OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS — 2615

Statistics 3

Monday **20 JUNE 2005** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 The weekly sales of petrol at a petrol station, measured in hundreds of thousands of litres, are modelled by the continuous random variable X with probability density function

$$f(x) = \begin{cases} 12x^2(1-x), & 0 \leqslant x \leqslant 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(i) Find the mode of X. [2]

(ii) Sketch the graph of $f(x)$. [3]

(iii) Find the mean and variance of X. [5]

(iv) Use a suitable approximation to find the probability that total sales in a year (52 weeks, all considered independent) exceed 3 million litres. [5]

2 As part of a fitness training programme, army recruits have to complete an exercise consisting of three components A, B and C. The times, in minutes, taken for recruits to complete these components are Normally distributed random variables as follows.

Component A	time X	mean 12.6	standard deviation 2.2
Component B	time Y	mean 8.8	standard deviation 1.6
Component C	time Z	mean 20.4	standard deviation 3.2

The components test different physical skills, so X, Y and Z may be taken as independent. Transfer from one component to another is considered as instantaneous.

(i) Find the probability that a randomly chosen recruit will complete the entire exercise in less than 40 minutes. [3]

(ii) One recruit, Graham, undertakes the components in the order A, B, C. Another, Hugh, starts with component C. They start the exercise simultaneously. Find the probability that Hugh completes component C before Graham starts component C. [4]

An instructor tries a new technique for training recruits for component C, hoping to reduce both the mean time to complete it and the standard deviation. The times, in minutes, taken to complete component C by the first 8 recruits trained by this new technique are as follows.

18.2 16.4 21.6 22.8 18.6 18.3 19.6 20.5

(iii) Regarding these 8 recruits as a random sample from the underlying population of recruits trained by the new technique, obtain a two-sided 95% confidence interval for the population mean time to complete component C. Hence comment on whether the technique appears to have been successful in reducing this mean time. [6]

(iv) Comment briefly on whether it is reasonable to regard these 8 recruits as a random sample from the underlying population. [2]

3 A manufacturer produces large electric fans designed for ventilating industrial premises. A standard measure of the efficiency of these fans has been devised by health and safety inspectors. It is specified that this efficiency measure should on average not be less than 540 units. Over the whole population of fans produced by this manufacturer, the efficiency measure is modelled by a Normally distributed random variable whose standard deviation is known to be 14 units.

(i) A random sample of 12 fans has the following efficiency measures.

531.2   529.4   548.6   537.5   522.2   534.6   556.0   533.8   544.1   535.6   526.3   559.7

Carry out a 5% test of significance to examine whether it appears that the fans are being made in compliance with the specification for this efficiency measure, stating clearly the null and alternative hypotheses and the conclusion. [10]

(ii) Calculate the probability of a Type II error for the test if in fact the population mean efficiency measure is 530 units. [5]

4 As part of an experiment in a nuclear research establishment, scientists need to study the amount of background radiation and the level of radiation from the experiment itself.

(i) Radiation counts of the background radiation are made during 100 separate 10-second periods (considered as a random sample) using a Geiger counter, with the following results.

Radiation count	Number of 10-second intervals
0	19
1	25
2	22
3	18
4	5
5	9
6	2
more than 6	0

(*A*) Show that the sample mean is 2. [1]

(*B*) Use a suitable statistical procedure and a 5% significance level to assess the goodness of fit of a Poisson distribution to the background radiation counts, and comment briefly. [10]

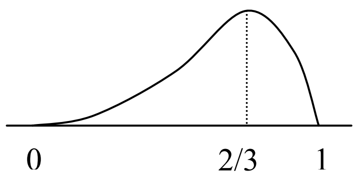
(ii) The radiation levels from the experiment itself during these 100 periods are measured on a continuous scale using a different instrument. These readings are summarised by

$$\sum x = 2426.4, \qquad \sum (x - \bar{x})^2 = 1216.68.$$

Provide a two-sided 95% confidence interval for the true mean radiation level from the experiment, as measured on this scale. [4]

