RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2603(A)

Pure Mathematics 3

Section A

Thursday **9 JANUARY 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

NOTE

- This paper will be followed by **Section B: Comprehension.**

This question paper consists of 4 printed pages.

1 **(a)** Find the coefficient of x^3 in the binomial expansion of $\sqrt{1+2x}$. [3]

(b) Solve the equation $\sin 2x = \cos x$ for $0 \leqslant x \leqslant 2\pi$, giving your answers in terms of π. [5]

(c) By using the substitution $u = \sin x$, or otherwise, find $\int \sin^2 x \cos x \, dx$.

Hence evaluate $\int_0^{\frac{1}{2}\pi} \sin^2 x \cos x \, dx$. [4]

(d) You are given that $y = 2^x$. By finding $\ln y$ in terms of x, and differentiating implicitly, or otherwise, find $\frac{dy}{dx}$ in terms of x. [4]

[Total 16]

2 Fig. 2 shows the graph of the curve with parametric equations

$$x = 4\cos\theta, \quad y = 3\sin\theta, \qquad 0 \leqslant \theta < 2\pi.$$

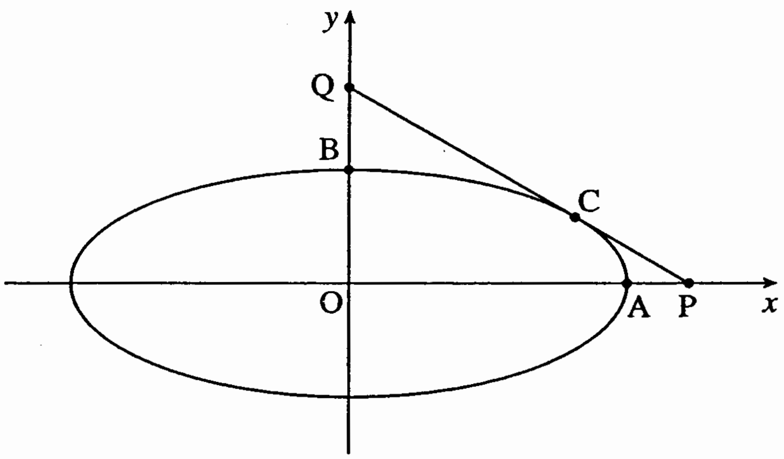


Fig. 2

(i) Find the coordinates of the points A and B. [2]

(ii) The point C has parameter $\theta = \frac{1}{4}\pi$. Find the exact coordinates of C. [2]

(iii) Find $\frac{dy}{dx}$ in terms of θ.

Hence show that the equation of the tangent to the curve at C is $3x + 4y = 12\sqrt{2}$. [5]

(iv) This tangent meets the x- and y-axes at P and Q respectively. Find the area of the triangle POQ. [2]

(v) Find the cartesian equation of the curve. [3]

[Total 14]

3 Fig. 3 shows the graph of $y = \frac{1}{x(2-x)}$.

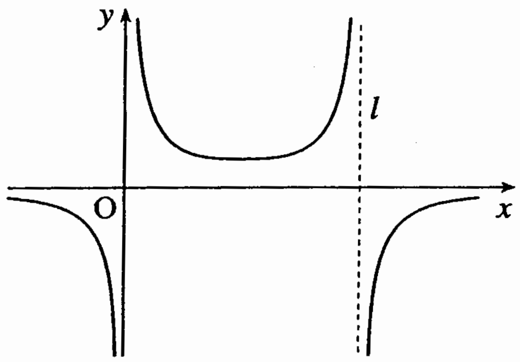


Fig. 3

(i) Write down the equation of the asymptote l. [1]

(ii) Express $\frac{1}{x(2-x)}$ in partial fractions.

Hence find the exact area enclosed by the graph of $y = \frac{1}{x(2-x)}$, the x-axis and the lines $x = 1$ and $x = 1.5$. Express your answer in the form $a \ln b$. [8]

You are given that z satisfies the differential equation

$$\frac{1}{z}\frac{dz}{dx} = \frac{1}{x(2-x)}$$

and that $z = 1$ when $x = 1$.

(iii) Use integration to solve this equation, and hence show that

$$z^2 = \frac{x}{2-x}.$$ [6]

[Total 15]

[Turn over

4 The lines l_1 and l_2 have the following vector equations.

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 1 \\ 8 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix}$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

(i) Verify that the point P (1, 2, 0) lies on both the lines l_1 and l_2. [3]

(ii) Find the acute angle between the two lines. [5]

A plane has cartesian equation $2x - y + z = 12$.

(iii) Write down a vector normal to the plane. Hence write down a vector equation for the line through P perpendicular to the plane. [2]

(iv) Find where this line meets the plane, and hence find the distance of P from the plane. [5]

[Total 15]

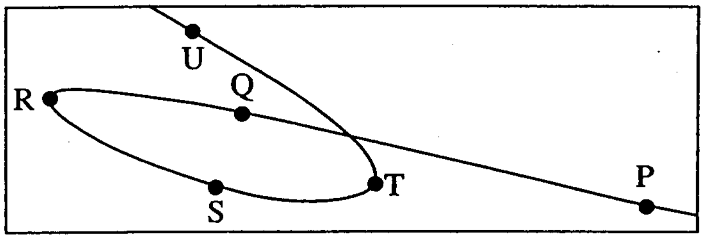
Why planets retrograde

Introduction

For a long time people believed that the Earth was the centre of the universe and that the Sun and other stars followed circular orbits round it. The only exceptions were the five "wandering stars", Mercury, Venus, Mars, Jupiter and Saturn. We now know that these are planets orbiting the Sun.

When observed from northern latitudes, the planets usually appear to move from right to left through the background of "fixed stars". However there are times when this motion goes into reverse. At such a time the planet is said to *retrograde*.

