

Monday 13 May 2013 – Afternoon

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Find the values of A , B , C and D in the identity $2x(x^2 - 5) \equiv (x - 2)(Ax^2 + Bx + C) + D$. [5]

2 You are given that $z = \frac{3}{2}$ is a root of the cubic equation $2z^3 + 9z^2 + 2z - 30 = 0$. Find the other two roots. [6]

3 You are given that $\mathbf{N} = \begin{pmatrix} -9 & -2 & -4 \\ 3 & 2 & 2 \\ 5 & 1 & 2 \end{pmatrix}$ and $\mathbf{N}^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ -\frac{7}{2} & p & -6 \end{pmatrix}$.

(i) Find the value of p . [2]

(ii) Solve the equation $\mathbf{N} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -39 \\ 5 \\ 22 \end{pmatrix}$. [4]

4 The complex number z_1 is $3 - 2j$ and the complex number z_2 has modulus 5 and argument $\frac{\pi}{4}$.

(i) Express z_2 in the form $a + bj$, giving a and b in exact form. [2]

(ii) Represent z_1 , z_2 , $z_1 + z_2$ and $z_1 - z_2$ on a single Argand diagram. [4]

5 You are given that $\frac{4}{(4n - 3)(4n + 1)} \equiv \frac{1}{4n - 3} - \frac{1}{4n + 1}$. Use the method of differences to show that

$$\sum_{r=1}^n \frac{1}{(4r - 3)(4r + 1)} = \frac{n}{4n + 1}. \quad [6]$$

6 The cubic equation $x^3 - 5x^2 + 3x - 6 = 0$ has roots α , β and γ . Find a cubic equation with roots $\frac{\alpha}{3} + 1$, $\frac{\beta}{3} + 1$ and $\frac{\gamma}{3} + 1$, simplifying your answer as far as possible. [7]

Section B (36 marks)

- 7 Fig. 7 shows an incomplete sketch of $y = \frac{cx^2}{(bx-1)(x+a)}$ where a , b and c are integers. The asymptotes of the curve are also shown.

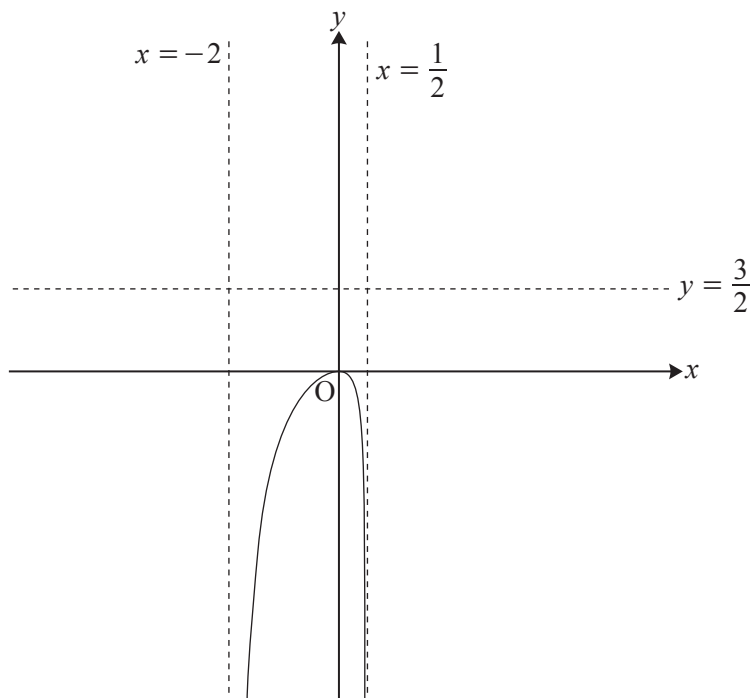


Fig. 7

- (i) Determine the values of a , b and c . [4]

Use these values of a , b and c throughout the rest of the question.

