OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2615

Statistics 3

Friday **13 JUNE 2003** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 An industry regulator is investigating the lengths of time that callers have to wait to be answered by a telephone enquiry bureau. The waiting times are always between 5 seconds and 60 seconds.

A model being used for the waiting times, in seconds, is the random variable T with the following cumulative distribution function.

$$\mathrm{F}(t)=\begin{cases}0 & t\leqslant 5\\ k\left(1-\dfrac{5}{t}\right) & 5<t\leqslant 60\\ 1 & t>60\end{cases}$$

(i) Show that $k=\frac{12}{11}$. [2]

(ii) Find the median waiting time. [3]

(iii) Find the probability that the length of time a caller waits is no more than twice the median. [2]

(iv) Derive the probability density function of T. Hence find the mean waiting time. Find also the probability that a caller waits longer than the mean waiting time. [8]

2 An industrial process requires the extrusion of a small amount of plastic on to a metal surface so that an air-tight sealed joint can be made with another surface. The mass of plastic extruded is critical and is specified as being 1.32 grams. If it is too small, a properly sealed joint cannot be made; if it is too large, bumps and ridges tend to form on the plastic so that again the joint is not properly sealed.

It is understood that the variations in the masses extruded are well modelled by a Normal distribution and that its standard deviation is 0.03 grams.

An inspector examines a random sample of ten extrusions and records their masses, which are as follows (in grams).

1.30 1.26 1.33 1.32 1.35 1.29 1.31 1.28 1.36 1.30

(i) Carry out a 5% significance test to examine whether the mean mass extruded is as specified, stating carefully your null and alternative hypotheses and your conclusion. [8]

(ii) Show that the critical region for the test consists of values of the sample mean outside the range (1.3014, 1.3386). [2]

(iii) Calculate the probability of a Type II error for the test if the mean mass extruded is in fact 1.33 grams. Explain briefly why the inspector will be interested in the Type II error probabilities for the test for a selection of values of the mean mass extruded. [5]

3 Two students who share a flat travel to and from the same building at their university. One cycles, the other travels by car. Journey times by cycle are Normally distributed with mean 25 and standard deviation 2. Journey times by car are Normally distributed with mean 23 and standard deviation 4. (Units are minutes throughout.) All journeys may be assumed to be independent of each other.

(i) One morning they set out simultaneously. Find the probability that the cyclist arrives at the university first. [5]

(ii) Find the probability that the total amount of time spent by the cyclist on ten journeys is less than four hours (240 minutes). [3]

(iii) A new subway for cyclists to cross a busy street is opened. The cyclist uses this new route and believes that it may have affected both the mean and the standard deviation of the journey times, though the times appear still to be Normally distributed. The journey times for a random sample of eight journeys using the new route are found to be as follows.

22.6 20.8 24.1 23.2 25.6 25.2 23.5 23.0

Test at the 5% level of significance whether the mean journey time by this route may be assumed still to be 25. [7]

4 At one of the stalls at a village fête, contestants are invited to guess the number of sixes that will be obtained when four ordinary six-sided dice are rolled. The stall-holder keeps notes of the Numbers of sixes obtained on 100 randomly selected occasions and finds the frequencies to be as follows.

Number of sixes	0	1	2	3	4
Frequency	57	34	5	2	2

(i) Assuming the dice are fair, explain why the number of sixes obtained should follow the $B(4, \frac{1}{6})$ distribution. [2]

(ii) Examine the fit of the $B(4, \frac{1}{6})$ distribution to the above data, using a 5% significance level. [8]

(iii) Contestants who successfully guess the number of sixes receive small cash prizes. The amounts of money x, in pence, paid to the contestants in the above sample are summarised as follows.

$$n = 100 \qquad \sum x = 780 \qquad \sum x^2 = 10100$$

Obtain a 95% confidence interval for the average pay-out to contestants during the entire fête. [5]

