RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4751/01**

MATHEMATICS (MEI)

Introduction to Advanced Mathematics (C1)

THURSDAY 15 MAY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

WARNING

You are not allowed to use a calculator in this paper.

This document consists of **4** printed pages.

Section A (36 marks)

1 Solve the inequality $3x - 1 > 5 - x$. [2]

2 **(i)** Find the points of intersection of the line $2x + 3y = 12$ with the axes. [2]

(ii) Find also the gradient of this line. [2]

3 **(i)** Solve the equation $2x^2 + 3x = 0$. [2]

(ii) Find the set of values of k for which the equation $2x^2 + 3x - k = 0$ has no real roots. [3]

4 Given that n is a positive integer, write down whether the following statements are always true (T), always false (F) or could be either true or false (E).

(i) $2n + 1$ is an odd integer

(ii) $3n + 1$ is an even integer

(iii) n is odd $\Rightarrow n^2$ is odd

(iv) n^2 is odd $\Rightarrow n^3$ is even [3]

5 Make x the subject of the equation $y = \dfrac{x+3}{x-2}$. [4]

6 **(i)** Find the value of $\left(\frac{1}{25}\right)^{-\frac{1}{2}}$. [2]

(ii) Simplify $\dfrac{(2x^2y^3z)^5}{4y^2z}$. [3]

7 **(i)** Express $\dfrac{1}{5+\sqrt{3}}$ in the form $\dfrac{a+b\sqrt{3}}{c}$, where a, b and c are integers. [2]

(ii) Expand and simplify $\left(3 - 2\sqrt{7}\right)^2$. [3]

8 Find the coefficient of x^3 in the binomial expansion of $(5 - 2x)^5$. [4]

9 Solve the equation $y^2 - 7y + 12 = 0$.

Hence solve the equation $x^4 - 7x^2 + 12 = 0$. [4]

Section B (36 marks)

10 **(i)** Express $x^2 - 6x + 2$ in the form $(x - a)^2 - b$. [3]

(ii) State the coordinates of the turning point on the graph of $y = x^2 - 6x + 2$. [2]

(iii) Sketch the graph of $y = x^2 - 6x + 2$. You need not state the coordinates of the points where the graph intersects the x-axis. [2]

(iv) Solve the simultaneous equations $y = x^2 - 6x + 2$ and $y = 2x - 14$. Hence show that the line $y = 2x - 14$ is a tangent to the curve $y = x^2 - 6x + 2$. [5]

11 You are given that $\mathrm{f}(x) = 2x^3 + 7x^2 - 7x - 12$.

(i) Verify that $x = -4$ is a root of $\mathrm{f}(x) = 0$. [2]

(ii) Hence express $\mathrm{f}(x)$ in fully factorised form. [4]

(iii) Sketch the graph of $y = \mathrm{f}(x)$. [3]

(iv) Show that $\mathrm{f}(x - 4) = 2x^3 - 17x^2 + 33x$. [3]

12 **(i)** Find the equation of the line passing through A $(-1, 1)$ and B $(3, 9)$. [3]

(ii) Show that the equation of the perpendicular bisector of AB is $2y + x = 11$. [4]

(iii) A circle has centre $(5, 3)$, so that its equation is $(x - 5)^2 + (y - 3)^2 = k$. Given that the circle passes through A, show that $k = 40$. Show that the circle also passes through B. [2]

(iv) Find the x-coordinates of the points where this circle crosses the x-axis. Give your answers in surd form. [3]