- (ii) Determine how the curve approaches the horizontal asymptote for large positive values of x , and for large negative values of x , justifying your answer. On the copy of Fig. 7, sketch the rest of the curve. [4]

- (iii) Find the x coordinates of the points on the curve where $y = 1$. Write down the solution to the inequality $\frac{cx^2}{(bx-1)(x+a)} < 1$. [4]

- 8 (i) Use standard series formulae to show that

$$\sum_{r=1}^n [r(r-1) - 1] = \frac{1}{3}n(n+2)(n-2). \quad (*) \quad [5]$$

- (ii) Prove (*) by mathematical induction. [7]

- 9 (i) Describe fully the transformation Q , represented by the matrix \mathbf{Q} , where $\mathbf{Q} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. [2]

The transformation M is represented by the matrix \mathbf{M} , where $\mathbf{M} = \begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$.

- (ii) M maps all points on the line $y = 2$ onto a single point, P . Find the coordinates of P . [2]
- (iii) M maps all points on the plane onto a single line, l . Find the equation of l . [2]
- (iv) M maps all points on the line n onto the point $(-6, 6)$. Find the equation of n . [2]
- (v) Show that \mathbf{M} is singular. Relate this to the transformation it represents. [2]
- (vi) R is the composite transformation M followed by Q . R maps all points on the plane onto the line q . Find the equation of q . [2]

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AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:

- Question Paper 4755/01 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



Candidate forename		Candidate surname	
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Centre number							Candidate number				
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Section A (36 marks)

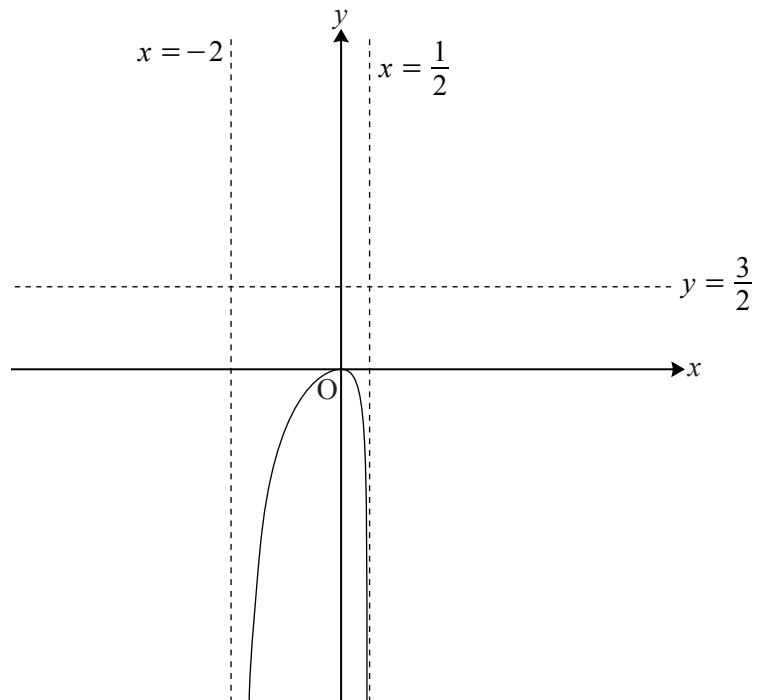
1	

3 (i)	
3 (ii)	

