OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2604

Pure Mathematics 4

Tuesday **3 JUNE 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 A curve has equation $y = \dfrac{x^2 - 9x}{x + 3}$.

(i) Write down the equation of the asymptote parallel to the y-axis. [1]

(ii) Find the equation of the oblique asymptote. [3]

(iii) Find $\dfrac{dy}{dx}$. Hence find the coordinates of the two stationary points. [6]

(iv) Sketch the curve. [4]

(v) Given that the curve has rotational symmetry, find the centre and state the order of symmetry. [3]

(vi) On a separate diagram, sketch the curve with equation $y^2 = \dfrac{x^2 - 9x}{x + 3}$. [3]

2 (a) Find $\displaystyle\sum_{r=1}^{n} (3r-1)(3r+5)$, giving your answer as simply as possible. [6]

(b) Express $\dfrac{1}{(3r-1)(3r+5)}$ in partial fractions, and hence find the sum of the series

$$\frac{1}{2\times 8} + \frac{1}{5\times 11} + \frac{1}{8\times 14} + \ldots + \frac{1}{(3n-1)(3n+5)}.$$ [7]

(c) Solve the inequality $\dfrac{2x+1}{x+3} > \dfrac{2}{x}$. [7]

3 (a) Find the complex number z which satisfies $(2+\mathrm{j})z + (3-2\mathrm{j})z^* = 32$. [5]

(b) The cubic equation $z^3 + z^2 + 4z - 48 = 0$ has one real root α and two complex roots β and γ.

(i) Verify that $\alpha = 3$ and find β and γ in the form $a + b\mathrm{j}$. Take β to be the root with positive imaginary part, and give your answers in an exact form. [5]

(ii) Find the modulus and argument of each of the numbers α, β, γ, $\dfrac{\beta}{\gamma}$, giving the arguments in radians between $-\pi$ and π.

Illustrate these four numbers on an Argand diagram. [8]

(iii) On your Argand diagram, draw the locus of points representing complex numbers z such that

$$\arg(z - \alpha) = \arg \beta.$$ [2]

4 (a) Four points have coordinates A(3, 9, 5), B(1, 14, 10), C(5, 0, −8) and D(13, −4, 4).

(i) Show that the lines AB and CD intersect, and find the coordinates of their point of intersection. [7]

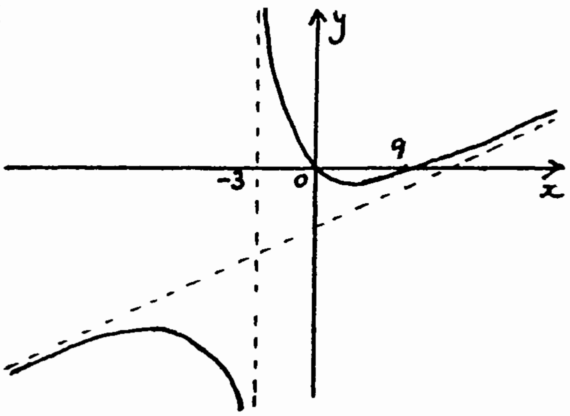
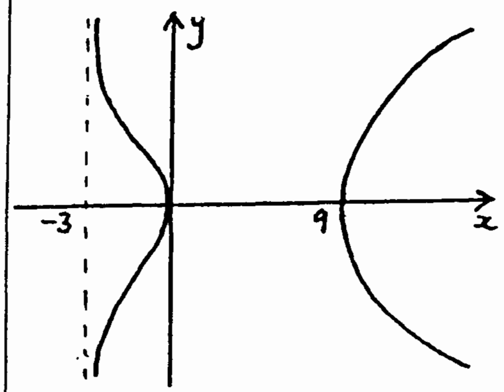
(ii) Find, in the form $ax + by + cz + d = 0$, the equation of the plane containing A, B, C and D. [5]

(b) A sequence of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots$ is defined by $\mathbf{v}_1 = \begin{pmatrix} 11 \\ 13 \\ -4 \end{pmatrix}$ and $\mathbf{v}_{n+1} = \mathbf{M}\mathbf{v}_n$ for $n \geqslant 1$, where

