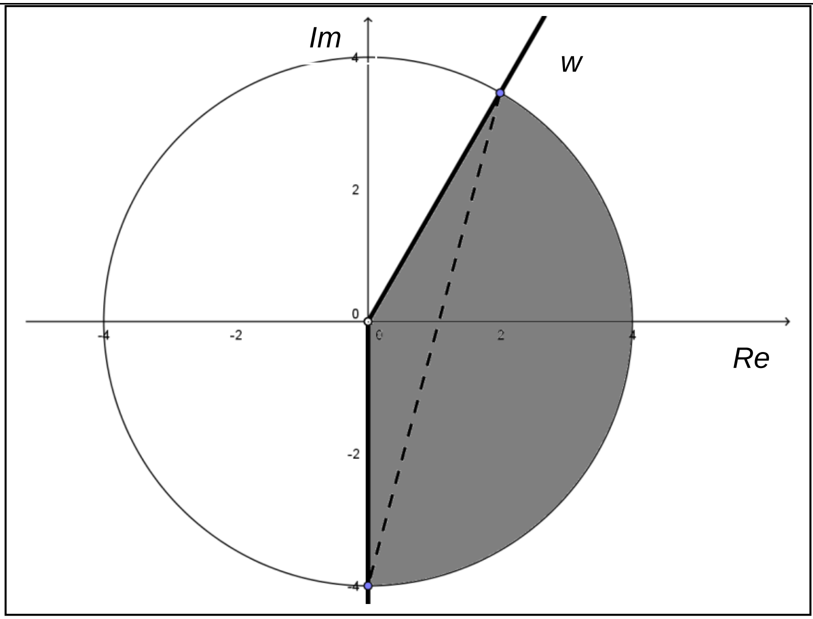
Question			Answer	Marks	Guidance
4			$\sum_{r=1}^{n}\frac{1}{(2r+3)(2r+5)}=\frac{1}{2}\sum_{r=1}^{n}\left[\frac{1}{2r+3}-\frac{1}{2r+5}\right]$	M1	Split to partial fractions. Allow missing $\frac{1}{2}$
			$=\frac{1}{2}\left[\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{7}\ldots\right)+\ldots..+\left(\ldots-\frac{1}{2n+5}\right)\right]$	M1 A1	Expand to show pattern of cancelling, at least 4 fractions All correct, allow missing $\frac{1}{2}$, condone r
			$=\frac{1}{2}\left[\frac{1}{5}-\frac{1}{2n+5}\right]=\frac{n}{5(2n+5)}$	M1 A1	Cancel to first minus last term must be in terms of n. oe single fraction
				[5]	

Question			Answer	Marks	Guidance
5			**Either**		
			$y = 3x - 1 \Rightarrow x = \dfrac{y+1}{3}$	M1*	Change of variable, condone $\dfrac{y-1}{3}, \dfrac{y}{3} \pm 1$.
			$\Rightarrow 3\left(\dfrac{y+1}{3}\right)^3 - 9\left(\dfrac{y+1}{3}\right)^2 + \left(\dfrac{y+1}{3}\right) - 1 = 0$	M1dep* A1	Substitute into cubic expression Correct
			Correct coefficients in cubic expression (may be fractions)	A3ft	ft their substitution (-1 each error)
			$\Rightarrow y^3 - 6y^2 - 12y - 14 = 0$	A1	cao. Must be an equation with integer coefficients
				[7]	
			Or		
			$\alpha + \beta + \gamma = \dfrac{9}{3} = 3$ $\alpha\beta + \alpha\gamma + \beta\gamma = \dfrac{1}{3}$	M1	All three root relations, condone incorrect signs
			$\alpha\beta\gamma = \dfrac{1}{3}$	A1	All correct
			Let new roots be k, l, m then $k + l + m = 3(\alpha + \beta + \gamma) - 3 = 6$ $kl + km + lm = 9(\alpha\beta + \alpha\gamma + \beta\gamma) - 6(\alpha + \beta + \gamma) + 3 = -12$	M1	Using (3α-1) etc in $\sum k, \sum kl, klm$, at least two attempted, and using $\sum \alpha, \sum \alpha\beta, \alpha\beta\gamma$
			$klm = 27\alpha\beta\gamma - 9(\alpha\beta + \beta\gamma + \beta\gamma) + 3(\alpha + \beta + \gamma) - 1 = 14$	A3ft	One each for 6, – 12, 14, ft their $3, \dfrac{1}{3}, \dfrac{1}{3}$.
			$\Rightarrow y^3 - 6y^2 - 12y - 14 = 0$	A1	cao. Must be an equation with integer coefficients
				[7]	