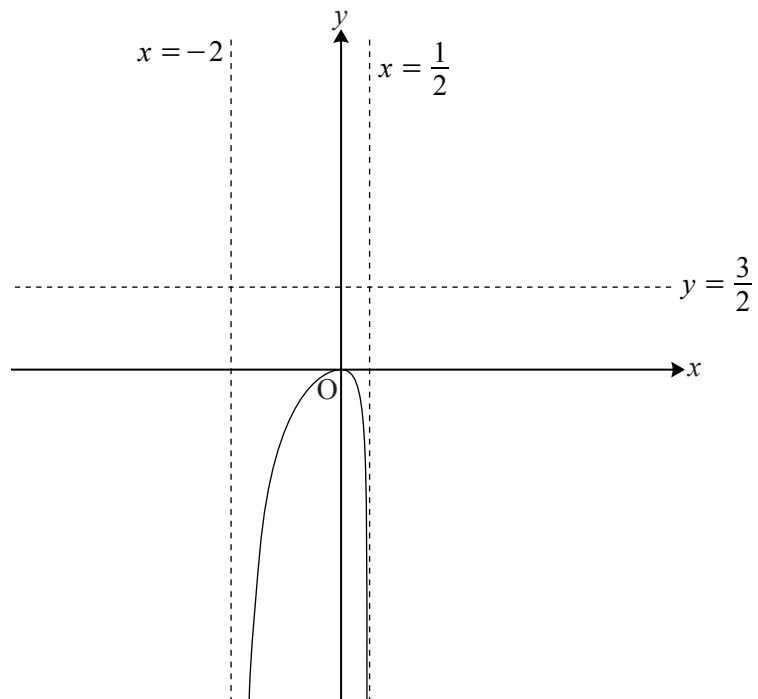
4 (i)	

4 (ii)	
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7 (ii) (continued)



Spare copy of graph for question 7(ii)



7 (iii)	

9 (i)	
9 (ii)	

9 (iii)	
9 (iv)	

9 (v)	
9 (vi)	



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Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4755**: Further Concepts for Advanced Mathematics

Mark Scheme for June 2013

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and *	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation *isw*. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

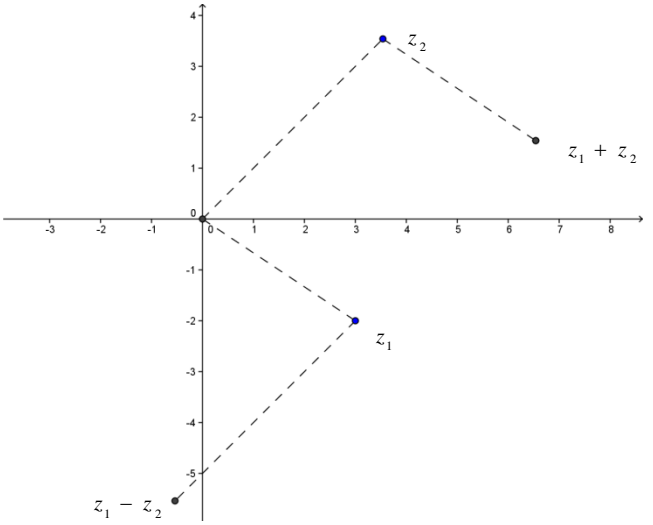
NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

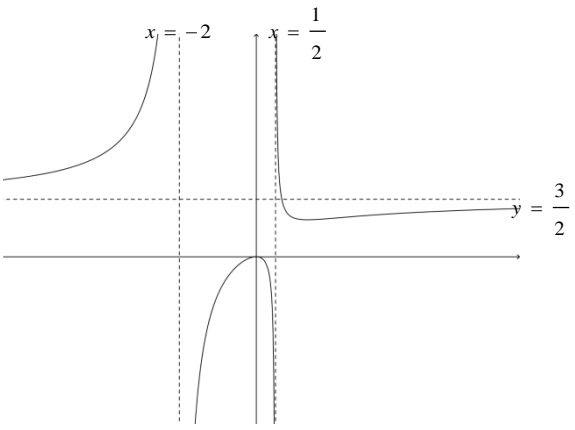
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