Mark Scheme

Q1	$F(t)=k\left(1-\frac{5}{t}\right),\ 5<t\le 60.$			
(i)	$1=F(60)$ $=k\left(1-\frac{5}{60}\right)=\frac{11}{12}k$ $\therefore k=\frac{12}{11}$	M1 A1	Set up requirement. c.a.o. ANSWER GIVEN.	 2
(ii)	Median of T, m, is given by $\frac{1}{2}=\frac{12}{11}\left(1-\frac{5}{m}\right)$ $\therefore 1-\frac{5}{m}=\frac{11}{24}$ $\therefore \frac{5}{m}=\frac{13}{24}$ $m=\frac{5\times 24}{13}=\frac{120}{13}=9\frac{3}{13}=9\cdot 23(07...)$	M1 M1 A1	Definition of median. Attempt to solve (as far as $m=\ldots$). (Condone 1 algebraic error.) Depends on previous M mark. (seconds) c.a.o. Accept any of these three forms.	 3
(iii)	F(2 × c's median) $=\frac{12}{11}\left(1-\frac{5\times 13}{2\times 120}\left[\text{or}\ \frac{5}{18\cdot 46}=0\cdot 271\right]\right)$ $=\frac{12}{11}\times\frac{35}{48}=\frac{35}{44}=0\cdot 79(\dot{5}\dot{4})$	M1 A1	 c.a.o.	 2
(iv)	$f(t)=\frac{d}{dt}F(t)$ $=\frac{12}{11}(-5)(-t^{-2})=\frac{60}{11t^2}$ for $5<t\le 60$ $\mu=\int_5^{60} t.\frac{60}{11}t^{-2}dt$ $=\frac{60}{11}[\ln t]_5^{60}$ $=\frac{60}{11}[4\cdot 094-1\cdot 609]\doteq 13\cdot 55$ Want 1 – F(candidate's mean) $=1-\frac{12}{11}\left(1-\frac{5}{13\cdot 55}\right)=1-0\cdot 688(36)=0\cdot 31(15)$	M1 A1 B1 M1 M1 A1 M1 A1	Shows intention to differentiate. c.a.o. Accept unsimplified equivalent forms. Allow ≤ or <. Do not insist on "0 elsewhere". N.B. This could appear before the differentiation. Set up integral together with correct limits, which may appear later. Successful integration of t × c's f(t), provided f(t) > 0. (seconds). ft c's f(t) > 0 with limits 5 and 60. Provided result is between 5 and 60. Must be finding the right-hand tail at some point. c.a.o.	 8
				15

Q2				
(i)	$H_0 : \mu = 1{\cdot}32$ $\quad H_1 : \mu \neq 1{\cdot}32$ (Do **not** allow $\bar{x} = \ldots$ or similar.) Where μ is the (population) mean mass extruded.	B1 B1	*Both* must be correct. Allow statements in words (see below). Must indicate "mean"; condone "average". If the symbol used is not μ, or no symbol is used, then insist on "population".	
	$\bar{x} = 1{\cdot}31$		ft candidate's $\bar{x}$ for all marks except the next A1.	
	Test statistic is $\dfrac{1{\cdot}31 - 1{\cdot}32}{\left(\frac{0{\cdot}03}{\sqrt{10}}\right)}$	M1	Given $\sigma = 0{\cdot}03$. May not use s_{n-1} (= 0·0309) or s_n (= 0·0293). Allow alternative: $1{\cdot}32 \pm$ (c's 1·96) $\times \frac{0{\cdot}03}{\sqrt{10}}$ (= 1·3014, 1·3386) for subsequent comparison with $\bar{x}$. (Or $\bar{x} \pm$ (c's 1·96) $\times \frac{0{\cdot}03}{\sqrt{10}}$ (= 1·2914, 1·3286) for comparison with 1·32.)	
	$= -1{\cdot}054(09)$	A1	c.a.o. (but ft from here if this is wrong.) Use of $1{\cdot}32 - \bar{x}$ scores M1A0, but next 4 marks still available.	
	Refer to N(0,1).	M1	No ft from here if wrong. BUT if $s_{n-1} = 0{\cdot}0309$ used above allow t_9 and 2·262 ONLY and ft as below.	
	Double-tailed 5% point is 1·96.	A1	No ft from here if wrong. $\Phi(-1{\cdot}054) = 0.146$ (> 0·025)	
	Not significant. ALLOW phrases such as "no evidence against …" "appears that …" "do not reject …"	A1 A1	ft only c's test statistic. ft only c's test statistic.	8
(ii)	H_0 is rejected if $\bar{x} > 1{\cdot}32 + 1{\cdot}96 \times \frac{0{\cdot}03}{\sqrt{10}}$ or if $\bar{x} < 1{\cdot}32 - 1{\cdot}96 \times \frac{0{\cdot}03}{\sqrt{10}}$	M1	Require some evidence here. Allow $1{\cdot}32 \pm k \times s / \sqrt{n}$. Also e.g. $1{\cdot}32 \pm 0{\cdot}01859(4\ldots)$ Condone incorrect k and/or s if consistent with part (i).	
	i.e. $\bar{x} > 1{\cdot}3386$ $\bar{x} < 1{\cdot}3014$	A1	c.a.o. ANSWER GIVEN.	2
(iii)	H_0 is accepted if $1{\cdot}3014 < \bar{x} < 1{\cdot}3386$ If $\mu = 1{\cdot}33$ then $\bar{X} \sim N\left(1{\cdot}33, \dfrac{0{\cdot}03^2}{10}\right)$	M1	Identifies or uses correct new distribution. Condone incorrect s if consistent with above.	
	So we want $P\left(1{\cdot}3014 < N\left(1{\cdot}33, \dfrac{0{\cdot}03^2}{10}\right) < 1{\cdot}3386\right)$	M1	Meaning of Type II error in this context. Needs to calculate with both end points. Independent of the previous mark.	
	$= P(-3{\cdot}0145 < N(0,1) < 0{\cdot}9065)$ $= 0{\cdot}8176 - 0{\cdot}0012 = 0{\cdot}8164$	A1	c.a.o.	
	Type II error probabilities are the probabilities of wrongly accepting the null hypothesis when in fact it is false – in a sense, the "sensitiveness" of the test. The inspector is interested in knowing how sensitive the test is in detecting departures from H_0.	E2		5
				15