Question			Answer	Marks	Guidance
6			When $n = 1$, $\frac{1}{1\times 3}=\frac{1}{3}$ and $\frac{n}{2n+1}=\frac{1}{3}$, so true for $n = 1$	B1	Condone eg "$\frac{1}{3}=\frac{1}{3}$ "
			Assume true for $n = k$	E1	Assuming true for k, (some work to follow) If in doubt look for unambiguous "if…then" at next E1 Statement of assumed result not essential but further work should be seen
			Sum of $k + 1$ terms		NB "last term = sum of terms" seen anywhere earns final E0
			$=\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}$	M1	Adding correct $(k + 1)$th term to sum for k terms
			$=\frac{k(2k+3)+1}{(2k+1)(2k+3)}$	M1	Combining their fractions
			$=\frac{2k^2+3k+1}{(2k+1)(2k+3)}$		
			$=\frac{(k+1)(2k+1)}{(2k+1)(2k+3)}=\frac{k+1}{2k+3}$	A1	Complete accurate work
			which is $\frac{n}{2n+1}$ with $n=k+1$		May be shown earlier
			Therefore if true for $n = k$ it is also true for $n = k+1$.	E1	Dependent on A1 and previous E1.
			Since it is true for $n = 1$, it is true for all positive integers, n.	E1	Dependent on B1 and previous E1 E0 if "last term"= "sum of terms " seen above
				[7]	

Question			Answer	Marks	Guidance
7	(i)		$\left(0, -\frac{5}{6}\right)$	B1	Allow for both $x = 0$ and $y = -\frac{5}{6}$ seen
			$\left(\sqrt{5},\ 0\right),\ \left(-\sqrt{5},\ 0\right)$	B1	(both) Allow $(\pm\sqrt{5}, 0)$ or for both $y = 0$ and $x = \pm\sqrt{5}$ seen
				[2]	
7	(ii)		$a = 2$	B1	
			$y = 0$	B1	
			$x = -3$, $x = 2$	B1	Must be two equations
				[3]	
7	(iii)			B1	Two outer branches correctly placed Inner branches correctly placed Correct asymptotes and intercepts labelled
				B1	For good drawing. Dep all 3 marks above
				B1	Look for a clear maximum point on the right-hand branch, (not really shown here).
				B1	Condone turning points in $-\sqrt{5} < x < \frac{1}{2}, y < 0$
				[4]	
	(iv)		$-3 < x < -\sqrt{5},\ \frac{1}{2} < x < 2,\ x > \sqrt{5}$	B3	One mark for each. Strict inequalities. Allow 2.24 for $\sqrt{5}$ (if B3 then – 1 if more than 3 inequalities)
				[3]	

Question			Answer	Marks	Guidance
8	(i)		$\lvert w \rvert = \sqrt{\left(2^2 + \left(2\sqrt{3}\right)^2\right)} = 4$	B1	
			$\arg w = \arctan \frac{2\sqrt{3}}{2} = \frac{\pi}{3}$	M1	
			$w = 4\left(\cos\frac{\pi}{3} + \mathrm{j}\sin\frac{\pi}{3}\right)$	A1	Accept $\left(4,\ \frac{\pi}{3}\right)$, 1.05 rad , 60° in place of $\frac{\pi}{3}$, or $4e^{j\frac{\pi}{3}}$
				[3]	

Question			Answer	Marks	Guidance
8	(ii)			B1	Circle, or arc of circle, centre the origin
				B1	Radius 4
				B1	Half line from origin $\frac{\pi}{4} < \text{angle} < \frac{\pi}{2}$ with positive real axis or acute angle labelled as $\pi/3$
				B1	Use of negative Im axis clearly indicated
				B1	Correct region indicated. Dependent on first 4 B marks Ignore placing of *w*.
				B1	*w* at intersection of $\frac{\pi}{3}$ line and circle (dep 1st 3 B marks)
			Maximum $\lvert z-w\rvert = \sqrt{\left(2^2+\left(4+2\sqrt{3}\right)^2\right)}$ = 7.73 (3 s.f.)	B1	Maximum $\lvert z-w\rvert$ indicated by chord on diagram oe or sight of $-4j-(2+2\sqrt{3}j)$ oe
			Or 2x 4cos15□ = $2\sqrt{6}+2\sqrt{2}$	M1	Valid attempt to calculate maximum $\lvert z-w\rvert$
				A1	allow $\sqrt{32+16\sqrt{3}}$ oe (accept 2 s.f. or better)
				[9]	

