

OCR

Oxford Cambridge and RSA

Friday 16 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

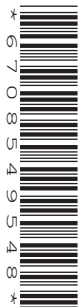
OCR supplied materials:

- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

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- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 The displacement, x m, of a particle at time t s is given by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 3x = 6 \cos t .$$

- (i) Find the general solution. [8]

The particle is initially at rest, and its displacement remains bounded as $t \rightarrow \infty$.

- (ii) Find the particular solution for x . [4]
- (iii) Show that for large values of t the motion of the particle is oscillatory. Find the approximate amplitude of the oscillations. [2]

On another occasion, the displacement of the particle satisfies the differential equation

$$\frac{d^3x}{dt^3} + 2\frac{d^2x}{dt^2} - 3\frac{dx}{dt} = 0 .$$

In this case, the particle is initially at the origin with acceleration 6 m s^{-2} and velocity $k \text{ m s}^{-1}$, where k is a positive constant.

- (iv) Find the particular solution for x in terms of t and k . [8]
- (v) Show that, whatever the value of k , the displacement of the particle cannot remain bounded for large values of t . [2]

- 2 (a) A small particle moving in a fluid satisfies the differential equation

$$\frac{dv}{dt} = -0.25(v^2 + 2v) ,$$

where $v \text{ m s}^{-1}$ is its velocity at time t s.

Given that $v = 20$ when $t = 0$, find the particular solution for v in terms of t . [8]

- (b) The differential equation

$$x \frac{dy}{dx} - 4y = x^3 \sqrt{x}$$

is to be solved for $x > 0$, subject to the initial condition $y = 0$ when $x = 1$.

- (i) Find the particular solution for y in terms of x . [10]
- (ii) Find the x -coordinate of the stationary point on this solution curve. [2]
- (iii) State the value of $\frac{dy}{dx}$ when $x = 1$. State also the limiting value of y as $x \rightarrow 0$. Hence, given that the stationary point is a minimum, sketch a graph of your solution to part (i). [4]

- 3 (a) The differential equation $\frac{dy}{dx} = \sqrt{x^2 + y^2}$ is to be investigated, firstly by means of a tangent field and then numerically.
- (i) Describe fully the isocline $\frac{dy}{dx} = m$ where m is a positive constant. [1]
- (ii) In the Answer Book, sketch on the given axes the isoclines $\frac{dy}{dx} = m$ for $m = \frac{1}{2}, 1, 2$ and 3 and hence draw a sketch of the tangent field. [4]
- (iii) Sketch on your tangent field the solution curve through the point $(0.5, 0.5)$ and the solution curve through the point $(2, 0)$. [3]

The differential equation is now to be solved numerically using Euler's method. The algorithm is given by

$$x_{r+1} = x_r + h, \quad y_{r+1} = y_r + hy'_r,$$

with $(x_0, y_0) = (0.5, 0.5)$.

- (iv) Use a step length of 0.05 to estimate y when $x = 0.65$. [4]
- (v) How might the accuracy of your estimate for y be improved? [1]
- (b) Consider the differential equation

$$\frac{dy}{dx} + y = x^2 - 1.$$

- (i) Find the complementary function and the particular integral and hence state the general solution for this differential equation. [6]
- (ii) Find the particular solution for which $y = 3$ when $x = 0$. [2]
- (iii) Show that y is always positive and sketch the solution curve for y . You do not need to find the coordinates of any stationary points. [3]

Question 4 begins on page 4

- 4 Two species of insects, X and Y, compete for survival on an island. The populations of the species are x and y respectively at time t , where t is measured in tens of years. The situation is modelled by the simultaneous differential equations

$$\frac{dx}{dt} = 2x + 2y,$$

$$\frac{dy}{dt} = 6y - 4x.$$

- (i) Eliminate y to obtain a second order differential equation for x in terms of t . Hence find the general solution for x . [7]

- (ii) Find the corresponding general solution for y . [4]

When $t = 0$, $\frac{dx}{dt} = 10$ and the population of species Y is k times the population of species X, where k is a positive constant.