Mark Scheme 2615
June 2005

2615 Statistics 3

Q1	$f(x) = 12x^2(1-x),\ 0 \le x \le 1.$			
(i)	Mode given by $f'(x) = 0$. $f'(x) = 24x - 36x^2$ Which = 0 (at $x = 0$ and) at $x = 2/3$. Mode is 2/3.	M1 A1	For attempting to find $f'(x)$ and set =0. c.a.o. No need to explicitly confirm maximum. Do NOT allow if it happens to ft from an incorrect $f'(x)$.	 2
(ii)		G1 G1 G1	Correct general shape (anything continuous, smooth and unimodal, in [0, 1]). Maximum at $x = 2/3$ (ft candidate's mode). Slope 0 at $x = 0$ <u>and</u> steeply descending at $x = 1$.	 3
(iii)	$E(X) = \int_0^1 12x^3(1-x)\,dx$ $= \left[12\frac{x^4}{4} - 12\frac{x^5}{5}\right]_0^1 = 3 - \frac{12}{5} = \frac{3}{5}$ $E(X^2) = \int_0^1 12x^4(1-x)\,dx$ $= \left[12\frac{x^5}{5} - 12\frac{x^6}{6}\right]_0^1 = \frac{12}{5} - \frac{12}{6} = \frac{2}{5}$ $\text{Var}(X) = E(X^2) - (E(X))^2 = \frac{2}{5} - \left(\frac{3}{5}\right)^2 = \frac{1}{25}$	M1 A1 M1 A1 A1	Integral for $E(X)$ including limits (which may appear later). Integral for $E(X^2)$ including limits (which may appear later). ft from candidate's values unless $\text{Var} \le 0$.	 5
(iv)	$X_1 + X_2 + \ldots + X_{52} \sim \text{approx N}(31\cdot 2,\ 2\cdot 08)$ $P(\text{this} > 30) = P\left(Z > \frac{30 - 31\cdot 2}{\sqrt{2\cdot 08}} = -0\cdot 832(05)\right)$ $= 0\cdot 797(3)$	B1 B1F B1F M1 A1	Normal. Mean; f.t. candidate's mean × 52. Variance; f.t. candidate's variance (>0) × 52. Accept sd if indicated clearly as such. If the name of the distribution is wrong or missing then allow the marks for the parameters either if they are the conventional parameters for the named distribution or they are named explicitly. For an attempt to standardise a reasonable Normal distribution. c.a.o. Accept 0·8, 0·80 if clearly correctly obtained.	 5
				15

Q2	$X \sim N(12{\cdot}6, \sigma = 2{\cdot}2)$ $Y \sim N(8{\cdot}8, \sigma = 1{\cdot}6)$ $Z \sim N(20{\cdot}4, \sigma = 3{\cdot}2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	$X + Y + Z \sim N(41{\cdot}8,$ $\sigma^2 = 2{\cdot}2^2 + 1{\cdot}6^2 + 3{\cdot}2^2 = 17{\cdot}64)$ $P(X+Y+Z<40)$ $= P\left(N(0,1) < \frac{40-41\cdot8}{4\cdot2} = -0\cdot4286\right)$ $= 1 - 0\cdot6659 = 0\cdot334(1)$	B1 B1 B1	Mean. Variance. Accept $\sigma = 4{\cdot}2$. c.a.o.	3
(ii)	Want $P(Z < X + Y)$ i.e. $P(Z - X - Y < 0)$ $Z - X - Y \sim N(-1,$ $17{\cdot}64)$ $\therefore P(\text{this} < 0)$ $= P\left(N(0,1) < \frac{0-(-1)}{4\cdot2} = 0\cdot2381\right) = 0\cdot594(1)$	M B1 B1 A1	Or $P(X + Y - Z > 0)$. Mean. Or "+1" for alternative method. Variance. Accept $\sigma = 4{\cdot}2$. N.B. Method and mean should be consistent with each other. c.a.o. Or $P\left(N(0,1) > \frac{0-1}{4\cdot2} = -0\cdot2381\right)$.	4
(iii)	Sample mean = 19·5, $s_{n-1} = 2{\cdot}065(36)$ CI is given by 19·5 ± 2·365 $\times \frac{2\cdot06536}{\sqrt{8}}$ = 19·5 ± 1·72(696) = (17·7(73), 21·2(27)) This interval contains the former mean (20·4), suggesting that there has been no improvement.	B1 M B1 M A1 E1	Allow $s_n = 1{\cdot}931(97)$ only if used correctly in sequel. Must be c's $\bar{x}$ ± … From t_7. Allow c's s_{n-1}, but not 3·2. Allow $s_n/\sqrt{7}$ (see above). c.a.o. Must be written as an interval. Non-assertive comment.	6