Question			Answer	Marks	Guidance
9	(i)		$\beta = (-1)(3\alpha-1)+5\alpha+(-1)(2\alpha+1)$ $= -3\alpha+1+5\alpha-2\alpha-1 = 0$	M1 A1 **[2]**	multiply second row of **A** with first column of **B** Correct
9	(ii)		$\gamma = (1)(3\alpha-1)+15+(-1)(2\alpha+1)$ $= \alpha+13$	M1 A1 **[2]**	Attempt to multiply relevant row of **A** with relevant column of **B.** Condone use of **BA** instead Correct
9	(iii)		When $\alpha = 2,\ \gamma = 15$ $\mathbf{A}^{-1} = \frac{1}{15}\begin{pmatrix} 5 & -8 & -1 \\ 5 & 1 & 2 \\ 5 & -5 & 5 \end{pmatrix}$ $\mathbf{A}^{-1}$ does not exist when $\alpha = -13$	M1 A1 B1ft **[3]**	Multiplication of **B** by $\frac{1}{\text{their } \gamma}$,($\gamma \neq 1$) using $\alpha = 2$ in both Correct elements in matrix and correct γ. ft their $\gamma = 0$. Condone "$\alpha \neq -13$"
9	(iv)		$\frac{1}{15}\begin{pmatrix} 5 & -8 & -1 \\ 5 & 1 & 2 \\ 5 & -5 & 5 \end{pmatrix}\begin{pmatrix} 25 \\ 11 \\ -23 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $= \frac{1}{15}\begin{pmatrix} 60 \\ 90 \\ -45 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix}$ $\Rightarrow x = 4,\ y = 6,\ z = -3$	M1 B1 A3 **[5]**	Set-up of pre-multiplication by their 3x3 $\mathbf{A}^{-1}$, or by **B** (using $\alpha = 2$) $(60\ \ 90\ \ -45)'$ soi need not be fully evaluated cao A1 for each explicit identification of *x*, *y*, *z* in a vector or a list. (-1 unidentified) Answers only or solution by other method, M0A0

4755 Further Concepts for Advanced Mathematics (FP1)

General Comments:

Most of the candidates in this examination showed that they were well prepared and familiar with the content of the specification. It did not appear that the majority of candidates ran short of time. There were many very good scripts submitted in which the quality of communication was also extremely good, however, a good number of candidates' scripts were much poorer in this respect. Many candidates who were less successful often demonstrated inadequate algebraic skills. It is appreciated that candidates have to work under pressure of time but attention to the placement of brackets, completing expressions and using correct notation could help to avoid errors.

Comments on Individual Questions:

Question 1
This gave most candidates a safe start to the paper and it was well answered by the great majority. The series was successfully split in nearly every case, and the sum of squares could be obtained from the MF2 booklet, for an easy mark. Surprisingly for this level, the second sum was not always correct. It is the wise course to learn the result by heart. Some candidates forgot the multiplier 2 and some thought that the sum $\sum r$ was n. The factorisation of the resulting expression was not always efficiently carried out. Careless use of brackets, or lack of them, could result in a sign error. The final result expected was for linear factors with integer coefficients and a numerical factor of 1/6.

Question 2
Again, many good answers were seen in all parts of this question. There were however some fairly common misconceptions, where in parts (i) and (iii) candidates confused a transformation matrix with the 2x4 matrix of position vectors for the quadrilateral. The 2x2 matrix given in part (ii) was not always correct and most candidates answered part (iii) by carrying out matrix multiplication, not always in the correct sequence. The easiest method, looking for the images of $\overrightarrow{OA'}$ and $\overrightarrow{OC'}$ after reflection, did not seem to be used. Maybe candidates were uncertain whether simply writing down the result would involve a penalty.

Question 3
This question was again usually well answered, and the best responses left the reader in no doubt of the values of the roots found. In many cases however the quality of communication was poor. For example, the complex conjugate was written down but rarely identified as a root. Solutions to a quadratic equation were obtained, but again it was often not stated that these were indeed the roots required. The most efficient solutions to this problem found the quadratic factor arising from the complex roots and then the second quadratic factor, either by long division or by equating coefficients. It was good to be informed when the process was carried out 'by inspection'. The two real roots then followed. Many solutions seen used the root relationships, and sometimes there was a mixture of the methods. The two real roots could be quite easily found by less sophisticated methods. These were given credit provided there was adequate justification.

Question 4
This was another question for which candidates were well prepared, but in which the unwary did encounter some hazards. A full solution was expected here, displaying how the method of differences worked. Most candidates did this well, showing clearly the cancellation pattern of

terms in the expanded series. Some erred in forgetting that the sum of n terms was required and left off at the r th term. Some failed finally to obtain a single fraction. Some either failed to incorporate the necessary factor of ½, or multiplied by 2.