4751 (C1) Introduction to Advanced Mathematics

Section A

1	$x > 6/4$ o.e. isw	2	M1 for $4x > 6$ or for 6/4 o.e. found or for their final ans ft their $4x > k$ or $kx > 6$	2
2	(i) (0, 4) and (6, 0) (ii) −4/6 o.e. or ft their (i) isw	2 2	1 each; allow $x = 0$, $y = 4$ etc; condone $x = 6$, $y = 4$ isw but 0 for (6, 4) with no working 1 for $-\frac{4}{6}x$ or 4/−6 or 4/6 o.e. or ft (accept 0.67 or better) 0 for just rearranging to $y = -\frac{2}{3}x + 4$	4
3	(i) 0 or −3/2 o.e. (ii) $k < -9/8$ o.e. www	2 3	1 each M2 for 3^2 (−)(−8k) < 0 o.e. or −9/8 found or M1 for attempted use of $b^2 - 4ac$ (may be in quadratic formula); SC: allow M1 for $9 - 8k < 0$ and M1 ft for $k > 9/8$	5
4	(i) T (ii) E (iii) T (iv) F	3	3 for all correct, 2 for 3 correct. 1 for 2 correct	3
5	$y(x - 2) = (x + 3)$ $xy - 2y = x + 3$ or ft [ft from earlier errors if of comparable difficulty – no ft if there are no xy terms] $xy - x = 2y + 3$ or ft $[x =]\dfrac{2y+3}{y-1}$ o.e. or ft <u>alt method:</u> $y = 1 + \dfrac{5}{x-2}$ $y - 1 = \dfrac{5}{x-2}$ $x - 2 = \dfrac{5}{y-1}$ $x = 2 + \dfrac{5}{y-1}$	M1 M1 M1 M1 M1 M1 M1 M1	for multiplying by $x - 2$; condone missing brackets for expanding bracket and being at stage ready to collect x terms for collecting x and 'other' terms on opposite sides of eqn for factorising and division for either method: award 4 marks only if fully correct	4

18

6	(i) 5 www (ii) $8x^{10}y^{13}z^4$ or $2^3x^{10}y^{13}z^4$	2 3	allow 2 for ±5; M1 for $25^{1/2}$ seen or for 1/5 seen or for using $25^{1/2} = 5$ with another error (ie M1 for coping correctly with fraction and negative index or with square root) mark final answer; B2 for 3 elements correct, B1 for 2 elements correct; condone multn signs included, but −1 from total earned if addn signs	5	
7	(i) $\frac{5-\sqrt{3}}{22}$ or $\frac{5+(-1)\sqrt{3}}{22}$ or $\frac{5-1\sqrt{3}}{22}$ (ii) 37 − 12√ 7 isw www	2 3	or $a = 5$, $b = -1$, $c = 22$; M1 for attempt to multiply numerator and denominator by $5-\sqrt{3}$ 2 for 37 and 1 for − 12√ 7 or M1 for 3 correct terms from 9 − 6√ 7 − 6√ 7 + 28 or 9 − 3√ 28 − 3√ 28 + 28 or 9 − √ 252 − √ 252 + 28 o.e. eg using 2√ 63 or M2 for 9 − 12√ 7 + 28 or 9 − 6√ 28 + 28 or 9 − 2√ 252 + 28 or 9 − √ 1008 + 28 o.e.; 3 for 37 − √ 1008 but not other equivs	5	
8	−2000 www	4	M3 for $10 \times 5^2 \times (-2[x])^3$ o.e. or M2 for two of these elements or M1 for 10 or (5×4×3)/(3×2×1) o.e. used [$^5C_3$ is not sufficient] or for 1 5 10 10 5 1 seen; or B3 for 2000; condone x^3 in ans; equivs: M3 for e.g $5^5 \times 10 \times \left(-\frac{2}{5}[x]\right)^3$ o.e. [5^5 may be outside a bracket for whole expansion of all terms], M2 for two of these elements etc similarly for factor of 2 taken out at start	4	
9	$(y-3)(y-4)$ [= 0] $y = 3$ or 4 cao $x = \pm\sqrt{3}$ or ± 2 cao	M1 A1 B2	for factors giving two terms correct or attempt at quadratic formula or completing square or B2 (both roots needed) B1 for 2 roots correct or ft their y (condone √ 3 and √ 4 for B1)	4	18

Section B

10	**i**	$(x-3)^2-7$	3	mark final answer; 1 for $a = 3$, 2 for $b = 7$ or M1 for $-3^2 + 2$; bod 3 for $(x - 3) - 7$	3	
	ii	(3, −7) or ft from (i)	1+1		2	
	iii	sketch of quadratic correct way up and through (0, 2)	G1	accept (0, 2) o.e. seen in this part [eg in table] if 2 not marked as intercept on graph		
		t.p. correct or ft from (ii)	G1	accept 3 and −7 marked on axes level with turning pt., or better; no ft for (0, 2) as min	2	
	iv	$x^2 - 6x + 2 = 2x - 14$ o.e.	M1	or their (i) = $2x - 14$		
		$x^2 - 8x + 16$ [= 0]	M1	dep on first M1; condone one error		
		$(x - 4)^2$ [= 0]	M1	or correct use of formula, giving equal roots; allow $(x + 4)^2$ o.e. ft $x^2 + 8x + 16$		
		$x = 4$, $y = -6$	A1	if M0M0M0, allow SC2 for showing (4, −6) is on both graphs (need to go on to show line is tgt to earn more)		
		equal/repeated roots [implies tgt] - must be explicitly stated; condone 'only one root [so tgt]' or 'line meets curve only once, so tgt' or 'line touches curve only once' etc]	A1	or for use of calculus to show grad of line and curve are same when $x = 4$	5	12