(iv)	Reward any reasonable discussion probably to the effect that the first 8 are unlikely to be a random sample.	E2	(E2, E1, E0). Could include discussion in context about how the sample might have been chosen.	2
				15

Q3				
(i)	$H_0 : \mu = 540$ $H_1 : \mu < 540$	B1 B1	Do **not** allow any other symbol, including $\overline{X}$ or similar, unless it is clearly and explicitly stated to be a <u>population</u> mean. Allow statements in words (see below).	
	Where μ is the (population) mean efficiency measure for the fans.	B1	μ must be defined verbally. Must indicate "mean"; condone "average". Allow absence of "population" if correct notation μ is used, otherwise insist on "population".	
	$n = 12$, $\Sigma x = 6459{\cdot}0$, $\bar{x} = 538{\cdot}25$ ($\sigma = 14$ is given.)	B1		
	Test statistic is $\dfrac{538{\cdot}25 - 540}{\left(\dfrac{14}{\sqrt{12}}\right)}$	M1	Allow c's $\bar{x}$. Use of s_{n-1} or s_n gets M0. Allow alternative: 540 – (c's 1·645) × $\frac{14}{\sqrt{12}}$ (= 533·35) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ + (c's 1·645) × $\frac{14}{\sqrt{12}}$ (= 544·90) for comparison with 540.)	
	$= -0{\cdot}433(01)$	A1	c.a.o. (but ft from here if this is wrong.) Use of $540 - \bar{x}$ scores M1A0, but next 4 marks still available.	
	Refer to N(0, 1). Lower 5% point is –1·645. $(\Phi(-0\cdot4330) = 0\cdot3325$, for comparison with $0\cdot05.)$	M1 A1	No ft from here if wrong. Must be <u>minus</u> 1·645 unless absolute values are being compared. No ft from here if wrong.	
	Not significant.	E1	ft only c's test statistic. Explicit comparison required.	
	Reasonable to suppose specification is being met.	E1	ft only c's test statistic. Should be in context with reference either to the mean or to the specification being met.	10
(ii)	If $\mu = 530$, $\overline{X} \sim \text{N}\left(530, \dfrac{14^2}{12}\right)$	M1	For the distribution of $\overline{X}$ with $\mu = 530$, and c's standard error from above.	
	H_0 is accepted if $\overline{X} > 540 - 1.645 \times \dfrac{14}{\sqrt{12}} = 533\cdot35(18)$	M1	For the critical point for the test above. Allow c's –1·645.	
	So P(Type II error) $= \text{P}\left(\text{N}\left(530, \dfrac{14^2}{12}\right) > 533\cdot35\right)$	M1	M0 if RHS = 540 or 538·25.	
	$= \text{P}(\text{N}(0, 1) > 0{\cdot}8289)$	m1	Standardising. Accept awrt 0·829. Depends on the first and third of the preceding M marks.	
	= 1 – (awrt 0·796 or 0·797) = awrt 0·203 or 0·204	A1	<u>This</u> mark is cao.	5
				15

Q4				
(i) (A)	Sample mean = $\frac{\sum fx}{\sum f} = \frac{200}{100} = 2$	B1	Beware printed answer.	1
(B)	(see table below)			

x	0	1	2	3	4	5	6	>6
o	19	25	22	18	5	9	2	0
p	0·1353	0·2707	0·2707	0·1804	0·0902	0·0361	0·0121	0·0045 *
e	13·53	27·07	27·07	18·04	9·02	3·61	1·21	0·45

Combining last 3 cells: o = 11, e = 5·27

* From Poisson(2). These are from cumulative tables. Might differ slightly if calculated directly.