- (iii) Find the particular solutions for x and y , in terms of t and k . [5]

Consider the case $k = 6$.

- (iv) Determine whether the model predicts that species X or species Y dies out first. State the value of t at which this first species dies out. [7]

- (v) Comment on why the time predicted by the model for the second species to die out is unreliable. [1]

END OF QUESTION PAPER

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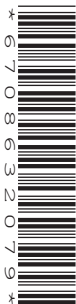
OCR supplied materials:

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Candidate forename		Candidate surname	
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Centre number						Candidate number				
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1 (ii)	

1 (iii)	

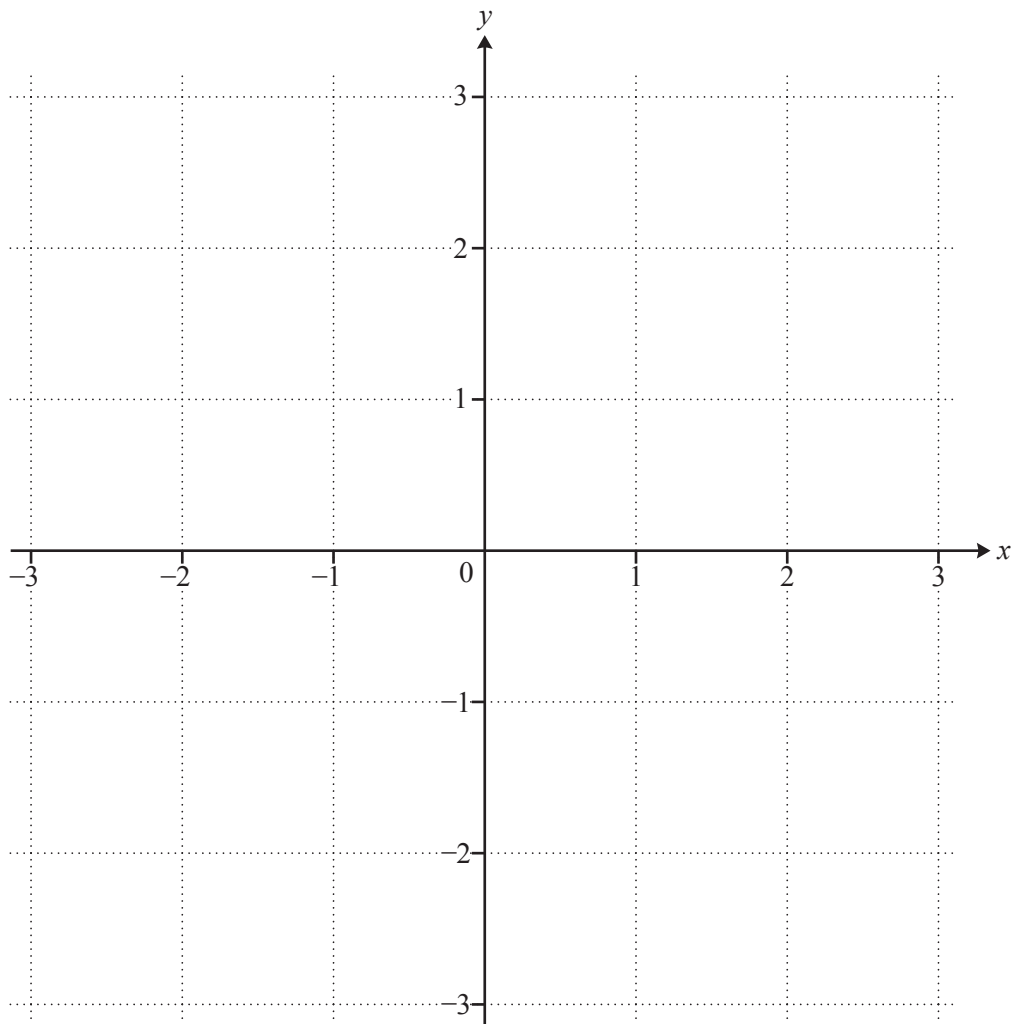
1 (iv)	
1 (v)	

2 (b)(i)	

2(b)(ii)	

2(b)(iii)	

3(a)(i)

**3(a)(ii)
& (iii)**

A spare copy of this diagram can be found on page 16.