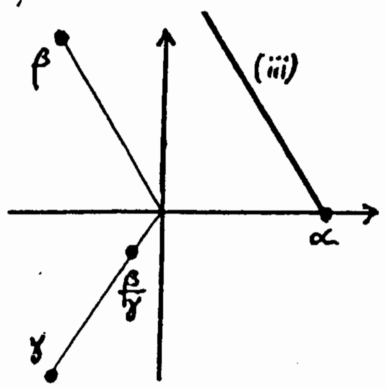
$$\mathbf{M} = \begin{pmatrix} 1 & 3 & 3 \\ 0 & 4 & 3 \\ -3 & 0 & -2 \end{pmatrix}.$$

Show by induction that $\mathbf{v}_n = \begin{pmatrix} 9n^2+2 \\ 9n^2+4 \\ -(3n-1)^2 \end{pmatrix}$. [8]

Mark Scheme

1 (i)	Vertical asymptote is $x=-3$	B1 1	
(ii)	$y=\dfrac{(x+3)(x-12)+36}{x+3}$ $=x-12+\dfrac{36}{x+3}$ Oblique asymptote is $y=x-12$	M1 B1A1 3	*Or* dividing x^2-9x by $x+3$ to obtain a linear quotient For $y=x\ldots$ and -12 *(must be the equation of a line)*
(iii)	$\dfrac{dy}{dx}=1-\dfrac{36}{(x+3)^2}$ $=0$ when $(x+3)^2=36$ $x=-9,\ 3$ Stationary points are $(-9,-27)$ and $(3,-3)$	M1A1 M1 M1 A1A1 6	*Or* $\dfrac{(x+3)(2x-9)-(x^2-9x)}{(x+3)^2}$ Obtaining quadratic (e.g. $x^2+6x-27$) Solving quadratic Give A1 for $x=-9,\ 3$
(iv)		B1 B1 B1 B1 4	LH branch: below x-axis, maximum RH branch crossing x-axis at 0 and 9 Minimum on RH branch Fully correct shape and approaching asymptotes correctly
(v)	Centre is where asymptotes meet i.e. at $(-3,-15)$ Order is 2	M1 A1 B1 3	*Or* midpoint of stationary points
(vi)		B1 ft B1 ft B1 cao 3	No curve in $x<-3$, $0<x<9$ and curve in $-3<x<0$, $x>9$ Symmetry in x-axis Fully correct shape, including infinite gradient at two points where curve crosses x-axis *(No need for any numbers on axes)*