Q3	Cycle ~ $N(25, 2^2)$, Car ~ $N(23, 4^2)$			
(i)	We want P(cycle < car) = P(cycle – car < 0) = P(N(2, 20) < 0) = P(N(0, 1) < $\frac{0-2}{\sqrt{20}}$ = – 0·4472) = 1 – 0·6726 = 0·3274	M1 B1 B1 M1 A1	Possibly implied. Accept "car – cycle > 0". Mean. (–2 for "car – cycle"). Variance. for correct standardising using c's parameters (consistent with above); may award in (ii) if not awarded here. c.a.o.	5
(ii)	$\text{cycle}_1 + \ldots + \text{cycle}_{10}$ ~ N(250, 40) P(this < 240) = P(N(0, 1) < $\frac{240-250}{\sqrt{40}}$ = – 1·581) = 1 – 0·9430 = 0·0570	B1 B1 A1	Mean. Variance. Interpret the situation using c's parameters. Can award M1 for correct standardising if not already earned in (i). c.a.o.	3
(iii)	t test of $H_0 : \mu = 25$ against $H_1 : \mu \neq 25$ $\bar{x} = 23{\cdot}5$, $s_{n-1} = 1{\cdot}5166$ ($s_{n-1}^2 = 2{\cdot}3$) Test statistic is $\frac{23{\cdot}5-25}{\left(\frac{1{\cdot}5166}{\sqrt{8}}\right)}$ = –2·79(75) Refer to t_7. Double-tail 5% point is 2·365. Significant. Appears mean journey time has changed (downwards).	B1 M1 A1 M1 A1 A1 A1	. Allow $s_n = 1{\cdot}4186$, ($s_n^2 = 2{\cdot}0125$) only if correctly used in sequel Allow alternative: 25 ± (c's 2·365) × $\frac{1{\cdot}5166}{\sqrt{8}}$ (= 23·732, 26·268) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ ± (c's 2·365) × $\frac{1{\cdot}5166}{\sqrt{8}}$ (= 22·232, 24·768) for comparison with 25.) c.a.o. (but ft from here if this is wrong.) Use of 25 – $\bar{x}$ scores M1A0, but next 4 marks still available. No ft from here if wrong. No ft from here if wrong, but ALLOW s-t 5% pt (1·895) and ft if candidate clearly thinks test should be one-sided. ft only c's test statistic. ft only c's test statistic. S.C. (t_7 and 1·895) or (t_8 and 2·306) can score max 1 of the last 2 marks if either form of conclusion is given, consistent with the test statistic and critical value.	7
				15