3(a)(iv)	
3(a)(v)	

3 (b)(i)	

3 (b)(ii)	

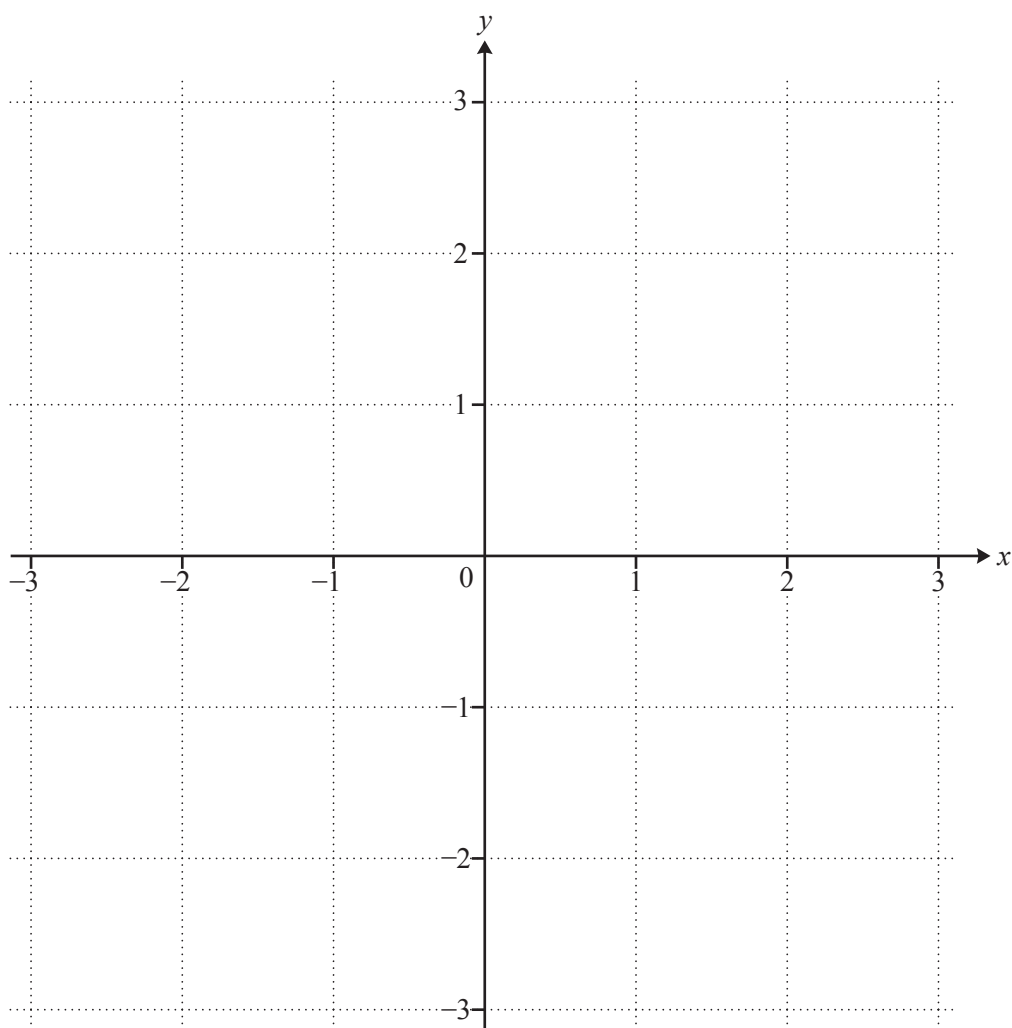
3 (b)(iii)	

4(i)	

4(ii)	

3(a)(ii)
& (iii)

Spare copy of diagram for questions 3(a)(ii) and 3(a)(iii)