2 (a)	$\sum_{r=1}^{n}(9r^2+12r-5)$	B1	
	$=\frac{3}{2}n(n+1)(2n+1)+6n(n+1)-5n$	B1B1B1 ft	
	$=\frac{1}{2}n(6n^2+9n+3+12n+12-10)$	M1	Multiplying out and collecting terms
	$=\frac{1}{2}n(6n^2+21n+5)\quad\left[=3n^3+\frac{21}{2}n^2+\frac{5}{2}n\right]$	A1 6	Give M1A0 for factorising the first two terms [e.g. $\frac{3}{2}n(n+1)(2n+5)-5n$]
(b)	$\frac{1}{(3r-1)(3r+5)}=\frac{1}{6}\left(\frac{1}{3r-1}-\frac{1}{3r+5}\right)$	M1 A1	Finding partial fractions
	$\text{Sum}=\frac{1}{6}\left(\frac{1}{2}-\frac{1}{8}\right)+\frac{1}{6}\left(\frac{1}{5}-\frac{1}{11}\right)+\frac{1}{6}\left(\frac{1}{8}-\frac{1}{14}\right)+\dots$	M1	Using partial fractions in at least two terms
	$+\frac{1}{6}\left(\frac{1}{3n-4}-\frac{1}{3n+2}\right)+\frac{1}{6}\left(\frac{1}{3n-1}-\frac{1}{3n+5}\right)$	A1 ft	At least three terms correct
		M1M1	Two fractions left at beginning, two fractions left at end
	$=\frac{1}{6}\left(\frac{1}{2}+\frac{1}{5}-\frac{1}{3n+2}-\frac{1}{3n+5}\right)$	A1 cao 7	
	$=\frac{7}{60}-\frac{1}{6(3n+2)}-\frac{1}{6(3n+5)}$		
	$\left(=\frac{7}{60}-\frac{6n+7}{6(3n+2)(3n+5)}=\frac{n(21n+29)}{20(3n+2)(3n+5)}\right)$		
(c)	$\frac{2x+1}{x+3}-\frac{2}{x}>0$	M1	*Or* multiplying by $x^2(x+3)^2$ *Or* solving $\frac{2x+1}{x+3}=\frac{2}{x}$
	$\frac{2x^2-x-6}{x(x+3)}>0$	M1	Obtaining quadratic *Dependent on previous M1*
	$\frac{(2x+3)(x-2)}{x(x+3)}>0$	M1	Factorising or solving quadratic *Dependent on previous M1M1*
		M1	Considering 5 intervals defined by $-3,\ -1.5,\ 0,\ 2$ (ft) *This will imply previous M3*
	$x<-3,\quad -1.5<x<0,\quad x>2$	A1A1A1 7	cao Give A1 for $-3<x<-1.5,\ 0<x<2$ *SR:* If M0, then B1B1 for -1.5, 2

3 (a)	Let $z = a + b\,\mathrm{j}$, $z^* = a - b\,\mathrm{j}$ $(2 + \mathrm{j})(a + b\,\mathrm{j}) + (3 - 2\mathrm{j})(a - b\,\mathrm{j}) = 32$ Equating real parts, $2a - b + 3a - 2b = 32$ $5a - 3b = 32$ Equating imaginary parts, $a + 2b - 2a - 3b = 0$ $-a - b = 0$ Solving, $a = 4$, $b = -4$ $z = 4 - 4\mathrm{j}$	B1 M1A1 A1 A1 5	
(b)(i)	$27 + 9 + 12 - 48 = 0$, so $\alpha = 3$ is a root	B1	
	$(z-3)(z^2 + 4z + 16) = 0$ $z = \dfrac{-4 \pm \sqrt{-48}}{2}$	M1 M1 M1	Obtaining quadratic Quadratic formula (or equivalent) Use of $\sqrt{-n} = \sqrt{n}\,\mathrm{j}$ *(M0 for $\sqrt{-48} = 48\mathrm{j}$ etc)*
	OR Complex roots are $a \pm b\mathrm{j}$ $3 + (a + b\mathrm{j}) + (a - b\mathrm{j}) = -1$ $3 + 2a = -1$ M2 $3(a + b\mathrm{j})(a - b\mathrm{j}) = 48$ $3(a^2 + b^2) = 48$ M1		Using $a \pm b\mathrm{j}$ and obtaining a real equation involving a, b Obtaining a second real equation
	$\beta = -2 + 2\sqrt{3}\,\mathrm{j}$, $\gamma = -2 - 2\sqrt{3}\,\mathrm{j}$	A1 5	Accept $\dfrac{-4 + \sqrt{48}\mathrm{j}}{2}$ etc
(ii)	$\lvert\alpha\rvert = 3$, $\arg\alpha = 0$ $\lvert\beta\rvert = \lvert\gamma\rvert = 4$ $\arg\beta = \frac{2}{3}\pi$ $\arg\gamma = -\frac{2}{3}\pi$ $\left\lvert\dfrac{\beta}{\gamma}\right\rvert = 1$ $\arg\dfrac{\beta}{\gamma} = -\frac{2}{3}\pi$ 	B1 B1 ft B1 cao B1 ft B1 cao B1 ft B2 cao 8	 Accept 2.1 ft $-\arg\beta$ *Max penalty of 1 mark for arguments in degrees and / or out of range* Four points on diagram. Requires β, γ to be reflections of each other and O, $\dfrac{\beta}{\gamma}$, γ in a straight line. Give B1 ft [from (i)] for two of β, γ, $\dfrac{\beta}{\gamma}$ in the correct quadrants. *Condone points not labelled* *SR:* If β, γ interchanged in (i), full marks can be earned (ft) in (ii) and (iii)
(iii)	Line on diagram (see above)	B2 cao 2	Give B1 for any half-line from α or (part of) line through α parallel to Oβ