Q4				
(i)	4 independent trials Constant probability (1/6) of "success".	E1 E1		2
(ii)	No. 6's: 0, 1, 2, 3, 4 Obs f's: 57, 34, 5, 2, 2 Probs: 0·4823, 0·3858, 0·1157, 0·0154, 0·0008 Exp f's: 48·23, 38·58, 11·57, 1·54, 0·08	 M1 A1	(from B(4,1/6).) (for n = 100.) ft from here provided Σe_i = 100. Min 1 d.p. required.	
	Groupings for $e \le 5$: 9; 13·19 [H_0 : B(4,1/6) fits. H_1 : B(4,1/6) does not fit.]	B1	Both correct or implied.	
	$X^2 = \frac{(57-48\cdot 23)^2}{48\cdot 23} + \ldots$ $= 1\cdot 5947 + 0\cdot 5437 + 1\cdot 3310$ $= 3\cdot 47$	B1	c.a.o. (but ft from here if this is wrong.) e_i to 1 d.p. gives $X^2 = 3\cdot 49$	
	Refer to χ_2^2.	M1	If grouping is wrong (or absent), allow this mark if it agrees with candidate's table, and then ft as below.	
	Upper 5% point 5·991.	A1	No ft from here if not correct point from candidate's χ^2.	
	Not significant. Appears that the fit is good.	A1 A1	 "Model fits data" NOT "data fits model".	8
(iii)	$\bar{x} = 7\cdot 80 \quad \sum(x_i - \bar{x})^2 = 10100 - \frac{780^2}{100} = 4016$			
	$s^2 = \begin{cases} \text{divisor } n-1 & 40\cdot 5\dot{6} \\ \text{divisor } n & 40\cdot 16 \end{cases} \quad s = \begin{cases} 6\cdot 369 \\ 6\cdot 337 \end{cases}$	B1	Allow either divisor.	
	CI is given by 7·80 ±	M1	Must be 7·80 ±	
	1·96	B1		
	$\times \frac{6\cdot 369 \text{ or } 6\cdot 337}{\sqrt{100}}$	M1	Allow c's s_n or s_{n-1}. Allow $\sqrt{99}$ if 6·337 or s_n used.	
	$= 7\cdot 80 \pm \begin{cases} 1\cdot 248 & (6\cdot 552, 9\cdot 048) \\ 1\cdot 242 & = (6\cdot 558, 9\cdot 042) \\ 1\cdot 248 & (6\cdot 552, 9\cdot 048) \end{cases}$	A1	Using 6·369 / $\sqrt{100}$. Using 6·337 / $\sqrt{100}$. Using 6·337 / $\sqrt{99}$. c.a.o. Must be written as an interval.	5
				15

Examiner’s Report

2615 Statistics 3

General Comments

There were about 1000 candidates for this paper, compared with about 1230 in June 2002. The general standard of many of the scripts seen was pleasing: many candidates were clearly well prepared for this paper and deservedly scored high marks.

Invariably all four questions were attempted. Marks for Question 3 were consistently high: many candidates worked through it quickly and easily. Question 1 was the next most successful question, but there was a very obvious split between those who knew clearly what to do and so scored high marks and those whose apparent lack of understanding left them floundering. In Question 4 candidates showed that they knew broadly what to do but were let down by careless errors. Candidates were least successful in Question 2.

There are a few general points giving some concern and which are felt to be worthy of mention.

Some candidates are rather casual with regard to the accuracy of their numerical work. This includes evidence of premature approximation and how carefully they look up probabilities in the Normal tables, interpolating when appropriate.

There are many candidates at this level who do not seem to have a correct understanding of basic inequalities. For instance, in Question 2(i), the assertion that "$-1{\cdot}054 < -1{\cdot}96$" was often seen. In Question 3(i) work containing incorrect and/or inconsistent use of inequalities (often leading, via two or more errors, to the right answer) was fairly commonplace.

In questions about hypothesis tests of the mean, candidates should cultivate the habit of defining the symbol, μ, as the *population* mean. It is frequently not clear that they realise that they are testing a population mean. In addition, the conclusion at the end of the test should also make explicit reference to this, as part of a statement set in the context of the question. The wording of conclusions needs to be much more circumspect and less assertive. Hypothesis tests do not *prove* conclusively that H_0 or H_1 are true or false. Furthermore a different level of significance could, on occasion, lead to a different conclusion.

Comments on Individual Questions

Q.1 Continuous random variables; telephone callers' waiting times

Good candidates understood the difference between a c.d.f. and a p.d.f. These candidates worked through this question easily and efficiently. However, candidates who thought that they had been given a p.d.f. experienced considerable problems. There were many false starts and much crossing out before candidates managed something that they were satisfied with.

(i) Many tried to integrate the given function, but got nowhere. Some differentiated and then integrated their result (which is rather a lot of work for just 2 marks), and many of these candidates then integrated their p.d.f. every time they needed to find a probability and so did much more work than was necessary. Among candidates who used the c.d.f. appropriately it was quite common to see F(5) included.

(ii) There was some better work seen here. Most seemed to know how to find the median. A few had difficulty with the algebra. Those who attempted to integrate made no progress.

(iii) Candidates who were successful in part (ii) were usually able to do this part.

(iv) Although the notation often left much to be desired, most candidates knew what they were supposed to do to find the p.d.f., but the standard of differentiation was rather mixed. The domain was very rarely included in their answers.