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GCE

Mathematics (MEI)

Unit **4758**: Differential Equations

Advanced GCE

Mark Scheme for June 2017

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the

establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper

Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for g . E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

- i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

1.	(i)	AE: $m^2 + 2m - 3 = 0$ $m = -3, 1$ CF: $x = Ae^{-3t} + Be^t$ PI: $x = p \cos t + q \sin t$ $2q - 4p = 6$ $4q + 2p = 0$ $q = 0.6, p = -1.2$ $x = Ae^{-3t} + Be^t + 0.6 \sin t - 1.2 \cos t$	M1 A1 F1 B1 M1 M1 A1 F1 [8]	Differentiate twice, substitute and equate coefficients Solve PI + CF with 2 arbitrary constants
	(ii)	Solution bounded implies $B = 0$ $\dot{x} = -3Ae^{-3t} + 0.6 \cos t + 1.2 \sin t$ $t = 0, \frac{dx}{dt} = 0 \quad 0 = -3A + 0.6$ $A = 0.2: x = 0.2e^{-3t} + 0.6 \sin t - 1.2 \cos t$	M1 M1 M1 A1 [4]	Use condition Differentiate Use condition
	(iii)	For large values of t : exponential term tends to zero: $x \rightarrow 0.6 \sin t - 1.2 \cos t$: (this is oscillatory motion) Amplitude = $0.6\sqrt{5}$ (= 1.34)	B1 B1 [2]	ft aef
	(iv)	$\frac{d}{dx}(\ddot{x} + 2\dot{x} - 3x) = 0$ $x = Pe^{-3t} + Qe^t + R$ $t = 0, x = 0: P + Q + R = 0$ $t = 0, \dot{x} = k: -3P + Q = k$ $t = 0, \ddot{x} = 6: 9P + Q = 6$	M1 A1 M1 M1 M1 M1	Recognise connection to d.e.in (i) or find and solve cubic auxiliary equation Correct or FT their CF from (i) + constant Use condition Differentiate TWICE Use condition Use condition

		$P = \frac{6-k}{12} : Q = \frac{3k+6}{4} : R = \frac{-(6+2k)}{3}$ $x = \frac{6-k}{12} e^{-3t} + \frac{3k+6}{4} e^t - \frac{2k+6}{3}$	M1 A1 [8]	Solve to find 3 constants cao aef
	(v)	To remain bounded, Q must be zero $3k + 6$ is always positive, so displacement cannot remain bounded.	M1 A1 [2]	Ft their Q from (iv) cao, dependent on final A1 in (iv)
2.	(a)	$\frac{dv}{v(v+2)} = -0.25dt$ $\frac{1}{2} \left(\frac{1}{v} - \frac{1}{v+2} \right) dv = -0.25dt$ $\ln v - \ln(v+2) = -0.5t + A$ $A = \ln \frac{10}{11}$ $v = \frac{20e^{-0.5t}}{11 - 10e^{-0.5t}} \text{ or } v = \frac{20}{11e^{0.5t} - 10}$	M1 M1* Dep *M1 A1 B1 M1 M1 A1 [8]	Separate variables Partial fractions Integrate both sides LHS correct RHS correct, including arbitrary constant on one side Use condition Rearrange to make v the subject in terms of exponentials cao
	(b)(i)	Divide through by x IF: $e^{\int -\frac{4}{x} dx}$ $= e^{-4 \ln x}$ $= x^{-4}$ $\frac{d}{dx} (yx^{-4}) = x^{-3/2}$	B1 M1 A1 A1 M1	Both sides Allow sign error cao Multiply both sides by IF and express LHS as differential of y times their IF