4(a)(i)	For a point of intersection, $3-2\lambda=5+8\mu$ (1) $9+5\lambda=-4\mu$ (2) $5+5\lambda=-8+12\mu$ (3) Solving (1) and (2), $\lambda=-2$, $\mu=0.25$ Check in (3), LHS $=5-10=-5$ RHS $=-8+3=-5$ Hence the lines intersect Point of intersection is $(7,-1,-5)$	M1 A1 M1A1 M1 A1 A1 7	Equating two components, *using different parameters* Two equations correct Checking consistency *Dependent on first M1A1M1*
(ii)	Normal is $\begin{pmatrix}-2\\5\\5\end{pmatrix}\times\begin{pmatrix}8\\-4\\12\end{pmatrix}=\begin{pmatrix}80\\64\\-32\end{pmatrix}=16\begin{pmatrix}5\\4\\-2\end{pmatrix}$ Equation is $5x+4y-2z=15+36-10$ i.e. $5x+4y-2z-41=0$	M1 A2 M1 A1 5	Vector product, or other method for finding normal vector Give A1 if just one error Finding constant Accept $80x+64y-32z=656$ etc
(b)	When $n=1$, $\begin{pmatrix}9n^2+2\\9n^2+4\\-(3n-1)^2\end{pmatrix}=\begin{pmatrix}11\\13\\-4\end{pmatrix}$ so it is true for $n=1$ Assuming it is true for $n=k$, $\mathbf{v}_{k+1}=\begin{pmatrix}1&3&3\\0&4&3\\-3&0&-2\end{pmatrix}\begin{pmatrix}9k^2+2\\9k^2+4\\-(3k-1)^2\end{pmatrix}$ $=\begin{pmatrix}9k^2+2+3(9k^2+4)-3(3k-1)^2\\4(9k^2+4)-3(3k-1)^2\\-3(9k^2+2)+2(3k-1)^2\end{pmatrix}$ $=\begin{pmatrix}9k^2+18k+11\\9k^2+18k+13\\-9k^2-12k-4\end{pmatrix}$ $=\begin{pmatrix}9(k+1)^2+2\\9(k+1)^2+4\\-(3k+2)^2\end{pmatrix}$ True for $n=k \Rightarrow$ True for $n=k+1$ (Hence true for all positive integers n)	B1 M1 A2 M1 A2 A1 8	Give A1 for one component correct Obtaining simple quadratic (one component sufficient) *Dependent on previous M1* Correctly obtained (Give A1 for two components) Stated or clearly implied *Dependent on previous 6 marks*

Examiner’s Report

2604 Pure Mathematics 4

General Comments

There were very many good scripts, with about a quarter of the candidates scoring 50 marks or more (out of 60). On the other hand, about a quarter scored less than 25 marks. Almost all candidates answered questions 1 and 2, then about two thirds chose question 3 and about one third chose question 4. Most candidates appeared to have sufficient time to complete the paper.

Comments on Individual Questions

Q.1 Curve sketching

This question was well answered, with half the attempts scoring 15 marks or more (out of 20), but very few scored full marks. In part (i) the vertical asymptote was almost always given correctly. Part (ii) was very often answered correctly, but many candidates did not appear to know a method for finding the oblique asymptote, with $y = x$ and $y = x - 9$ being commonly given. In part (iii) most candidates found the derivative (usually by the quotient rule) and the stationary points correctly. The graph in part (iv) was very often drawn correctly, although a wrong oblique asymptote usually resulted in a curve of the wrong shape. In part (v), the centre of rotational symmetry was usually found as the point of intersection of the asymptotes (some used the midpoint of the stationary points), but not many candidates gave the order of symmetry correctly; this was frequently given as 180° or π and was often omitted altogether. The square root graph in part (vi) was well understood, but the great majority of candidates lost a mark for not showing a curve with infinite gradient where it crosses the x-axis.