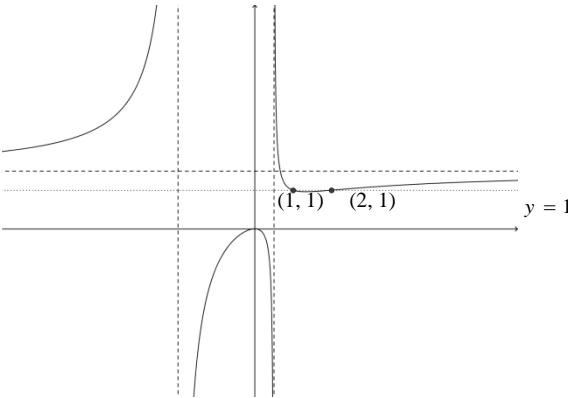
Question	Answer	Marks	Guidance
1	$2x(x^2 - 5) \equiv (x - 2)(Ax^2 + Bx + C) + D$ <p>Comparing coefficients of x^3, $A = 2$ Comparing coefficients of x^2, $B - 2A = 0 \Rightarrow B = 4$ Comparing coefficients of x, $C - 2B = -10 \Rightarrow C = -2$ Comparing constants, $D - 2C = 0 \Rightarrow D = -4$</p>	M1 B1 B1 B1 B1 [5]	Evidence of comparing coefficients, or multiplying out the RHS, or substituting. May be implied by $A = 2$ or $D = -4$ Unidentified, max 4 marks.
2	$z = \frac{3}{2} \text{ is a root } \Rightarrow (2z - 3) \text{ is a factor.}$ $\Rightarrow (2z - 3)(z^2 + bz + c) = (2z^3 + 9z^2 + 2z - 30)$ <p>Other roots when $z^2 + 6z + 10 = 0$</p> $z = \frac{-6 \pm \sqrt{36 - 40}}{2}$ $= -3 + j \text{ or } -3 - j$ <p>OR $\frac{3}{2} + \beta + \gamma = -\frac{9}{2}$, $\frac{3}{2}\beta\gamma = 15$, or $\frac{3}{2}\beta + \beta\gamma + \frac{3}{2}\gamma = 1$</p> $\beta + \gamma = -6, \beta\gamma = 10$ $z^2 + 6z + 10 = 0,$ $z = \frac{-6 \pm \sqrt{36 - 40}}{2}$ $= -3 + j \text{ or } -3 - j$ <p>or roots must be complex, so $a \pm bj$, $2a = -6, 9 + b^2 = 10$</p> $z = -3 + j, z = -3 - j$	M1 M1 M1 A1 M1 A1 M1 M1 A1 M1 A1 M1 A1 M1 A1 [6]	Use of factor theorem, accept $2z + 3, z \pm \frac{3}{2}$ Attempt to factorise cubic to linear x quadratic Compare coefficients to find quadratic (or other valid complete method leading to a quadratic) Correct quadratic Use of quadratic formula (or other valid method) in their quadratic oe for both complex roots FT their 3-term quadratic provided roots are complex. Two root relations (may use α) leading to sum and product of unknown roots and quadratic equation which is correct Use of quadratic formula (or other valid method) in their quadratic oe For both complex roots FT their 3-term quadratic provided roots are complex. SCM0B1 if conjugates not justified

Question		Answer	Marks	Guidance	
3	(i)	$-2 - 4p = 0$ $\Rightarrow p = -\frac{1}{2}$	M1 B1 [2]	Any valid row x column leading to p	
3	(ii)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{N}^{-1} \begin{pmatrix} -39 \\ 5 \\ 22 \end{pmatrix}$ $= \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ -\frac{7}{2} & -\frac{1}{2} & -6 \end{pmatrix} \begin{pmatrix} -39 \\ 5 \\ 22 \end{pmatrix}$ $= \begin{pmatrix} 5 \\ -7 \\ 2 \end{pmatrix}$	M1 M1 A1 A1 [4]	Attempt to use \mathbf{N}^{-1} Attempt to multiply matrices (implied by 3x1 result) One element correct All 3 correct. FT their p	Correct solution by means of simultaneous equations can earn full marks. M1 elimination of one unknown, M1 solution for one unknown A1 one correct, A1 all correct
4	(i)	$z_2 = 5 \left(\cos \frac{\pi}{4} + j \sin \frac{\pi}{4} \right)$ $= \frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2} j$	M1 A1 [2]	May be implied oe (exact numerical form)	