		$yx^{-4} = -2x^{-\frac{1}{2}} + C$ $C = 2$ $y = 2x^4 - 2x^{7/2}$	M1 A1 M1 M1 A1 [10]	Integrate both sides Including + C Use condition Divide BOTH sides by their IF (including the C) cao, aef
	(ii)	$8x^3 - 7x^{5/2} = 0$ $x = 49/64$	M1 A1 [2]	Differentiate and equate to zero Solve
	(iii)	When $x = 1$, $\frac{dy}{dx} = 1$ As $x \rightarrow 0$, $y \rightarrow 0$ Sketch with curve approaching (0, 0) and going through (1, 0) with gradient 1 Correct sketch, including behaviour for large x	B1ft B1ft B1 B1 [4]	 cao for graph of correct solution to (i) cao for graph of correct solution to (i)
3.	(a)(i)	Circle, centre (0, 0) radius m	B1 [1]	
	(ii)	Sketch circles with correct radii Direction arrows: at least 4 on each of at least 2 isoclines. All correct	B1 B1 B1 B1 [4]	2 correct Other 2 correct
	(iii)	Attempt at one solution curve One correct curve	M1 A1	Must use their tangent field

		Second correct curve	A1																				
			[3]																				
(iv)	$\frac{dy}{dx} = \sqrt{x^2 + y^2}$ <table border="1" style="margin-left: 20px;"> <thead> <tr> <th>x</th> <th>y</th> <th>y'</th> <th>hy'</th> </tr> </thead> <tbody> <tr> <td>0.5</td> <td>0.5</td> <td>0.7071076...</td> <td>0.035355...</td> </tr> <tr> <td>0.55</td> <td>0.535355...</td> <td>0.767531...</td> <td>0.038376...</td> </tr> <tr> <td>0.6</td> <td>0.573731...</td> <td>0.830161...</td> <td>0.041508...</td> </tr> <tr> <td>0.65</td> <td>0.615239...</td> <td></td> <td></td> </tr> </tbody> </table> <p>0.615</p>	x	y	y'	hy'	0.5	0.5	0.7071076...	0.035355...	0.55	0.535355...	0.767531...	0.038376...	0.6	0.573731...	0.830161...	0.041508...	0.65	0.615239...			M1 A1 A1 A1	Use algorithm (evidenced by numerical values) 0.535355... 0.573731... 0.615 or better
x	y	y'	hy'																				
0.5	0.5	0.7071076...	0.035355...																				
0.55	0.535355...	0.767531...	0.038376...																				
0.6	0.573731...	0.830161...	0.041508...																				
0.65	0.615239...																						
(v)	Reduce step length		B1 [1]																				
(b)(i)	Solve auxiliary equation: $m+1=0, m=-1$ CF: Ae^{-x} PI: $y = px^2 + qx + r$ $p=1, 2p+q=0, q+r=-1$ $q=-2, r=1$ $y = Ae^{-x} + x^2 - 2x + 1$	M1 A1 B1 M1 A1 F1	Differentiate, substitute and equate coefficients FT their CF with one arbitrary constant + their PI																				
(ii)	$A=2: y = 2e^{-x} + x^2 - 2x + 1$	M1 A1	Use condition [2]																				
(iii)	$y = 2e^{-x} + (x-1)^2$	B1	First term is always positive; second term is non-negative (must see perfect square)																				

		Correct shaped curve, always positive Correct curve through (0, 3)	B1 B1 [3]	
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Question	Answer	Marks	Guidance
4. (i)	$\ddot{x} = 2\dot{x} + 2\dot{y}$ $\ddot{x} = 2\dot{x} + 2(6y - 4x)$ $2y = \dot{x} - 2x$ $\ddot{x} - 8\dot{x} + 20x = 0$ AE $m^2 - 8m + 20 = 0$ $m = 4 \pm 2i$ CF $x = e^{4t}(A \cos 2t + B \sin 2t)$	M1 M1 M1 A1 M1 A1 F1 [7]	Differentiate Substitute for \dot{y} Substitute for y oe
(ii)	$\dot{x} = e^{-4t}(-2A \sin 2t + 2B \cos 2t + 4A \cos 2t + 4B \sin 2t)$ Use $2y = \dot{x} - 2x$ Substitute x and \dot{x} $y = e^{4t}((A + B) \cos 2t + (B - A) \sin 2t)$	M1 M1 M1 A1 [4]	Differentiate x as a product Substitute into expression for y Simplify
(iii)	$t = 0, \dot{x} = 10: 2B + 4A = 10$ $A + B = kA$ $x = e^{4t} \left(\frac{5}{k+1} \cos 2t + \frac{5(k-1)}{k+1} \sin 2t \right)$ $y = e^{4t} \left(\frac{5k}{k+1} \cos 2t + \frac{5(k-2)}{k+1} \sin 2t \right)$	M1 M1 M1 A1 A1 [5]	Use condition Use condition $y = kx$ when $t = 0$ Solve equations
(iv)	A species dies out when x or $y = 0$	B1	