Question 5

Algebraic mistakes were seen quite often in the solutions to this question. In general the substitution method was the most successful method used, very few candidates used the wrong one, and nearly all dealt with the cubic expansion required. There were some sign errors, usually from lack of brackets, when dealing with the squared term. Another common mistake was to forget the constant term when clearing fractions. Candidates who had a strategy for carefully setting out their work fared best. The method of using the root relationships was popular, and usually successful when careful attention was paid to the signs required in the coefficients of the new equation. Some candidates lost a mark by leaving an expression, not an equation.

Question 6

It was good to see many extremely well argued proofs. Very many candidates showed that they appreciated the need for clarity and logical statements, but the three explanation marks were not always earned. Proof by induction has a standard format:

prove for $n=1$;
assume the conjecture is true for $n=k$ and hence show it is true for $n=k+1$;
state "if it is true for $n=k$ then it is true for $n=k+1$; since it is true for $n=1$ …" etc.

The mark schemes of past years set out the argument clearly but there are still those who believe that "$n=k$" is sufficient to tell the reader that a result is being assumed to be true, and that "true for "$n=1, n=k$ and $n=k+1$ is adequate to replace "if…then…since…".

This particular series should not have been difficult to deal with algebraically. Candidates could help themselves by choosing the simplest denominator when combining fractions. When this was not done and a cubic obtained by expanding the numerator, it was not always convincingly re-factorised. Some candidates made the mistake of adding $\frac{1}{2k(2k+2)}$ as the $(k+1)$th term. Some lost a mark because, in their work, the last term of the series was equated to the sum of the terms.

Question 7

Part (i) This part was well answered. Some candidates lost marks by giving a line equation rather than co-ordinates. Each point requires two line equations, not one. Wrapping the two numbers up in brackets is much quicker!

Part (ii) was also well answered. A common error was to omit the horizontal asymptote $y=0$. The majority of candidates knew how to find both intercepts and asymptotes.

Part (iii) was less well done. It was quite easy to forget that $x=\frac{1}{2}$ was also an asymptote and to produce a curve with three branches instead of four. There was uncertainty about the behaviour of the right hand branch above the x- axis, where a turning point was not shown. A graphical calculator is obviously a valuable tool but there is hardly time for detailed examination of the curve, and an understanding of how the asymptotes are approached is essential. Adequate labelling is also important, for both intercepts and asymptotes.

In the final part (iv), many candidates answered well, using their graphs and not delving into algebra. Some candidates showed that they didn't understand the idea of an asymptote by inserting £ or ³ in their inequalities.

Question 8
Nearly all candidates answered this well. A few forgot to insert j in their final answer. There were some incorrect values for the modulus, $|w| = \sqrt{10}$ being most frequently seen. Some candidates thought it sufficient to find the modulus and argument and to stop there, although the question asks for w expressed in modulus-argument form.
Part (ii) proved to be more difficult. The marks relating to the Argand diagram were on the whole easily obtained, and a convincing sector indicated. It was necessary to give some clear indication of the radius of the circle. It was surprisingly common to see the circle centred at w. It was also not uncommon to see $\theta = -\frac{\pi}{2}$ drawn in the wrong place, usually looking more like $\theta = -\frac{\pi}{4}$. Both the boundary half-lines should have had labels, but some benefit of the doubt was given if they were shown in appropriate positions. Many candidates failed to understand the connection between part (i) and the position of w in their diagram.
The last three marks were very rarely earned. The most popular choice for the requested z was diametrically opposite w, and therefore not in the region. Other positions were also popular, for example on the real axis. Many candidates omitted any attempt at this. When the correct location was found, it was good to see a few candidates using purely geometric calculations of the required distance, showing an ability to think outside the "tunnel" of complex numbers, and providing a very neat exact solution.

Question 9
After the last question candidates found this one straightforward and many earned full marks. In part (i) some candidates used the wrong row and column, (1)(8) + (3)(1) + (-1)(-5) was seen a number of times. There was also a bit of fiddling to get zero, "$-3\alpha + 1 + 5\alpha - 2\alpha + 1 = 0$" was seen quite often.
Part (ii) proved very straightforward.
In part (iii) not all candidates were able to make the link between (i) and (ii). Many evaluated B using $\alpha = 2$ but omitted 1/15 from their answer. It is likely that many candidates overlooked the second request in part (iii) where there was no answer or work shown relating to it.
Part (iv) was again straightforward and very few candidates failed to use a matrix method to solve the equations. A few placed the inverse of A in the wrong position, thereby raising the suspicion that their equations had been solved effortlessly on a calculator and without the understanding of how the matrix algebra worked.