OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2601

Pure Mathematics 1

Wednesday **14 JANUARY 2004** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

[Turn over

Section A (30 marks)

1 Differentiate $3x^6 + 2x + 7$. [2]

2 Sketch the graph of $y = |x - 3|$, showing the values of x and y where it meets the axes. [3]

3 Find the coefficient of x^2 in the binomial expansion of $(2 + 3x)^5$. [3]

4 Sketch the graph of $y = \cos x$ for $0° < x < 360°$.

Solve the equation $\cos x = 0.4$ for $0° < x < 360°$. Give your answers correct to 1 decimal place. [3]

5 Show that, for a point (x, y) on a circle with centre (a, b) and radius r,

$$(x - a)^2 + (y - b)^2 = r^2.$$

Use a sketch copy of Fig. 5 as part of your explanation. [3]

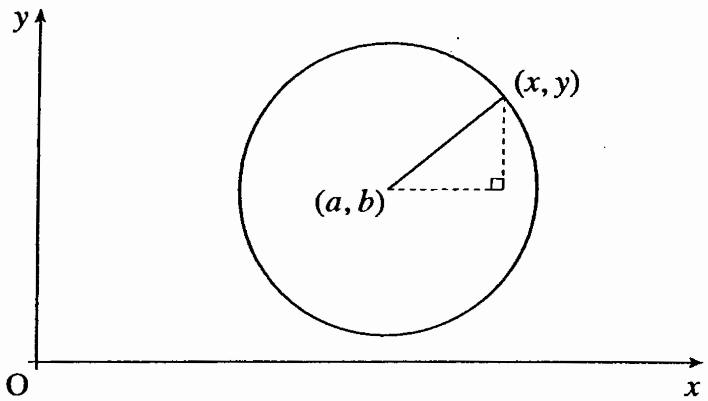
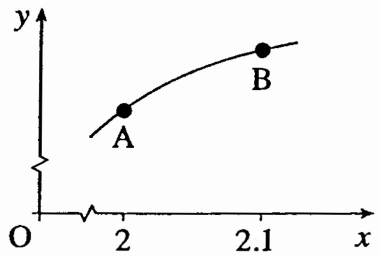


Fig. 5

6 Show that $(x + 3)$ is a factor of $x^4 + 25x - 6$ but that $(x + 2)$ is not. [4]

7 In Fig. 7, A and B are points on the curve $y = \sqrt{x}$ with x-coordinates 2 and 2.1 respectively.



Not to scale

Fig. 7

Find the gradient of the chord AB.

Stating the points you use, find the gradient of another chord which will give a closer approximation to the gradient of the tangent to $y = \sqrt{x}$ at $x = 2$. [4]

8 Fig. 8 shows a sector OPQ of a circle with centre O and radius 5 cm. Angle POQ = 0.3 radians.

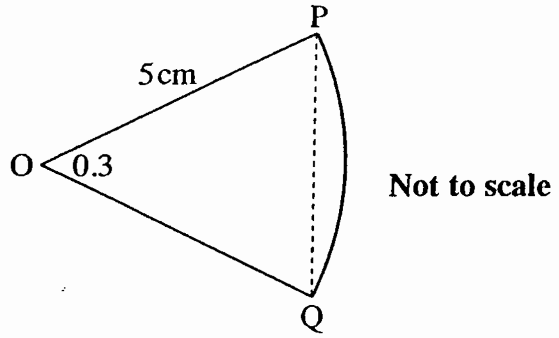


Fig. 8

Find the length of the arc PQ.

Find also the area of triangle POQ. [4]

9 The region bounded by the curve $y = x^2$, the lines $x = 1$, $x = 3$ and the x-axis is rotated through 360° about the x-axis.

Find the volume of revolution generated. [4]

TURN OVER FOR SECTION B

Section B (30 marks)

10 Water is delivered through three hoses, P, Q and R.

(i) Hose P delivers water at a constant rate of 0.2 litres per second.

(*A*) What volume of water is delivered in 40 seconds? [1]

(*B*) Sketch the graph of V against t, where V litres is the volume of water delivered by this hose in t seconds.

State the equation for V in terms of t. [2]

(ii) Hose Q delivers water at a variable rate, as shown in Fig. 10.1.