Fig. 1(a) shows the path of the planet Mars through the sky between 1st June and 31st October 2003. You can see that although its general motion is from right to left, there is a period during which it retrogrades. Around this time you can find Mars by looking towards the south soon after dark. It is near the Square of Pegasus, as shown in Fig. 1(b).



Positions of Mars, 2003

P	*June 1st*
Q	*June 29th*
R	*July 27th*
S	*August 31st*
T	*September 28th*
U	*October 31st*

Fig. 1(a)

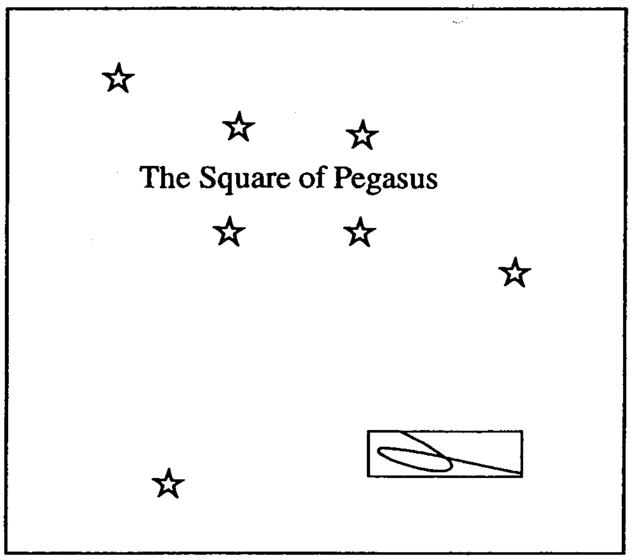


Fig. 1(b)

Apparent planetary motion

This article concentrates on the motion of two planets, Mercury and Mars, as seen from the Earth. Mercury is described as an *inferior planet* because it is closer to the Sun than the Earth. Mars, being further from the Sun than the Earth, is a *superior planet.*

In order to produce an initial model for this apparent motion, the following simplifying assumptions are made.

- All the planets (including the Earth) have circular orbits, and they travel at constant speeds with the Sun at the centre.
- The radii of the orbits of Mercury, the Earth and Mars are 58, 150 and 228 million kilometres respectively. The orbits are all in the same plane (this is nearly true).
- The times taken by Mercury, the Earth and Mars to complete one orbit of the Sun are 90, 360 and 720 days respectively. (The actual figures are 88 days for Mercury, 365 for the Earth, and 687 for Mars.)

Take the case of Mercury first. Figs. 2(a) and 2(b) show the positions of the Earth, the Sun and Mercury at two times, Day 0 and Day 20. Coordinate axes have been drawn with the Sun at the origin.

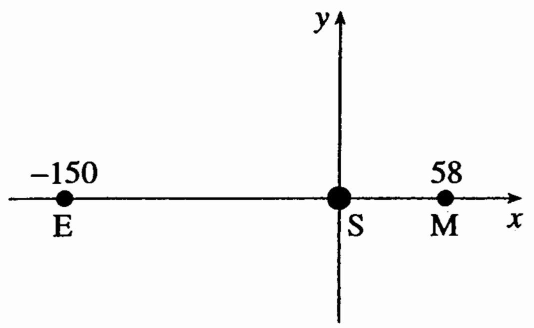


Fig. 2(a)

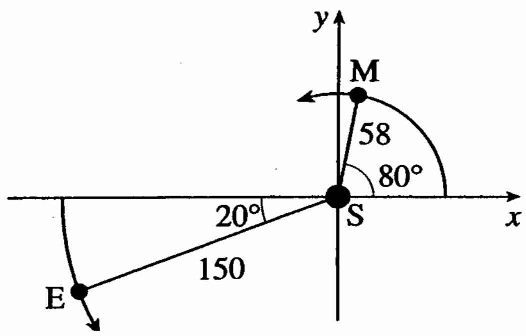


Fig. 2(b)

On Day 0, the three bodies are in line, on the x-axis, with the Earth and Mercury on opposite sides of the Sun. See Fig. 2(a).

By Day 20, Mercury has turned through an angle of $\frac{20}{90} \times 360° = 80°$.

Similarly by Day 20, the Earth has turned through an angle of $\frac{20}{360} \times 360° = 20°$.

On Day 20, the coordinates of Mercury are $(58\cos 80°, 58\sin 80°)$ and the coordinates of the Earth are $(-150\cos 20°, -150\sin 20°)$. See Fig. 2(b).

Figs. 3(a) and 3(b) show the same situation, but now the origin is at the Earth, which is taken to be stationary. These give the view as it appears from the Earth. The Sun appears to have a circular orbit, centre the Earth.

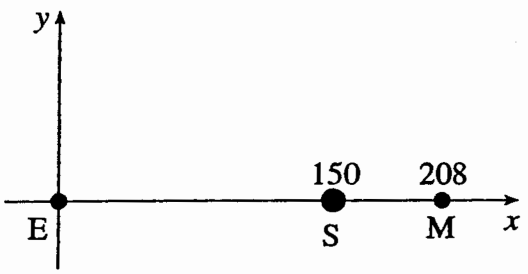


Fig. 3(a)

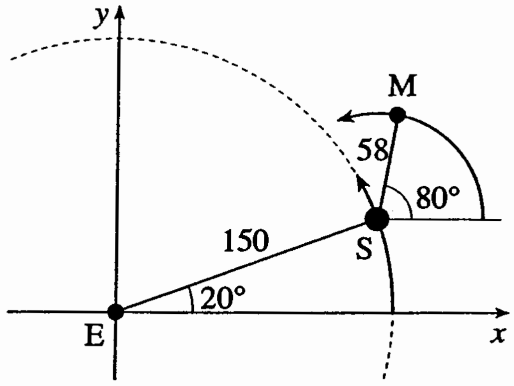


Fig. 3(b)

On Day 0, the Sun is at (150, 0) and Mercury is at (208, 0). See Fig. 3(a).