11	**i**	f(−4) used	M1			
		$-128 + 112 + 28 - 12$ [= 0]	A1	or B2 for $(x + 4)(2x^2 - x - 3)$ here; or correct division with no remainder	2	
	ii	division of f(x) by (x + 4)	M1	as far as $2x^3 + 8x^2$ in working, or two terms of $2x^2 - x - 3$ obtained by inspection etc (may be earned in (i)), or f(−1) = 0 found		
		$2x^2 - x - 3$	A1	$2x^2 - x - 3$ seen implies M1A1		
		$(x + 1)(2x - 3)$	A1			
		[f(x) =] $(x + 4)(x + 1)(2x - 3)$	A1	or B4; allow final A1 ft their factors if M1A1A0 earned	4	
	iii	sketch of cubic correct way up	G1	ignore any graph of $y = f(x - 4)$		
		through −12 shown on y axis	G1	or coords stated near graph		
		roots −4, −1, 1.5 or ft shown on x axis	G1	or coords stated near graph if no curve drawn, but intercepts marked on axes, can earn max of G0G1G1	3	
	iv	$x(x - 3)(2[x - 4] - 3)$ o.e. or $x(x - 3)(x - 5.5)$ or ft their factors	M1	or $2(x - 4)^3 + 7(x - 4)^2 - 7(x - 4) - 12$ or stating roots are 0, 3 and 5.5 or ft; condone one error eg $2x - 7$ not $2x - 11$		
		correct expansion of one pair of brackets ft from their factors	M1	or for correct expn of $(x - 4)^3$ [allow unsimplified]; or for showing g(0) = g(3) = g(5.5) = 0 in given ans g(x)		
		correct completion to given answer	M1	allow M2 for working backwards from given answer to $x(x - 3)(2x - 11)$ and M1 for full completion with factors or roots	3	12

12	**i**	grad AB = $\frac{9-1}{3--1}$ or 2 $y - 9 = 2(x - 3)$ or $y - 1 = 2(x + 1)$ $y = 2x + 3$ o.e.	M1 M1 A1	ft their *m,* or subst coords of A or B in y = their $m\,x + c$ or B3	3	
	ii	mid pt of AB = (1, 5) grad perp = −1/grad AB $y - 5 = -\frac{1}{2}(x - 1)$ o.e. or ft [no ft for just grad AB used] at least one correct interim step towards given answer $2y + x = 11$, and correct completion NB ans $2y + x = 11$ given alt method working back from ans: $y = \frac{11-x}{2}$ o.e. grad perp = −1/grad AB and showing/stating same as given line finding intn of their $y = 2x + 3$ and $2y + x = 11$ [= (1, 5)] showing midpt of AB is (1, 5)	M1 M1 M1 M1 M1 M1 M1 M1	condone not stated explicitly, but used in eqn soi by use eg in eqn ft their grad and/or midpt, but M0 if their midpt not used; allow M1 for $y = -\frac{1}{2}x + c$ and then their midpt subst no ft; correct eqn only mark one method or the other, to benefit of cand, not a mixture eg stating $-\frac{1}{2} \times 2 = -1$ or showing that (1, 5) is on $2y + x = 11$, having found (1, 5) first [for both methods: for M4 must be fully correct]	4	
	iii	showing $(-1 - 5)^2 + (1 - 3)^2 = 40$ showing B to centre = √40 or verifying that (3, 9) fits given circle	M1 M1	at least one interim step needed for each mark; M0 for just $6^2 + 2^2 = 40$ with no other evidence such as a first line of working or a diagram; condone marks earned in reverse order	2	
	iv	$(x - 5)^2 + 3^2 = 40$ $(x - 5)^2 = 31$ $x = 5 \pm \sqrt{31}$ or $\frac{10 \pm \sqrt{124}}{2}$ isw	M1 M1 A1	for subst $y = 0$ in circle eqn condone slip on rhs; or for rearrangement to zero (condone one error) <u>and</u> attempt at quad. formula [allow M1 M0 for $(x - 5)^2 = 40$ or for $(x - 5)^2 + 3^2 = 0$] or $5 \pm \frac{\sqrt{124}}{2}$	3	12

4751 Introduction to Advanced Mathematics (C1)

General Comments

The usual spread of candidates, from very good to extremely weak, was seen. Time did not appear to be a problem, even for weak candidates, with most parts of the last question usually being attempted.