Question	Answer	Marks	Guidance
4 (ii)	$z_1 + z_2 = 3 + \frac{5\sqrt{2}}{2} + \left(-2 + \frac{5\sqrt{2}}{2}\right)j = 6.54 + 1.54j$ $z_1 - z_2 = 3 - \frac{5\sqrt{2}}{2} + \left(-2 - \frac{5\sqrt{2}}{2}\right)j = -0.54 - 5.54j$ 	<p>M1</p> <p>B3</p> <p>[4]</p>	<p>Attempt to add and subtract z_1 and their z_2 - may be implied by Argand diagram</p> <p>For points cao, -1 each error – dotted lines not needed.</p>
5	$\sum_{r=1}^n \frac{1}{(4r-3)(4r+1)} = \frac{1}{4} \sum_{r=1}^n \left[\frac{1}{4r-3} - \frac{1}{4r+1} \right]$ $= \frac{1}{4} \left[\left(\frac{1}{1} - \frac{1}{5} \right) + \left(\frac{1}{5} - \frac{1}{9} \right) + \dots + \left(\frac{1}{4n-3} - \frac{1}{4n+1} \right) \right]$ $= \frac{1}{4} \left[1 - \frac{1}{4n+1} \right]$ $= \frac{1}{4} \left[\frac{4n+1-1}{4n+1} \right] = \frac{n}{4n+1}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[6]</p>	<p>For splitting summation into two. Allow missing 1/4</p> <p>Write out terms (at least first and last terms in full)</p> <p>Allow missing 1/4</p> <p>Cancelling inner terms; SC insufficient working shown above, M1M0M1A1 (allow missing 1/4)</p> <p>Inclusion of 1/4 justified</p> <p>Honestly obtained (AG)</p>

Question	Answer	Marks	Guidance
6	$w = \frac{x}{3} + 1 \Rightarrow 3(w - 1) = x$ $x^3 - 5x^2 + 3x - 6 = 0$ $\Rightarrow (3(w - 1))^3 - 5(3(w - 1))^2 + 3(3(w - 1)) - 6 = 0$ $\Rightarrow 27(w^3 - 3w^2 + 3w - 1) - 45(w^2 - 2w + 1) + 9w - 15 = 0$ $\Rightarrow 27w^3 - 126w^2 + 180w - 87 = 0$ $\Rightarrow 9w^3 - 42w^2 + 60w - 29 = 0$ <p>OR</p> <p>In original equation $\sum \alpha = 5, \sum \alpha\beta = 3, \alpha\beta\gamma = 6$</p> <p>New roots A, B, Γ</p> $\sum A = \frac{\sum \alpha}{3} + 3, \sum AB = \frac{\sum \alpha\beta}{9} + \frac{2}{3} \sum \alpha + 3$ $AB\Gamma = \frac{\alpha\beta\gamma}{27} + \frac{\sum \alpha\beta}{9} + \frac{\sum \alpha}{3} + 1$ <p>Fully correct equation</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A3</p> <p>A1</p> <p>M1A1</p> <p>M1</p> <p>A3</p> <p>A1</p> <p>[7]</p>	<p>Substituting</p> <p>Correct</p> <p>FT $x = 3w + 3, 3w \pm 1$, -1 each error cao</p> <p>all correct for A1</p> <p>At least two relations attempted Correct -1 each error FT their 5,3,6 Cao, accept rational coefficients here</p>