	M	For apparently correct method for e_i's. (>6 cell must be present and not empty, or equivalent if candidate obviously realises to group cells earlier.)
	A1	If all correct <u>or</u> if $\Sigma e_i = 100$. (But A0 if rounded to integers.)
	M	For grouping (cells where $e_i \le 5$).
$X^2 = 2{\cdot}21145 + 0{\cdot}15829 + 0{\cdot}94957 + 0{\cdot}00009 + 1{\cdot}79162 + 6{\cdot}23015 = 11{\cdot}34(12)$	M1	For evidence of correct method for X^2.
	A1	c.a.o. (but ft from here if this is wrong.) e_i to 1 d.p. gives $X^2 = 11{\cdot}27(12)$.
Refer to χ_ν^2, where ν = no of cells in candidate's table – 2 (ideally, $\nu = 4$).	M1	Allow this mark if it agrees with candidate's table, and then ft as below. Accept anything that implies use of this distribution.
For $\nu = 4$ upper 5% point is 9·488. (If ungrouped $\nu = 6$ upper 5% point is 12·59.)	A1	Allow candidate's ν if preceding M1 awarded. No ft from here if not correct point from candidate's χ^2.
Significant.	E1	No f.t. of above M1 or A1 if wrong, except for <u>Special Case:</u> $\nu + 1$ and its 5% point can get EITHER (but not both) of these 2 marks for the conclusion. ($\nu = 5$, cv = 11·07)

	Seems Poisson does not fit.	E1	"Model does not fit data" NOT "data do not fit model".	
	The main discrepancy is in the "top" cell, where there are substantially too many observations for the model to explain. Other discrepancies are comparatively small.	E1	Accept any reasonable descriptive comment e.g. about discrepancies.	10
(ii)	$\bar{x} = 24 \cdot 264 \quad s^2 = \frac{1216 \cdot 68}{99} = 12 \cdot 2897 = 3 \cdot 50566..^2$		Accept divisor 100: $s^2 = 12{\cdot}1668 = 3{\cdot}48809..^2$.	
	CI is given by 24·264 ±	M	Must be c's $\bar{x}$ ± …	
	1·96	B1	Must be from N(0, 1).	
	$\times \frac{\sqrt{12 \cdot 2897 \text{ or } 12 \cdot 1668}}{\sqrt{100}}$	M	Allow c's s_{n-1} or s_n. Accept ../$\sqrt{99}$ if 12·1668 used $\left(\frac{s_n}{\sqrt{n}}\right)$.	
	= 24·264 ± 0·6871 = (23·577, 24·951) or 24·264 ± 0·6837 = (23·580, 24·948)	A1	c.a.o. Must be written as an interval.	4
				15

2615 - Statistics 3

General Comments

There were slightly fewer than 800 candidates for this paper, compared with about 1000 in June 2004. Once again the overall standard of the scripts seen was pleasing: many candidates appeared well prepared for this paper. However, as in the past, comments and explanations were a consistent weakness.

Invariably all four questions were attempted. However, Questions 1 and 2 were well answered, with many candidates scoring full or nearly full marks. On the other hand the marks scored in Questions 3 and 4 seemed to be more uniformly spread across the range. There was evidence to suggest that candidates found themselves short of time at the end: in many cases Question 4 appeared rushed or unfinished.

Comments on Individual Questions

1) **Continuous random variables; sales of petrol.**

(i) On the whole this part was well answered, although there were a number of candidates who appeared less familiar with how to find the mode than they were with other parts of the question.

(ii) The quality of sketching was felt to be quite poor. Many candidates' curves were sloppy and careless. The most common failing was neglecting to show a gradient of zero at $x = 0$, a feature that should have been obvious from a careful analysis in part (i).

(iii) The mean and variance were found correctly in the vast majority of cases, but the examiners would have liked to see better presentation and attention to detail, and correct notation.

(iv) There were many good, completely correct answers to this part too. The errors that occurred were usually to do with the variance. Some candidates tried to work in litres or millions of litres but they inevitably came unstuck because they could not get the variance to agree. As above, correct and consistent notation (such as using $52X$ when they mean $X_1 + X_2 + \ldots + X_{52}$) was in fairly short supply.