Compared to some recent past papers, there were perhaps fewer parts in this paper that hindered good candidates from obtaining high marks, so that more candidates gained over 60 marks, for instance, than compared with last June. Some topics, such as using the discriminant to determine when the roots of a quadratic equation are real, remain poorly done by many candidates.

Some centres continue to issue graph paper to their candidates in the examination. This is to their disadvantage, when some with graph paper then spend time attempting plotted graphs (often with inappropriate scales) when all that is required is a sketch graph.

The examiners are also concerned that some candidates may not be sufficiently practised in non-calculator work. There were fewer fractions used in this paper than in last January's, but nonetheless poor arithmetic can lower the mark considerably for some candidates.

Comments on Individual Questions

Section A

1) This ought to have been an easy starter. Many picked up both marks, but some did not reach $4x > 6$ and some who did then gave $x > \frac{2}{3}$.

2) Most knew what to do and many gained full marks. To find the intercepts, those who rearranged to $y = -\frac{2}{3}x + 4$ before substituting $y = 0$ often gave themselves too hard a task in finding x. Some gave just the y- intercept. The usual errors were seen in the gradient, with $\frac{2}{3}$ and $-\frac{2}{3}x$ being seen more than occasionally. Thankfully, the gradient was rarely inverted.

3) Although many were able quickly to factorise and solve $2x^2 + 3x = 0$, weaker candidates often found this difficult, with the formula being frequently used and an error often made such as $4 \times 2 \times 0 = 8$ or in not knowing what to do with $\frac{0}{4}$. Some rearranged the equation then divided by x and found only one root.

   In the second part, many did not know the condition for real roots. A common error was to substitute k instead of $-k$ for c, often then making errors with the resulting negative coefficient in the inequality.

4) This was reasonably well-answered, with parts (i) and (iii) usually correct; part (ii) presented the most problems to candidates, with 'false' instead of 'either' being a common response. Many candidates showed no working.

5) Many candidates made a good start on rearranging the equation by correctly multiplying by (x − 2), with relatively few omitting the brackets. Those who knew the strategy to use often proceeded to gain the rest of the marks, but some floundered from here. Some weaker candidates started by multiplying the right-hand side by $\frac{x+2}{x+2}$, then multiplying out and often 'cancelling' the x^2 terms. Some candidates used spare time at the end of the examination to have another attempt at this question after failing to manage it first time round – some of these later attempts were successful.

6) In this question on indices, the correct answer of 5 in the first part was common, sometimes given with no working shown. Of those who did not gain 2 marks, many knew that a square root was involved, often reaching $\frac{1}{5}$ or $-\frac{1}{5}$, but not managing to cope correctly with the reciprocal. A few thought the index was also inverted and calculated 25^2.

In the second part, those who did not gain full marks often had partially correct answers, with the most common error being to think that $\left(x^2\right)^5 = x^7$ and so on, as expected.

7) Most of the candidates made an attempt at the first part and realised that they needed to multiply the numerator and denominator by 5 - √3. A few used 5 + √3. However many of the candidates made an error in determining the new denominator, with expressions such as 25 + 3 or 25 − 9 being used.

In the second part, most candidates obtained a mark for getting 9 – 6√7 – 6√7 ….. . However the final term was often wrong (such as 28, 14 or 4√7). Some errors were made in combining the terms involving √7.

8) Although most candidates had some idea about what was required here, many of them were unable to cope with the $(-2)^3$ element. It often appeared with the x included and in the form $-2x^3$; even if it was written as $(-2x)^3$, it was frequently evaluated as $-2x^3$. The negative sign was also often dropped. There were also a number of candidates who only included two of the required elements (such as 10 x 25, or 10 x -2^3). A few candidates tried to take a factor of 5 outside the brackets but they commonly then made errors. Very few candidates tried to multiply out $(5 - 2x)^5$, but of those who did most failed to do so correctly.