On Day 20, the Sun is at $(150\cos 20°, 150\sin 20°)$ and Mercury is at $(150\cos 20° + 58\cos 80°, 150\sin 20° + 58\sin 80°)$. See Fig. 3(b).

This expression for the position of Mercury on Day 20 can be generalised to give the position on Day t as

$$(150\cos t° + 58\cos 4t°, 150\sin t° + 58\sin 4t°).$$

Another way of expressing this is to say that, relative to the Earth, the position of Mercury is given by the parametric equations

$$x = 150\cos t° + 58\cos 4t°, \quad y = 150\sin t° + 58\sin 4t°.$$

This curve, together with the circular orbit of the Sun, is shown in Fig. 4. (The curve is called an epicycloid.)

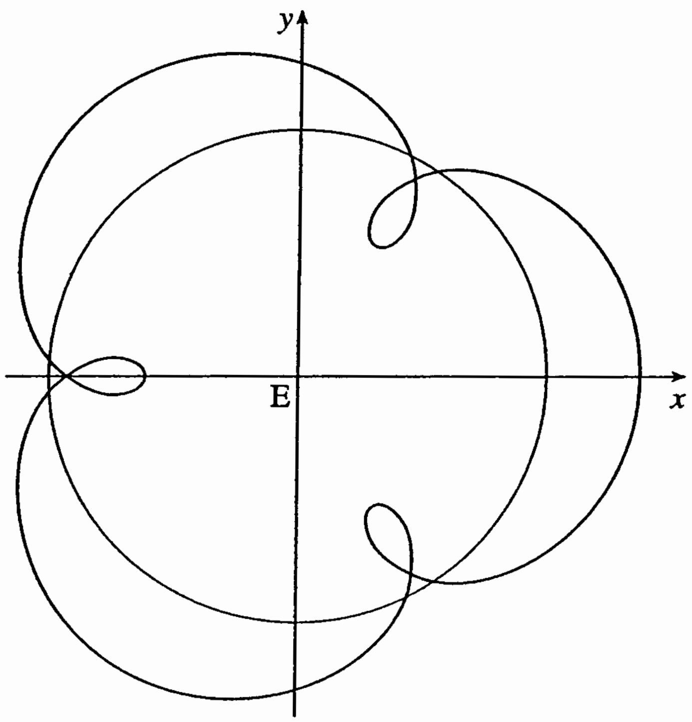


Fig. 4

You can see that this curve has three loops, each corresponding to a time when Mercury is retrograding. So Mercury retrogrades three times a year. However this retrograde motion cannot be observed because it occurs at a time when Mercury lies between the Earth and the Sun and so the planet is in the daytime sky.

The equivalent parametric coordinates for the position of Mars at time t days are given by

$$x = 150\cos t^\circ + 228\cos\tfrac{1}{2}t^\circ, \quad y = 150\sin t^\circ + 228\sin\tfrac{1}{2}t^\circ$$

and these give the curve in Fig. 5. (This too is an epicycloid.)

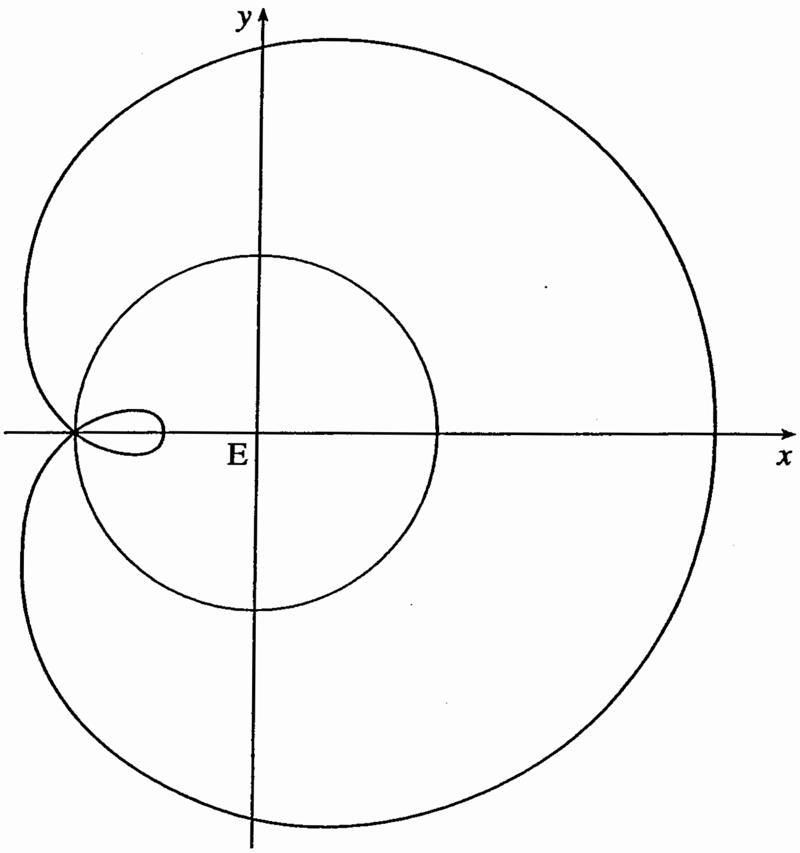


Fig. 5

This curve also has a loop but in this case it occurs when Mars and the Sun are on opposite sides of the Earth. This means that Mars is in the night sky and so its retrograding can be observed provided there are suitable clear nights.

The modelling assumptions

The work leading up to the curves in Figs. 4 and 5 depended on a number of modelling assumptions. What would be the effect of making these assumptions more realistic?