Question	Answer	Marks	Guidance
7 (i)	<p>Vertical asymptotes at $x = -2$ and $x = \frac{1}{2}$ occur when</p> $(bx - 1)(x + a) = 0$ $\Rightarrow a = 2 \text{ and } b = 2$ <p>Horizontal asymptote at $y = \frac{3}{2}$ so when x gets very large,</p> $\frac{cx^2}{(2x - 1)(x + 2)} \rightarrow \frac{3}{2} \Rightarrow c = 3$	<p>M1</p> <p>A1 A1</p> <p>A1</p> <p>[4]</p>	<p>Some evidence of valid reasoning – may be implied</p>
7 (ii)	<p>Valid reasoning seen</p> <p>Large positive x, $y \rightarrow \frac{3}{2}$ from below</p> <p>Large negative x, $y \rightarrow \frac{3}{2}$ from above</p> 	<p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>[4]</p>	<p>Some evidence of method needed e.g. substitute in 'large' values with result</p> <p>Both approaches correct (correct b,c)</p> <p>LH branch correct</p> <p>RH branch correct</p> <p>Each one carefully drawn.</p>

Question	Answer	Marks	Guidance
7 (iii)	$\frac{3x^2}{(2x-1)(x+2)} = 1 \Rightarrow 3x^2 = (2x-1)(x+2)$ $\Rightarrow 0 = (x-2)(x-1)$ $\Rightarrow x = 1 \text{ or } x = 2$ From the graph $\frac{3x}{(2x-1)(x+2)} < 1$ for $-2 < x < \frac{1}{2}$ or $1 < x < 2$	M1 A1 B1 B1 [4]	Or other valid method, to values of x (allow valid solution of inequality) Explicit values of x  FT their $x=1,2$ provided $>1/2$.

Question	Answer	Marks	Guidance
8 (i)	$\sum_{r=1}^n [r(r-1) - 1] = \sum_{r=1}^n r^2 - \sum_{r=1}^n r - n$ $= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - n$ $= \frac{1}{6}n[(n+1)(2n+1) - 3(n+1) - 6]$ $= \frac{1}{6}n[2n^2 - 8]$ $= \frac{1}{3}n[n^2 - 4]$ $= \frac{1}{3}n(n+2)(n-2)$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>Split into separate sums</p> <p>Use of at least one standard result (ignore 3rd term)</p> <p>Correct</p> <p>Attempt to factorise. If more than two errors, M0</p> <p>Correct with factor $\frac{1}{3}n$ oe</p> <p>Answer given</p>
8 (ii)	<p>When $n = 1$,</p> $\sum_{r=1}^n [r(r-1) - 1] = (1 \times 0) - 1 = -1$ <p>and $\frac{1}{3}n(n+2)(n-2) = \frac{1}{3} \times 1 \times 3 \times -1 = -1$</p> <p>So true for $n = 1$ Assume true for $n = k$</p> $\sum_{r=1}^k [r(r-1) - 1] = \frac{1}{3}k(k+2)(k-2)$ $\Rightarrow \sum_{r=1}^{k+1} [r(r-1) - 1] = \frac{1}{3}k(k+2)(k-2) + (k+1)k - 1$ $= \frac{1}{3}k^3 + k^2 - \frac{4}{3}k + k - 1$ $= \frac{1}{3}(k^3 + 3k^2 - k - 3)$	<p>B1</p> <p>E1</p> <p>M1*</p>	<p>Or “if true for $n=k$, then...”</p> <p>Add $(k+1)$th term to both sides</p>