Question	Answer	Marks	Guidance
	$\cos 2t + 5\sin 2t = 0$ or $6\cos 2t + 4\sin 2t = 0$ $\tan 2t = -\frac{1}{5}$ or $\tan 2t = -\frac{6}{4}$ Solves one of these equations: $t = 1.47$ or $t = 1.08$ Species Y dies out first, $t = 1.08$	M1 A1 M1 A1 A1 F1 [7]	Uses $k = 6$ to obtain one of these equations Only one expression required Second value of t Correct conclusion for their values of t
(v)	Model does not hold once one species has died out	B1 [1]	

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4758 Differential Equations (Written Examination)

General Comments:

Candidates performed well on this paper and the majority of the responses were of a high standard. The level of accuracy displayed by most candidates was commendable. The methods required to solve the second order differential equations in Questions 1 and 4 were known by almost all candidates and these two questions were attempted by the majority of the candidates. Question 3 was the least popular question. Very few candidates attempted more than three questions.

Comments on Individual Questions:

Question No. 1

Second order linear differential equations

- (i) All candidates were familiar with the method of solution required in this part and there were very few arithmetical errors.
- (ii) A common error in this part was to interpret the condition “the particle is initially at rest” as meaning that the particle was initially at the origin.
- (iii) Almost all candidates noted that for large values of t the exponential term in the solution to part (ii) tended to zero, leaving a trigonometric expression.
- (iv) Very few candidates spotted the connection between this part and the previous parts of the question. Most candidates worked from scratch, finding a new auxiliary equation and complementary function. The only errors that occurred were arithmetical or algebraic in solving the simultaneous equations involving k .
- (v) A significant number of candidates did not provide a convincing explanation. It was necessary to highlight the coefficient, $3k + 6$, of the exponential term with a positive power and use the positivity of k to show that this coefficient was never zero.

Question No. 2

First order differential equations

- (a) Almost all candidates recognised that the given differential equation required the application of the separation of variables method. A pleasing number went on to use partial fractions to enable integration and worked with accuracy to find v in terms of t .
- (b)(i) Almost all candidates recognised that the given differential equation required the application of the integrating factor method and most began correctly by dividing through by x , the coefficient of $\frac{dy}{dx}$. Most candidates found the correct integrating factor and worked through accurately to find the particular solution.
- (ii) This part was well-answered.
- (iii) The majority of candidates stated the values that were requested and then used them to produce a good sketch.

Question No. 3

First order differential equations

This was the least popular choice of question.

- (a)(i)** This was well-answered.
- (ii)** Most candidates realised that the isoclines were circles, and were given credit in spite of the inaccuracy of their sketches. The tangent field sketches were less successful, often with only a couple of lines drawn on each of the circles. Candidates are advised to draw a good number of these lines, not least because it will help them in drawing solution curves.
- (iii)** The solution curves were usually drawn well by those candidates who used their tangent fields from part (ii). A minority of candidates drew curves that had no connection with what they had already done.
- (iv)** Almost all candidates obtained the correct estimate, and it was pleasing to see that their work was usually presented neatly in a table.
- (v)** This was almost always correctly answered.
- (b)(i)** Most candidates earned full marks in this part. The only errors were arithmetical slips in finding the particular integral.
- (ii)** This was well-answered.
- (iii)** This part discriminated well between the candidates. Some produced very convincing arguments for y being always positive, by expressing the quadratic term as a perfect square. Others looked at each term individually and rarely made any real progress. The sketch graphs were variable in quality. It is worth noting that in such requests for sketches candidates are not expected to do any extra calculations. They have been given or asked to find sufficient information for the sketch.