The orbits of the planets around the Sun are not circles but ellipses. The Earth's orbit is between 147 and 152 million kilometres from the Sun and so is nearly circular anyway. Those of Mercury (46 to 70 million km) and Mars (207 to 249 million km) are more elliptical. Dealing with ellipses rather than circles would make the expressions for the parametric coordinates much more complicated, but the curves would still have similar features.

The times taken by the planets to orbit the Sun were rounded to 90, 360 and 720 days, giving convenient numbers of degrees that the planets move in a day: 4°, 1° and $\frac{1}{2}$°. With these numbers the curves repeat themselves exactly, so that it appears that retrograding always occurs at the same times of year. With the actual numbers, the times of retrograding vary from year to year.

For Examiner's Use

1 In line 32 the article reads "By Day 20, Mercury has turned through an angle of

$$\frac{20}{90} \times 360° = 80°."$$

Explain briefly where the fraction $\frac{20}{90}$ comes from. [2]

....

....

....

2 The parametric coordinates of a curve are

$$x = 2\cos t + \cos 3t, \quad y = 2\sin t + \sin 3t,$$

with the angle t measured in degrees.

(i) Complete this table, giving values correct to 1 decimal place. [2]

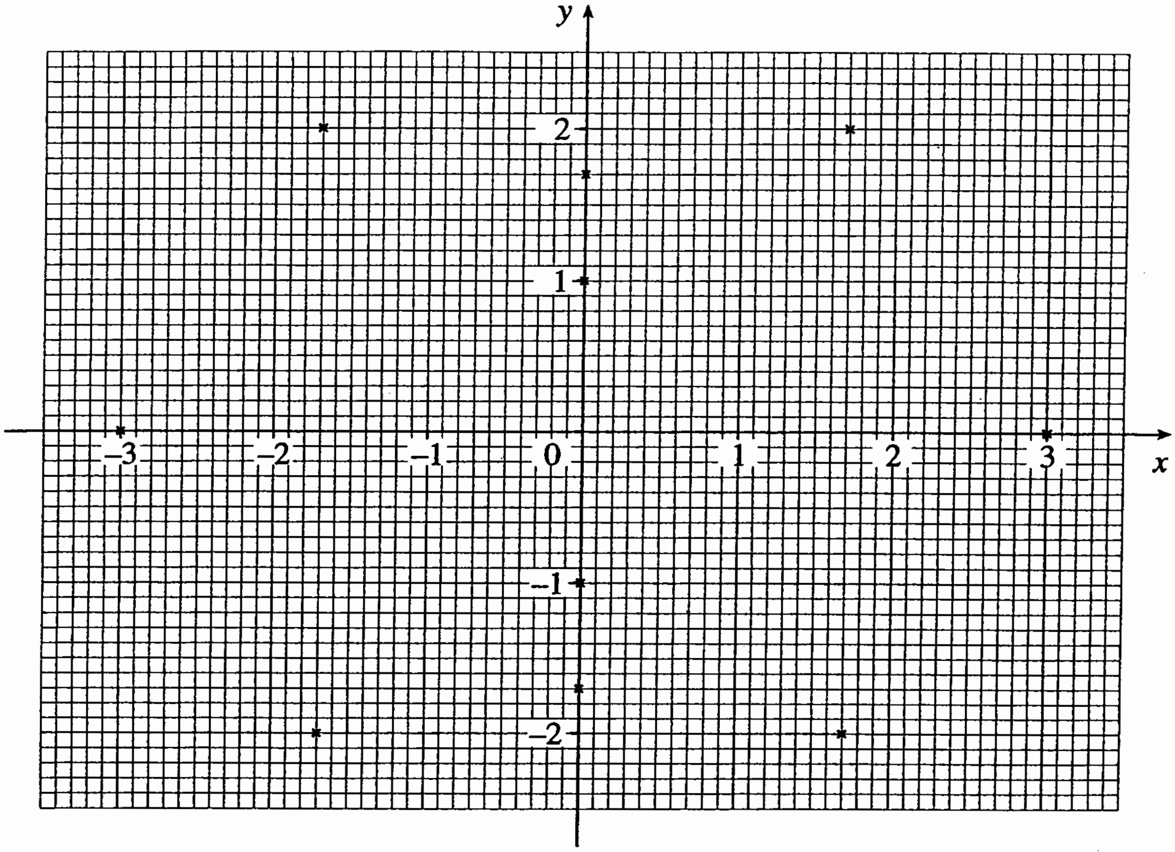
t	0	30	60	75	90	105	120	150	180
x	3	1.7	0		0		0	–1.7	–3
y	0	2	1.7		1		1.7	2	0

t	210	240	255	270	285	300	330	360
x	–1.7	0		0		0	1.7	3
y	–2	–1.7		–1		–1.7	–2	0

Parts (ii) and (iii) of this question are on page 3.

For Examiner's Use

(ii) Most of the points from your table are already marked on the axes below. Mark in the others. Then sketch the curve. [3]



(iii) Suppose that this curve illustrates the apparent motion of a planet. How many times a year would this planet retrograde? [1]

....

3 The planet Jupiter takes approximately 12 years to complete one orbit of the Sun. The radius of its orbit, which is assumed to be circular, is 778 million kilometres.

Write down parametric equations, like those in line 55, for the position of Jupiter relative to the Earth. [2]

....

....

....

[Turn over

For Examiner's Use

4 **(i)** According to the model described in the article, there are three periods in a year when Mercury retrogrades. For each of these state the Day number when Mercury is in the middle of its retrograde motion. [1]

....

....

....