Question	Answer	Marks	Guidance
	$= \frac{1}{3}(k+1)(k^2+2k-3)$ $= \frac{1}{3}(k+1)(k+3)(k-1)$ $= \frac{1}{3}(k+1)((k+1)+2)((k+1)-2)$ <p>But this is the given result with $n = k + 1$ replacing $n = k$. Therefore if the result is true for $n = k$, it is also true for $n = k+1$. Since it is true for $n = 1$, it is true for all positive integers, n.</p>	M1dep * A1 E1 E1 [7]	Attempt to factorise a cubic with 4 terms Or $= \frac{1}{3}n(n+2)(n-2)$ where $n = k + 1$; or target seen Depends on A1 and first E1 Depends on B1 and second E1
9 (i)	Q represents a rotation 90 degrees clockwise about the origin	B1 B1 [2]	Angle, direction and centre
9 (ii)	$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ <p>P = (-2, 2)</p>	M1 A1 [2]	Allow both marks for P(-2, 2) www
9 (iii)	$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ y \end{pmatrix}$ <p><i>l</i> is the line $y = -x$</p>	M1 A1 [2]	Or use of a minimum of two points Allow both marks for $y = -x$ www
9 (iv)	$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}$ <p><i>n</i> is the line $y = 6$</p>	M1 B1 [2]	Use of a general point or two different points leading to $\begin{pmatrix} -6 \\ 6 \end{pmatrix}$ $y=6$; if seen alone M1B1

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Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

June 2013

4755 Further Concepts for Advanced Mathematics (FP1)

General Comments

There were more candidates for this examination than has been usual for summer session; this may be the result of the winter session no longer being available. There were, as usual, many very good scripts, where the candidates produced accurate work which was well expressed, and scored high marks. There seemed to be a greater proportion of candidates who were less confident, and who may have found that they had insufficient time to do themselves justice. As has happened in the past, there were scripts which could have showed more attention to presentation. In many cases, marks were forfeited through insufficient algebraic competence, from simple mistakes over signs to more fundamental errors, such as in finding factors. There were many cases of wishful thinking in dealing with some of the lengthy expressions.

Comments on Individual Questions

- 1) This question provided a straightforward beginning to the question paper in which nearly all candidates did well. Any errors were mostly due to inattention either to signs or to the coefficients of x being matched, with C and D in error as a result. Finding D by setting $x = 2$ was not often used, and this could have provided a quick check on accuracy.
- 2) This question was also well done with the vast majority of candidates earning at least the first four marks. Most chose to use direct factorisation by inspection or to use division, and surprisingly, most used the linear factor $(z - \frac{3}{2})$ instead of $(2z - 3)$.
Candidates who used the root relationships were also frequently successful, but more often made an error with the signs in the resulting quadratic factor. The few candidates who assumed at the outset that the roots would be complex failed to justify this.
- 3) (i) Nearly all candidates were able to show a valid row by column multiplication leading to the correct value of p .
(ii) Most candidates used the inverse matrix successfully to solve the equation. Some chose to solve three simultaneous equations, and not many managed to do this without error.
- 4) (i) There was a good response from most candidates but a surprising number believed that z_2 was either $3 + 4j$ or $4 + 3j$. Some candidates forgot that exact expressions were requested.
(ii) An incorrect z_2 allowed the method mark to be earned but as the position of z_2 could be shown from the information given, the remaining marks were easily lost. $z_1 + z_2$ was usually well positioned, $z_1 - z_2$ was often seen in a strange place. Candidates who worked the sum and difference in terms of the exact expressions did not always appreciate the size and sign of the real and imaginary parts.