2) **Combinations of Normal distributions; confidence interval for the population mean using the *t* distribution; the times taken to complete components of a fitness training programme.**

In this question some candidates appeared not to understand the context: their answers seemed to suggest that they thought that they were dealing with the *manufacture* of components. Also it was very widespread to see candidates using *A*, *B* and *C* as the random variables rather than *X*, *Y* and *Z* given in the question.

(i) This part was usually correct, although a few candidates added the standard deviations rather than the variances.

(ii) This part was often correct too. The difficulties encountered resulted from an incorrect formulation of the requirement of the question (leading to the complement of the right answer) or from the wrong variance for the difference in times used. Once again the use of notation left much to be desired: it seemed that many candidates do not handle inequalities well, sometimes preferring to omit them altogether. A surprising error which happened sufficiently often to draw comment was "21·4 – 20·4 = 1·4".

(iii) There were many correct answers for the confidence interval. It was pleasing to see so many candidates identify correctly the appropriate percentage point from the *t* distribution. But there were those who used 1·96, from the Normal distribution, instead, and/or the wrong standard deviation.
The greatest difficulty in this part of the question was the interpretation of the interval. Some candidates ignored the interval altogether, arguing that 19·5 is less than 20·4 therefore there must have been a reduction in the training time. Others came to the same conclusion by saying that 20·4 was in the upper half of the interval. Others simply omitted to make any comment.
Some candidates set up their entire answer to this part of the question as a hypothesis test.

(iv) This was badly answered. Candidates had not read the preamble to parts (iii) and (iv) carefully enough, and their answers failed to address the question of whether *these* (first) 8 recruits could be regarded as a *random* sample. Two common misconceptions were that sample size was a relevant issue and that for "random" one could substitute "representative".

3) **Hypothesis test for the population mean using the Normal distribution; Type II error; efficiency measures for electric fans.**

(i) The hypotheses were usually stated correctly but many candidates neglected to define the symbol μ.
The test statistic was often worked out correctly. Most, but not all, appreciated that they were given the standard deviation for the population and that it did not require any adjustment. However the small sample size caused some to use the *t* distribution.
Despite the fact that they had given a correct alternative hypothesis earlier, the sign of the critical value quoted by many candidates did not always agree with it. One wondered if they properly understood that they were (or should have been) carrying out a 1-tail test at the lower tail.

(ii) On the whole a greater proportion of candidates than in the past showed that they understood something about Type II errors. However significant numbers of candidates worked out their critical point using the sample mean and/or used the distribution $N(530, 14^2)$ even when they had used the correct standard error in part (i).

4) **Chi-squared hypothesis test for the goodness of fit of a Poisson model; confidence interval for the population mean using the Central Limit Theorem and the Normal distribution; monitoring radiation levels.**

As mentioned above, many of the answers to this question contained careless errors or were incomplete, suggesting that candidates were running out of time at this point.

(i)(A) Hardly any candidates failed to earn the mark for this part, though, worryingly, when a sample mean other than 2 was found the candidate concerned was likely to persist into part (B) with his/her incorrect mean.

(B) Most candidates found the correct expected frequencies using the model, although, despite the prompt in the table, many neglected to either include the class “more than 6” or to check that their expected frequencies added up to 100. There then followed some uncertainty about the criterion for combining classes: there were those who decided to combine on the basis of low observed (rather than expected) frequencies. Nonetheless the correct test statistic was obtained in the majority of cases.
Some candidates identified the wrong number of degrees of freedom and hence the wrong critical value. This was usually because they did not allow for the estimated parameter (the mean) and/or for having combined classes.
After stating a conclusion to the hypothesis test carried out, almost all candidates omitted to go beyond and “comment briefly”.

(ii) There were many good answers to this question, but also there were many that showed signs of being rushed. Most realised, even if they did not say so, that the CLT allowed them to use the Normal distribution here. However some wanted to use a percentage point from t_{99} or t_{100}. Quite a few candidates were unable to cope with the summary information in the form supplied, particularly when trying to estimate the standard deviation (many thought that 1216·68 *was* the variance).