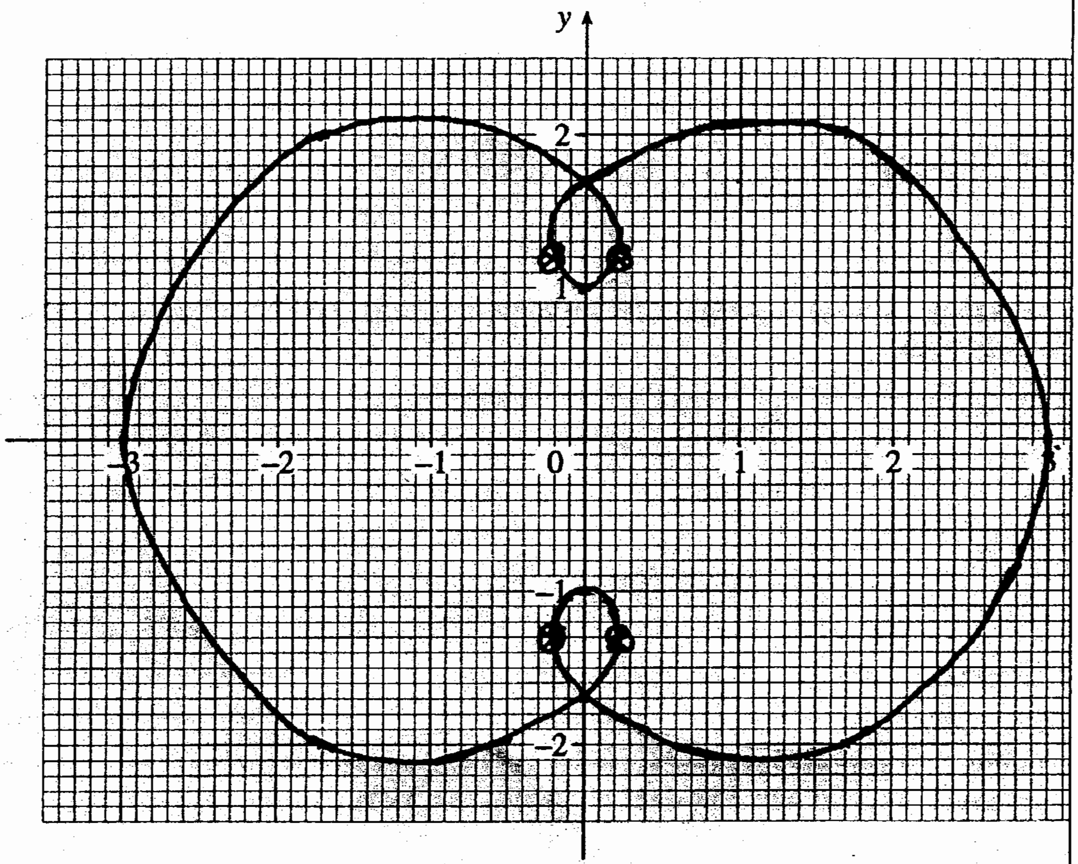
(ii) Draw a diagram showing the relative positions of the Earth, the Sun and Mercury at the first of these times. [2]

5 The curves in Fig. 5 on page 5 illustrate the motions of the Sun and Mars relative to the Earth. The curves intersect. Explain why this does not imply that Mars collides with the Sun. [2]

....

....

....

1	20 is the number of days since Day 0 90 is the number of days taken to complete an orbit	B1 B1 [2]
2.(i)	$t =$ 75 105 255 285 $x = -0.2$ 0.2 0.2 -0.2 $y =$ 1.2 1.2 -1.2 -1.2	x values B1 y values B1
(ii)		One new point correct B1 One loop attempted B1ft All correct B1cao
(iii)	2	B1 [6]
3	$x = 150 \cos t° + 778 \cos 1/12\, t°$ $y = 150 \sin t° + 778 \sin 1/12\, t°$	150 and 778 B1 Angles B1 [2]
4 (i) (ii	At days 60, 180, 300. For three bodies in a straight line with any orientation. For the correct order EMS or SME. Note: Incorrect answers to part (i) do not invalidate the marks for part (ii). Do not ft incorrect diagrams in part (ii)	B1 M1 A1 [3]

5	Give B2, 1 or 0, according to the quality of the answer . Guidelines; **Answers deserving B2** are those which say, in some way or other, that the Sun and Mars are never at the same point relative to the Earth. e.g. 'When one body is at the point of intersection the other is somewhere else.' 'The two bodies are never simultaneously at the point of intersection.' 'The curves are formed by varying the parameter *t*. The bodies don't have to be in the same place at the same time.' 'Although the two bodies pass through the same point relative to the Earth they don't have to do it at the same time.' **Answers deserving B1** are those which assert the above but for a wrong reason, **or** those which give a correct reason why the Sun and Mars do not collide without explaining why the orbits in figure 5 intersect. e.g. 'The curves intersecting doesn't imply that the Sun and Mars cross at the same point in time, because Mars reaches the point of intersection when the Sun is on the opposite side of the Earth .' 'Mars is orbiting the Sun which means that it cannot possibly collide with the Sun.' 'The curves illustrate relative motion and not the actual path taken.' 'Because the Earth and Mars orbit the Sun. This diagram shows how things appear from the Earth but they are actually miles apart.' **Answers not worth a mark .** Those which say no more than that the motion illustrated is that of the Sun and Mars relative to the Earth, (as stated in the question), or answers that are much too vague or imprecise. e.g. 'This does not imply that Mars collides with the Sun because they are both different distances away from the Earth.' 'They would be on the same plane relative to the Earth, but only appear to collide when Mars goes the other side of the Sun, only apparent on a 3-D diagram.'	B 2, 1 or 0 Total [15]