- 5) This question was well done by many candidates, but there were also many instances of poor written presentation. Summation sigmas could usefully be employed to make sense of the work. It was asked that a given result be shown: this indicates that a thorough and complete solution is necessary for full marks. In particular the provenance of the factor $\frac{1}{4}$ should be made clear from the outset or by demonstration at the end of the series summation. Responses where this factor appeared at a seemingly random place, or as an afterthought, lost a mark.
- 6) There was an almost equal split between those who tackled this question by substitution and those who used root relationships. In the former case there were some erroneous substitutions, usually $(3w-1)$, but also $(3w+3)$, $(3w+1)$ and $(\frac{w}{3}+1)$. Both methods required careful algebraic work that was not always forthcoming, in particular in developing the sum of products of the new roots, taken two at a time.
- 7) (i) Without giving a method in every case, most candidates showed the insight necessary to achieve full marks.
- (ii) A few candidates convincingly argued this from an algebraic viewpoint. Most substituted a large number for x . This needed evaluation, at least to the point where the relative sizes of numerator and denominator could be seen. It is insufficient to discuss the signs of the constituent parts of the expression for y in the case when the asymptote is other than $y=0$. The sketches were mostly carefully drawn, but some candidates believe that a sketch can be a rough one, and fail to indicate clearly the salient features of the curve.
- (iii) Many candidates found the correct intersections with $y=1$ and wrote down the relevant inequality, but many forgot the obvious inequality arising from the given graph. Some candidates initially tried to solve $\frac{3x^2}{(2x-1)(x+2)} < 1$, which was unnecessary given the wording of the question, and lost marks by proceeding to multiply by $(2x-1)(x+2)$ without justifying that this was a positive quantity.
- 8) (i) This was usually answered well. The most common error was to write $\sum_{r=1}^n 1 = 1$ and this led to difficulty with earning the next mark, especially when compounded by trying to work back from the given result. This question was also subject to some careless notation; too few sigmas and missing brackets.
- (ii) Most candidates knew how to earn the first three marks, but again the written work was frequently scruffy with missing sigmas in particular. It is nonsense to write the sum of a series as equal to its last term. In some scripts, the added term in the series was the k th, not the $(k+1)$ th. The following algebra proved too much for quite a few candidates, again not helped by missing brackets. It was just about possible to believe that the correct four term cubic could be instantly factorised, and some benefit of the doubt was given here.

Inevitably, marks were lost in the details of the induction argument. Initially, “assume $n=k$ ” does not state what is being assumed. “True for $n=1$ and $n=k$ ” is not so, when the latter is conditional. The language must be precise, and many candidates displayed only half remembered sentences, indicating that they did not fully understand the induction argument.

- 9) This question was one in which many candidates gave no response to some or all sections, whether through lack of confidence or from running out of time.
- (i) There were several reflections mentioned, but the most common error was to omit to give the centre of the rotation.
- (ii) This was done fairly well, most used a point on $y = 2$, usually $(0, 2)$.
- (iii) Many instances of confused notation were seen. When $\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ is evaluated the result is not $\begin{pmatrix} x \\ y \end{pmatrix}$ but $\begin{pmatrix} -y \\ y \end{pmatrix}$. The first use of this style of notation was penalised, but not subsequently. A minority of candidates did distinguish between the original point and its transform, usually as $\begin{pmatrix} x' \\ y' \end{pmatrix}$. A safer route was to transform particular points and to recognise the relation between the x and y co-ordinates.
- (iv) Some candidates confused the object and image, writing $\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -6 \\ 6 \end{pmatrix}$ which lost the mark for method, even if $y = 6$ was recovered.
- (v) Many candidates found that the determinant was zero, although not always through a correct expression. Not many were able to give a coherent interpretation, without describing what was already given in the question. Several confused this with the role played by a zero determinant in solving a set of equations, which was not relevant here.
- (vi) Ignoring unfortunate notation, many candidates scored both marks for combining the matrices in the correct sequence and for deducing the equation of the line. It was pleasing that a few did consider the clockwise rotation of the line $y = -x$ through 90° about the origin.

Unit level raw mark and UMS grade boundaries June 2013 series
AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)		Max Mark	a	b	c	d	e	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	62	56	51	46	41	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	54	48	43	38	33	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	46	40	33	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	66	59	53	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	61	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	60	52	44	36	28	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	62	56	51	46	40	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	33	25	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	50	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	64	56	48	41	34	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	55	49	44	39	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	58	52	46	40	35	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	44	38	31	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
GCE Statistics (MEI)		Max Mark	a	b	c	d	e	u
G241/01 (Z1) Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0