Similarly, all but the weaker candidates could set up the right sort of integral for the mean, but carrying out the integration was another matter.

For the final probability the usual mistake was to find the wrong tail.

(ii) 9·23; (iii) 0·795; (iv) $60/(11t^2)$ for $5<t\leq60$, Mean = 13·55, 0·312.

Q.2 Hypothesis test for the population mean using the Normal distribution; Type II errors; the mass of plastic extrusions

(i) The hypotheses were correct most of the time. Occasionally something other than μ was used. In contrast, however, the definition of μ was rarely given.

There was much good work seen in carrying out the test; this came from candidates who were well versed in what to do. However far too many candidates did not read the question sufficiently carefully and so a variety of serious errors were made. The question stated that the population could be taken to be Normally distributed and the population standard deviation was given. This meant that candidates should not estimate the standard deviation from the sample. It also meant that, even though the sample was small, the Normal distribution was the correct distribution for the test. Many candidates used the given standard deviation but then performed a t test. Many estimated the standard deviation from the sample but then used a Normal test (an inconsistency which emphasised that they did not understand the circumstances under which different tests are appropriate).

On this occasion the result obtained should have been "Not significant". Final conclusions stated by candidates were, on the whole, much too assertive: "The mean is 1·32g, as specified". More appropriate wording might be couched in terms such as "It appears that there is insufficient evidence at the 5% level to conclude that the mean mass of plastic extruded is not 1·32g, as specified." Here the conclusion is qualified, is clearly about the mean and is put in the context of "mass extruded".

(ii) While many candidates were able to set up the calculations for the two boundaries, hardly any described the critical region as "outside the range ...". It was worrying that so few appeared to know what a critical region is. Some said it was between the boundaries, some called it a confidence interval and some just performed the calculations with no further explanation. Curiously, while some who had made mistakes in part (i) persisted with the wrong standard deviation and/or distribution, there were many who suddenly discovered the need to use 1·96 and σ = 0·03 to achieve the given values.

(iii) Many candidates could quote the definition of a type II error, but very few could apply it. The usual mistakes involved not using the correct new distribution for the sample means, not using the correct acceptance region (from part (ii)) and not using both ends of the acceptance region.

Comments on the inspector's interest were disappointing. There was some attempt to put the Type II error into context but hardly anybody seemed able to look beyond that. The matter of a selection of values was not considered. Some candidates brought in the idea of the Power of a test, but did not discuss its relevance or meaning.

(i) test statistic –1·054, critical value –1·96; (iii) 0·8164

Q.3 Combinations of Normal distributions; Hypothesis test for the population mean using the t distribution; journey times

This question was well answered by very many candidates. Fully or nearly fully correct answers were often seen.

(i) Apart from some careless presentation and treatment of inequalities, this part of this question was done well. But the lack of care and attention to detail, particularly in the use of inequalities, displayed by some candidates turned out to be costly here. Candidates need to realise that in work such as this both the logic and the algebra must be seen to be correct.

(ii) This part was usually answered correctly. By far the most common mistake seen was the use of a variance of 400 (= $2^2 \times 10^2$).

(iii) There were many good answers seen here also. This time most candidates were safe in deciding that a small sample required a *t* test. As with the test in Question 2 the conclusions need to be more cautious. The test statistic was "significant" and so "it appears, at the 5% level, that the mean journey time has changed."

(i) 0·3274; (ii) 0·0569; (iii) test statistic –2·797, critical value –2·365

Q.4 Chi-squared test for goodness of fit; Central Limit Theorem and confidence interval for the population mean; a dice game at a village fête

(i) Most candidates answered this question by explaining the parameters 4 and 1/6. However a more sophisticated discussion was expected, explaining why the distribution should be binomial.

(ii) This part of the question was usually well done and many candidates scored full marks. It was clear that there were also many who were capable of scoring full marks but who did not do so for a variety of reasons which included: carelessness with regard to the accuracy of expected frequencies and/or in the subsequent calculations; failing to ensure that the expected frequencies added up to 100; failing to combine cells correctly; having the wrong number of degrees of freedom (number of cells used – 1).

As with the other hypothesis tests in the paper, a similarly cautious conclusion is expected. Furthermore candidates need to be aware that they are testing how well a model fits some observed data and not the other way round. So conclusions such as "the data fit the model" are not going to be acceptable.

(iii) There were many correct and efficiently presented answers to this part. There were a few errors with the mean (78 instead of 7·8) and either a value of 1·984 (from *t* tables) or 1·645 as an alternative to 1·96.

(ii) test statistic 3·469, critical value 5·991; (iii) (6·552, 9·048).