Mark Scheme

1 (a) $\dfrac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!}(2x)^3$ $= \frac{1}{2}x^3$, so $\frac{1}{2}$	M1 A1 A1 cao [3]	Binomial series with index ½. Correct expression for term in x^3. Simplified. Accept ½ x^3.
(b) $\sin 2x = \cos x$ $\Rightarrow 2\sin x\cos x = \cos x$ $\Rightarrow 2\sin x\cos x - \cos x = 0$ $\Rightarrow \cos x(2\sin x - 1) = 0$ $\Rightarrow \cos x = 0, x = \pi/2, 3\pi/2$ or $\sin x = 1/2, x = \pi/6, 5\pi/6$	M1 B1 ,B1 B1 , B1 [5]	$\sin 2x = 2\sin x\cos x$ used in the equation. Answers w.w. B1,2,3 or 5. Answers as decimals or in degrees- follow M.S. and –1
(c) let $u = \sin x$, $du = \cos x\,dx$ $\Rightarrow \int \sin^2 x\cos x dx = \int u^2 du$ $= u^3/3 = \frac{1}{3}\sin^3 x + c$ $\int_0^{\pi/2} \sin^2 x\cos x dx = \left[\frac{1}{3}\sin^3 x\right]_0^{\pi/2}$ $= \frac{1}{3}\sin^3\frac{\pi}{2} - \frac{1}{3}\sin^3 0$ $= \frac{1}{3}$	M1 A1 M1 A1 cao [4]	$\int u^2 du$. For I by P, or by inspection, giveB2 for correct result or 0. $\frac{1}{3}\sin^3 x + c$ Condone no c. .Accept $\frac{1}{3}u^3 + c$. substituting correct limits c.a.o. refers to whole of **(c)**.
(d) $y = 2^x$ $\Rightarrow \ln y = x\ln 2$ $\Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln 2$ **OR** $y = e^{x\ln 2}$ $\Rightarrow \frac{dy}{dx} = y\ln 2$ **OR** $\frac{dy}{dx} = \ln 2\, e^{x\ln 2}$ $= 2^x \ln 2$	B1 B1, B1 B1 cao [4] Total [16]	$x\ln 2$ $\frac{1}{y}\frac{dy}{dx}$, $\ln 2$ **OR** B1 $e^{x\ln 2}$ **OR** B2 cao $\ln 2e^{x\ln 2}$

2 (i) A is (4, 0) , B is (0, 3)	B1, B1 [2]	
(ii) $\theta = \pi/4 \Rightarrow x = 4 \times \sqrt{2}/2 = 2\sqrt{2}$ $y = 3\sqrt{2}/2$	B1 B1 [2]	Allow $4/\sqrt{2}$, $3/\sqrt{2}$ SCB1 both correct but approximate. 2.83, 2.12.
(iii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= -\frac{3\cos\theta}{4\sin\theta}$ When $\theta = \pi/4$ $dy/dx = -3/4$. Equation of tangent is $y - \frac{3\sqrt{2}}{2} = -\frac{3}{4}(x - 2\sqrt{2})$ $\Rightarrow 4y - 6\sqrt{2} = -3x + 6\sqrt{2}$ $\Rightarrow 3x + 4y = 12\sqrt{2}$ * Accept verification of the given equation, i.e. the equation is satisfied by $(2\sqrt{2}, 3\sqrt{2}/2)$ and the gradient is $-3/4$, from both the equation and dy/dx.	M1 A1 A1ft M1 E1 [5] M1 A1 E1	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ Seen or implied in subsequent work ft their x and y from (ii) and their gradient. No θ seen. Both attempted $dy/dx = -3/4$ All detail correct
(iv) When $y = 0$, $x = 4\sqrt{2}$ when $x = 0$, $y = 3\sqrt{2}$ $\Rightarrow$ area of POQ $= 1/2 \times 4\sqrt{2} \times 3\sqrt{2}$ $= 12$ square units	B1 B1ft [2]	Accept decimal equivalents 5.66, 4.24 Accept answer rounding to 12
v) $\cos\theta = x/4$, $\sin\theta = y/3$ s.o.i. $\Rightarrow \cos^2\theta + \sin^2\theta = (x/4)^2 + (y/3)^2 = 1$ $\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$ Accept $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$	M1 M1 A1 [3] Total [14]	Allow the first M1 for $\cos\theta = x/4$ and $y = 3\sin(\cos^{-1}x/4)$. $y = 3/4\sqrt{(16 - x^2)}$ may then gain M1 A1

3 (i) $x = 2$ Allow $l = 2$	B1 [1]	
(ii) $\frac{1}{x(2-x)} = \frac{A}{x} + \frac{B}{2-x}$ $\Rightarrow \quad 1 = A(2-x) + Bx$ $x = 0 \Rightarrow 1 = 2A \Rightarrow A = 1/2$ $x = 2 \Rightarrow 1 = 2B \Rightarrow B = 1/2$ $\Rightarrow \frac{1}{x(2-x)} = \frac{1}{2x} + \frac{1}{2(2-x)}$	M1 A1 A1	s.o.i.
$\text{A} = \int_1^{1.5} (\frac{1}{x(2-x)}) dx$	M1	Integral and limits
$= \frac{1}{2}\int_1^{1.5} \frac{1}{x} + \frac{1}{2-x} dx$ $= \frac{1}{2}$ [ln x − ln (2−x)]$_1^{1.5}$	A1ft	$\frac{1}{2}[\ln x - \ln(2-x)]$ or equivalent. ft their PFs
$= \frac{1}{2}(\ln 1.5 - \ln 0.5)$	M1 A1ft	substituting their limits ft their integration.
$= \frac{1}{2}\ln\frac{1.5}{0.5} = \frac{1}{2}\ln 3$ or $\ln\sqrt{3}$	A1 cao[8]	
(iii) $\frac{1}{z}\frac{dz}{dx} = \frac{1}{x(2-x)}$ $\Rightarrow \int \frac{1}{z} dz = \int \frac{1}{x(2-x)} dx$	M1	Separating variables
$\Rightarrow \ln z = \frac{1}{2}[\ln x - \ln(2-x)] + c$	A1ft	ft their integration in (ii) Condone the absence of c
When $x = 1, z = 1 \Rightarrow 0 = 0 - 0 + c$	B1ft	evaluating c
$\Rightarrow \ln z = \frac{1}{2}[\ln x - \ln(2-x)] = \frac{1}{2}\ln\frac{x}{2-x}$	M1	combining lns May be given at other stages in (ii) or (iii)
$\Rightarrow 2\ln z = \ln\frac{x}{2-x}$ $\Rightarrow \ln z^2 = \ln\frac{x}{2-x}$	M1	$2 \ln z = \ln z^2$ or $\frac{1}{2}\ln\frac{x}{x-2} = \ln\sqrt{\frac{x}{x-2}}$
$\Rightarrow z^2 = \frac{x}{2-x}$ *	E1 cso [6] Total [15]	