9) Most candidates were able to factorise (or use the formula) and of these most went on to arrive at the values of y. Only a minority realised that there was a connection between this equation and the next. As a consequence few candidates gained full marks on this question as they were unable to determine four roots from the quartic equation. Of those who did realise that x^2 was equal to 3 or 4 many only gave the two positive roots. 2 was sometimes left in the form √4.

Section B

10 (i) Completing the square was done better than expected, aided by the fact that no fractions were involved this time. Many candidates had clearly learned a formula for this. Some got as far as $(x - 3)^2$ but did not know how to proceed from there.

(ii) Although some realised the relevance to part (i) and just wrote down the answer as expected, a surprising number started again and used calculus to obtain the result, sometimes making errors in the process.

(iii) In sketching the graph, most knew the general shape of a parabola but many omitted the fact that it went though (0, 2), often having a graph with a negative *y*-intercept from estimating the general direction of the curve. Some did not use their turning point from part (ii) but instead had the minimum at (0, 2) or (3, 0).

(iv) Most candidates equated the two expressions for *y* and many then rearranged successfully and went on to obtain $x = 4$ as the only root. Forgetting to then go on to obtain the *y* value was a frequent error. Many did not realise (or failed to state) that the equal roots implied that the line was a tangent. Instead, some successfully used calculus to show that the gradients of the curve and the line were the same at (4, −6). Weaker candidates who had got this far sometimes thought that showing that (4, −6) satisfied both equations was sufficient to imply the line was a tangent. Candidates who started this part by substituting $\frac{y+7}{2}$ for *x* rarely made progress.

11 In general, this was the section B question which caused most difficulty to candidates, particularly part (iv).

(i) Many gained both marks, with calculating f(−4) being the most common method. Working out 7 × 16 proved beyond some, whilst the other main error was in −7(−4) becoming −28. More able candidates often divided by $(x + 4)$ and positioned themselves well for part (ii).

(ii) Many coped correctly with the division and achieved $2x^2 - x - 3$, with fewer sign errors being seen than in some past papers. Some obtained this by inspection. Although many proceeded correctly from here, some made errors in factorising the quadratic factor, or resorted to using the formula, often failing to cope with the fraction in factorising after this.

(iii) Most candidates realised the link with part (ii), with follow-through marks from their factors or roots allowing some to obtain full marks in spite of errors in part (ii). Omission of the *y*-intercept label was the most common error. Those who used graph paper often struggled with the scales, with the *y*-intercept of −12 producing a graph that they then found difficult to draw with a smooth curve.

(iv) The easiest way of showing the required result was to realise that the roots found in previous parts were increased by 4 and to work with $x(x + 3)(2x - 11)$. Some of the most able candidates did this, although some did not cope correctly with the 2 and used $(x - 5.5)$ or $(2x - 7)$. The majority who attempted a correct method went down the route of substituting $(x - 4)$ for *x* in the non-factorised form of f(*x*). This then meant they had to work out an involved algebraic expression, including expanding $2(x - 4)^3$. Some achieved this correctly, but often sign errors at various stages stopped progress. Weaker candidates often did not know what to do, with some attempting to divide the given expression by ($x - 4$) or substituting 4 or -4 in it, making no progress and gaining no marks.

12 (i) Finding the equation of the line joining the two points was done well by most candidates.

(ii) This was done less well than part (i), with many candidates failing to give complete reasoning. Most knew the condition for the gradients of perpendicular lines, but many failed to appreciate that the line went through the midpoint of AB. Many candidates worked backwards from the given equation in order to determine its gradient and used the perpendicularity condition, which afforded only a partial solution. Those using the 'backwards' method who went on to show that the intersection of the lines was the midpoint of AB were able to gain full marks.

(iii) Many candidates successfully showed that $k = 40$ and that B was on the circle. There was some confusion between 40 and $\sqrt{40}$. Some used long methods such as expanding $(-1 - 5)^2$ term by term.

(iv) Only the better candidates gained full marks here. Many candidates realised that they needed to put $y = 0$ into the equation of the circle (although a few put $x = 0$). Some thought that $(0 - 3)^2 = 0$. Candidates did not always use the equation in its simplest form, or did not realise the easy route of solution from having reached $(x - 5)^2 = 31$ and as a consequence they then needed to use the quadratic formula.