Question No. 4

Simultaneous second order linear differential equations

- (i)** There were many excellent responses to this part and the majority of the candidates scored full marks.
- (ii)** Almost all candidates gained the three method marks and the majority also gained the accuracy mark. The only errors were arithmetical.
- (iii)** There were many good accurate solutions to this part. Some candidates had difficulty in knowing how to apply the condition “the population of species Y is k times the population of species X when $t = 0$.” Other candidates made arithmetical and algebraic errors when solving their simultaneous equations.
- (iv)** Most candidates realised that they needed to equate each of their expressions for x and y to zero and obtained at least the method marks.
- (v)** The model used in this question only applied while species X and Y were competing for survival. A comment that acknowledged that the model was no longer valid when one species had died was required here.

Unit level raw mark and UMS grade boundaries June 2017 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)			Max Mark	a	b	c	d	e	u
4751	01 C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	49	45	0
		UMS	100	80	70	60	50	40	0
4752	01 C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	55	49	44	39	34	0
		UMS	100	80	70	60	50	40	0
4753	01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	49	45	41	36	0
4753	02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
		UMS	100	80	70	60	50	40	0
4754	01 C4 – MEI Applications of advanced mathematics (A2)	Raw	90	67	61	55	49	43	0
		UMS	100	80	70	60	50	40	0
4755	01 FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	57	52	47	42	38	0
		UMS	100	80	70	60	50	40	0
4756	01 FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
		UMS	100	80	70	60	50	40	0
4757	01 FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	64	56	48	41	34	0
		UMS	100	80	70	60	50	40	0
4758	01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758	02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
		UMS	100	80	70	60	50	40	0
4761	01 M1 – MEI Mechanics 1 (AS)	Raw	72	57	49	41	34	27	0
		UMS	100	80	70	60	50	40	0
4762	01 M2 – MEI Mechanics 2 (A2)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
4763	01 M3 – MEI Mechanics 3 (A2)	Raw	72	58	50	43	36	29	0
		UMS	100	80	70	60	50	40	0
4764	01 M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
		UMS	100	80	70	60	50	40	0
4766	01 S1 – MEI Statistics 1 (AS)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
4767	01 S2 – MEI Statistics 2 (A2)	Raw	72	56	50	45	40	35	0
		UMS	100	80	70	60	50	40	0
4768	01 S3 – MEI Statistics 3 (A2)	Raw	72	63	57	51	46	41	0
		UMS	100	80	70	60	50	40	0
4769	01 S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771	01 D1 – MEI Decision mathematics 1 (AS)	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0
4772	01 D2 – MEI Decision mathematics 2 (A2)	Raw	72	53	48	43	39	35	0
		UMS	100	80	70	60	50	40	0
4773	01 DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776	01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	58	53	48	43	37	0
4776	02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
4777	01 NC – MEI Numerical computation (A2)	Raw	72	55	48	41	34	27	0

		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2017 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

Level 3 Certificate Mathematics for Engineering				Max Mark	a*	a	b	c	d	e	u
H860	01	Mathematics for Engineering		This unit has no entries in June 2017							
H860	02	Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers				Max Mark	a*	a	b	c	d	e	u
H865	01	Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H866	02	Critical maths	Raw	60*	48	42	36	30	24	0
			Overall	144	112	97	83	70	57	0

*Component 02 is weighted to give marks out of 72

Level 3 Certificate Mathematics - Quantitative Problem Solving (MEI) (GQ Reform)				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H867	02	Statistical problem solving	Raw	60*	41	36	31	27	23	0
			Overall	144	103	90	77	66	56	0

*Component 02 is weighted to give marks out of 72

Advanced Free Standing Mathematics Qualification (FSMQ)				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	72	63	55	47	39	0

Intermediate Free Standing Mathematics Qualification (FSMQ)				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0