4 (i) $\begin{pmatrix}1\\8+3\lambda\\8+4\lambda\end{pmatrix}=\begin{pmatrix}1\\2\\0\end{pmatrix}$ when $\lambda=-2$ $\begin{pmatrix}3+2\mu\\3+\mu\\-1-\mu\end{pmatrix}=\begin{pmatrix}1\\2\\0\end{pmatrix}$ when $\mu=-1$ So both lines contain the point (1, 2, 0)	M1 A1 A1 [3]	Equating $\lambda=-2$ $\mu=-1$
(ii) Angle between $\begin{pmatrix}0\\3\\4\end{pmatrix}$ and $\begin{pmatrix}2\\1\\-1\end{pmatrix}$ $\text{Cos }\theta=\dfrac{0\times 2+3\times 1+4\times -1}{\sqrt{25\times 6}}$ $=-\dfrac{1}{5\sqrt{6}}$ $\Rightarrow \theta=94.7°$ so $85.3°$	M1 M1 A1,A1 A1 [5]	Give M0 for other vectors but allow the second M1 and A1,A1 for $\cos\ \theta=$ $(1.3+8.3+8.-1)/\sqrt{(129.19)}$ -1 or 1, $5\sqrt{6}$ 85.3°, condone 94.7°. Accept 1.49 radians, condone 1.65 radians
(iii) $\mathbf{n}=\begin{pmatrix}2\\-1\\1\end{pmatrix}$ $\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$	B1 B1ft [2]	 Condone the omission of **r**. ft their **n**
(iv) Substituting $x=1+2\lambda$, $y=2-\lambda$, $z=\lambda$; $\Rightarrow 2(1+2\lambda)-(2-\lambda)+\lambda=12$ $\Rightarrow 2+4\lambda-2+\lambda+\lambda=12$ $\Rightarrow \lambda=2$ So line meets plane at (5, 0, 2) Perp distance $=\sqrt{\{(5-1)^2+(0-2)^2+(2-0)^2\}}$ $=\sqrt{24}$ or $2\sqrt{6}$ or 4.90	M1 A1 ft A1 ft M1 A1c.a.o [5] Total [15]	 ft their line from (iii) or l_1 or l_2 $\lambda=2$ Distance formula.

Examiner’s Report

2603 Pure Mathematics 3

General Comments

Candidates generally performed well on this paper, a very pleasing number scoring marks in the range 65 to 75 with rather more candidates than usual scoring full marks. Very few candidates scored less than ten marks, indeed the number in the range 0 to 20 was quite small. There were some very well presented scripts, neat and accurate, and generally there was less evidence of careless mistakes and very poor algebra.

All questions contained parts which were accessible to all candidates and these parts were answered well; the binomial theorem, coordinates from parametric equations, partial fractions, the angle between two lines and the equation of a line perpendicular to a plane.

In Section B, too, the first three questions presented candidates with few problems except, perhaps, the difficulty of drawing a smooth curve through the given and the calculated points.

Comments on Individual Questions

Question 1

(a) The third term of the binomial series for $(1 + 2x)^{1/2}$ was found correctly by most candidates. Occasional errors were an incorrect index $-1/2$; $2x^3$ or x^3 instead of $(2x)^3$; or an incorrect product in the numerator of the coefficient $1/2(-3/2)(-5/2)$ instead of $1/2(-1/2)(-3/2)$.

(b) Almost all candidates were able to use the double angle formula for sin $2x$ correctly to obtain $2 \sin x \cos x = \cos x$, but very many cancelled $\cos x$ and solved $\sin x = ½$, thus obtaining only two of the four possible solutions. A few candidates went from the above equation to $2 \sin x = 0$. Another occasional error was to consider only the range $0 \leq \theta \leq \pi$.

(c) Many candidates are not confident in the use of integration by substitution and the direct statement let $u = \sin x$, $\mathrm{d}u = \cos x \Rightarrow \int \sin^2 x \cos x \, \mathrm{d}x = \int u^2 \mathrm{d}u$ was only seen from the most able candidates. However, other candidates arrived at the correct result by less direct means. Most candidates getting this far, then went on to complete the question correctly, often changing the limits of the integral to limits for the variable u. Unfortunately a few of these candidates failed to return at all to the variable x and so did not answer the first part of the question. However this omission was condoned.

Some candidates confused integration by substitution and integration by parts, having put $u = \sin x$ they attempted to use integration by parts on the integral $\int u^2 \cos x \, \mathrm{d}x$.

(d) This question was completed successfully only by the most able candidates.

Most candidates were able to take the first step to obtain $\ln y = x \ln 2$, but although some were able to differentiate $\ln y$ implicitly many differentiated $x \ln 2$ using the product formula and differentiating ln2 as ½. A small number of candidates went from the above line to $y = e^{\ln x}$ then differentiated correctly to obtain $\mathrm{d}y/\mathrm{d}x = \ln 2 \, e^{x \ln 2}$.

(a) ½ ; (b) $\pi/6$, $\pi/2$, $5\pi/6$, $3\pi/2$; (c) $1/3 \sin^3 x + c$, $1/3$; (d) $2^x \ln x$.

Question 2 (Parametric equations)

(i) Most candidates obtained the correct coordinates for the points A and B in a variety of ways.