(i) $x = -3$; (ii) $y = x - 12$; (iii) $\frac{dy}{dx} = 1 - \frac{36}{(x+3)^2}$, $(-9, -27)$ and $(3, -3)$;

(v) $(-3, -15)$, order 2.

Q.2 Series and Inequalities

This question was also well answered, with most attempts scoring 14 marks or more, and it was the one which most frequently earned full marks.

In part (a), most candidates knew that they should multiply out $(3r - 1)(3r + 5)$ and apply the formulae for $\sum r^2$ and $\sum r$. However, the constant term -5 was often left as -5 instead of being summed to give $-5n$. Most responded correctly to the request to give their answer 'as simply as possible' by multiplying out the brackets and collecting terms. Some factorised the first two terms, leaving the $-5n$ hanging at the end; this lost the final mark. Some thought they should clear fractions, and multiplied their answer by 2 at the end.

In part (b) a suitable method for finding partial fractions was well known but was by no means always accurately applied. Some attempts ended at this point, but most candidates understood the method of differences and the correct answer was frequently obtained. Common errors were having the wrong denominator in the penultimate term, and leaving the final answer in terms of *r* instead of *n*.

In part (c) a variety of correct methods were used to solve the inequality, and it was often answered confidently and efficiently; the most successful method seemed to be that of rewriting the inequality as $\frac{(2x+3)(x-2)}{x(x+3)} > 0$. Those who multiplied by $x^2(x+3)^2$ were prone to lose a factor of x, or to multiply everything out and then be unable to re-factorise. There were many candidates who ignored the critical values 0 and -3 completely.

(a) $\frac{1}{2}n(6n^2+21n+5)$; (b) $\frac{1}{6}\left(\frac{1}{3r-1}-\frac{1}{3r+5}\right)$, $\frac{7}{60}-\frac{1}{6(3n+2)}-\frac{1}{6(3n+5)}$;
(c) $x<-3$, $-1.5<x<0$, $x>2$.

Q.3 Complex numbers

This question was also generally well answered, with an average mark of about 13.

Part (a) seemed to be unfamiliar to many candidates, who were unable to start it. Nevertheless, a good proportion did substitute $z=a+b\text{j}$ and $z^*=a-b\text{j}$ and often obtained the correct answer, although minor slips were quite common here.

Parts (b)(i) and (ii) were well answered. There was a noticeable improvement since last year in the standard of work on these topics, solving a quadratic equation with complex roots and finding moduli and arguments. A surprisingly common error was $\arg 3=\pi$. In part (iii) the locus was rarely drawn correctly; common errors were to give a complete line instead of a half-line, or to draw the line joining α to β.

(a) $z=4-4\text{j}$;
(b)(i) $\beta=-2+2\sqrt{3}\,\text{j}$, $\gamma=-2-2\sqrt{3}\,\text{j}$; (ii) Moduli $3, 4, 4, 1$, Arguments 0, $\frac{2}{3}\pi$, $-\frac{2}{3}\pi$, $-\frac{2}{3}\pi$.

Q.4 Vectors and Matrices

This was the worst answered question, with an average mark of about 11.

Part (a)(i) was answered well and the correct point of intersection was very often found. However, many candidates did not include a convincing check that the lines do indeed intersect. Part (ii) was also well understood, with almost all candidates using the vector product to find a normal vector.

Part (b) was an unfamiliar induction exercise, and some candidates omitted it altogether. Those who did attempt it generally demonstrated that they understood the process. The induction was often started at $n=2$ instead of $n=1$, and there were many errors in the algebraic manipulation, even though this only involved quadratic polynomials.

(a)(i) $(7,-1,-5)$; (ii) $5x+4y-2z-41=0$.