(ii) The coordinates of C were also found correctly by many candidates although some found approximate instead of exact values.

(iii) Almost all candidates were able to differentiate the parametric equations correctly and find an expression for $\mathrm{d}y/\mathrm{d}x$, although just a few had this expression upside down. However many candidates made very heavy weather of the remainder of this part of the question. Some did not substitute the value of the parameter, $\theta = \pi/4$, immediately, to obtain $dy/dx = -3/4$, or if they did the result was often left as $\frac{-3/\sqrt{2}}{4/\sqrt{2}}$ for substitution into the equation of the tangent.

Just a few candidates attempted to verify that the given equation was the equation of the tangent at C but most of these candidates failed to check both the gradient and the point of contact.

(iv) Despite some difficulty in manipulating surds in part (iii) almost all candidates were able to obtain the correct coordinates for P and Q, and find the area of the triangle OPQ. Just a few candidates resorted to approximate values but still obtained the correct area.

(v) There was a mixed response to this question. Some candidates quoted the general equation of an ellipse and then substituted, usually correctly, for the values of '*a*' and '*b*' using their results from part (i); a few candidates had already recognised the parametric coordinates of an ellipse in part (i) and used the equation there to find the coordinates of A and B. It was usually the more able candidates who eliminated θ from the parametric equations using the identity $\sin^2\theta + \cos^2\theta = 1$, and the weaker candidates who attempted to proceed by squaring and adding the equations as they stand obtaining $x^2 + y^2 = 16\cos^2\theta + 9\sin^2\theta$. Just a few candidates felt that it was sufficient to write $\theta = \cos^{-1}x/4$, and hence $y = 3\sin(\cos^{-1}x/4)$.

(i) (4,0), (0,3); (ii) $2\sqrt{2}, 3\sqrt{2}/2$; (iv) 12 sq. units; (v) $x^2/16 + y^2/9 = 1$.

Question 3 (Partial fractions, integration and differential equations)

(i) Most candidates gave the correct equation although just a few did not answer this part.

(ii) The partial fractions were found correctly by almost all the candidates.

(iii) Again, almost all candidates wrote down the correct integral and limits and used the partial fractions to enable them to integrate, but there were two common errors.

$$\int \frac{1}{2x} + \frac{1}{2-x}\,dx = 2\ln x - 2\ln(2-x) \text{ or } \frac{1}{2}\ln x + \frac{1}{2}\ln(2-x)$$

Most candidates made correct substitutions into their integrals.

(iii) The above errors were usually repeated in solving the differential equation after candidates had separated the variables correctly. Also, very many candidates dealt correctly with the combination of $\ln x$ and $\ln(2-x)$ and with $\frac{1}{2}\ln\frac{x}{2-x} = \ln\left(\frac{x}{2-x}\right)^{1/2}$. However, those candidates who delayed finding the value of the arbitrary constant until this stage often wrote

$$\ln z = \ln\left(\frac{x}{2-x}\right)^{1/2} + c \Rightarrow z = \left(\frac{x}{2-x}\right)^{1/2} + e^c.$$

(i) $x = 2$; (ii) $\frac{1}{x(2-x)} = \frac{1}{2x} + \frac{1}{2(2-x)}$, $\frac{1}{2}\ln x$.

Question 4. Vectors.

This question as a whole was very well done by the majority of candidates.

(i) Candidates who dealt separately with the two lines almost always found the correct values for λ and μ and checked that their values satisfied the equations from each of the coordinates. Candidates using the fact that if P lies on both lines then it is the point of intersection, usually solved the simultaneous equations $8 + 3\lambda = 3 + \mu$, and, $8 + 4\lambda = -1 - \mu$, and only had to check their values of λ and μ in one other equation. A few candidates solved just the one equation, $3 + 2\mu = 1$,and failed to find the value of λ.

(ii) This question was done very well except that a surprising number of candidates gave the obtuse angle between the lines. A small number used the wrong vectors, $\begin{pmatrix}1\\8\\8\end{pmatrix}$ and $\begin{pmatrix}3\\3\\-1\end{pmatrix}$.

(iii) Also very well done. Just a few candidates interchanged the position vector and the direction vector in their equation of the line.

(iv) Candidates were confident in finding the intersection of the line and the plane and most used their point of intersection to find the perpendicular distance required. A few candidates used the formula for the distance of a point from a plane to do the last part.

Section B. Comprehension.

The performance of candidates on the comprehension was good, especially in questions 1, 2 and 3. There were a few numerical or sign errors in calculating the missing values in the table and some graphs were not very well drawn with straight sections instead of smooth curves. Occasionally candidates failed to get their graphs to cross at the points (0, 1.7) and (0, −1.7).

In question 3 (a) small number of candidates confused Jupiter with Mars and wrote the angle $t/2$ instead of $t/12$ and there were occasional slips of cos for sin or vice-versa. Also an occasional error in the radius 778.

Question 4 was not well done, many candidates giving the Day numbers as 15, 45 and 75 instead of 60, 180 and 300. In addition, although the relative positions of the Earth, the Sun and Mercury were often shown in the correct order on a straight line the answer would have been more convincing if the line had been shown at an angle of 60° to an initial line.

Question 5. A fairly small number of candidates showed clearly that they understood the diagram of the motion of the Sun and Mars relative to the Earth and gave excellent answers to this question. The majority of candidates appeared to be confused, many by lines 57-59 in the insert. Candidates often referred to this statement and came to the wrong conclusion, that the Sun and Mars could not collide because, when Mars was at the point of intersection the Sun was on the other side of the Earth. Other candidates said that the Sun and Mars appeared from Earth to be in line but were in fact far apart, whilst others said that 'things were different in three dimensions'. A considerable number of candidates avoided an explanation of the intersection of the apparent orbits by stating simply, and correctly, that the Sun and Mars could not collide because Mars was in an orbit round the Sun.