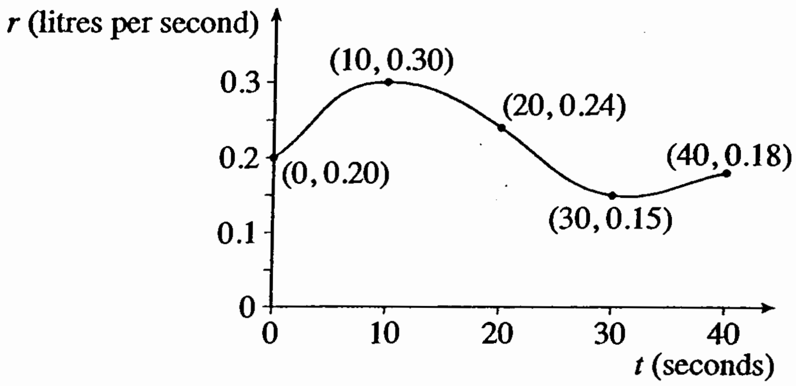


Fig. 10.1

Use the trapezium rule with 4 strips to estimate the volume of water delivered by this hose in 40 seconds. [3]

(iii) Fig. 10.2 shows the rate of flow delivered by hose R in the first 40 seconds.

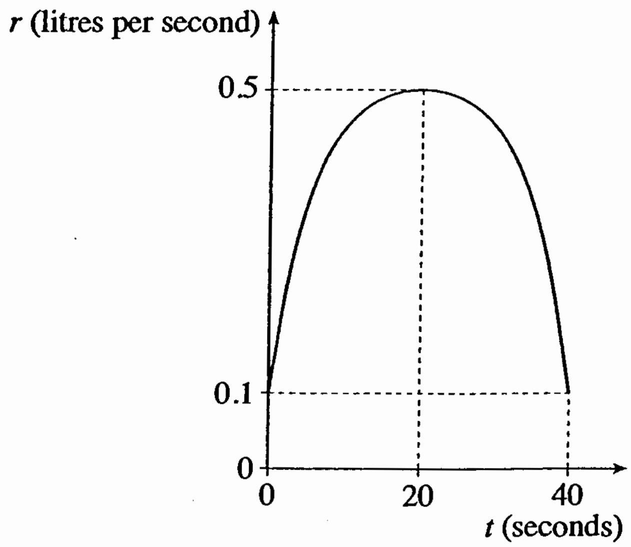


Fig. 10.2

A model is proposed for this rate of flow. In this model,

$$r = 0.001(100 + 40t - t^2)$$

where r is the rate of flow in litres per second at time t seconds.

(*A*) Calculate $0.001\int_0^{40}(100 + 40t - t^2)\,dt$ to find the volume of water delivered in the first 40 seconds according to this model. [3]

(*B*) Write $100 + 40t - t^2$ in the form $a - (t - b)^2$. [3]

(*C*) Give two features which make this model appropriate for $0 \leqslant t \leqslant 40$. [2]

(*D*) Comment on the suitability of this model for $t = 50$. [1]

11 The equation of a curve is $y = x^2 - 8x + 12$.

(i) Find the points of intersection of this curve with the x- and y-axes. [3]

(ii) Show that the equation of the tangent to the curve at the point with x-coordinate 5 is $y = 2x - 13$. [4]

(iii) Show that this line is also a normal to the curve $y = -2x^2 + 23.5x - 70$ at the point $(6, -1)$. [4]

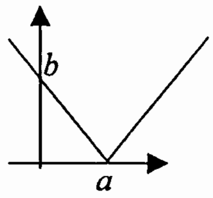
(iv) Find the x-coordinates of the points of intersection of the two curves

$$y = x^2 - 8x + 12,$$

$$y = -2x^2 + 23.5x - 70.$$

Give your answers correct to 2 decimal places. [4]

Mark Scheme

	Section A			
1	$18x^5 + 2$	B2	1 for each term; −1 if extra term	2
2	sketch of line of positive or negative gradient from $(a, 0)$, $a>0$ [ignore continuing for $y < 0$]; second arm of V (accept for any a) [ignore continuing below x axis; $a = 3$and $b = 3$ shown, cao – V clearly chosen and reasonably symmetrical gradients	G1 G1 G1	allow this mark for V with vertex on y axis 	3
3	720	B3	M2 for ${}^5C_2 \times 2^3 \times (3x)^2$ or better, M1 for two of these, <u>or</u> B1 for 1 5 10 10 5 1 seen or 10 used or final answer of 72, 90, 80 or 240	3
4	curve of cos shape, one period amplitude 1 shown 66.4 and 293.6 and no other solns in range	G1 G1 B1	or 360 shown in correct position [allow from eg sin curve] condone other r.o.t. versions of 66.42... and 293.578... [wrong rounding penalised in q11]	3
5	distances $x - a$ and $y - b$ clearly shown in correct places Pythagoras theorem quoted completion from Pythagoras, with length and length^2 used correctly	B1 B1 B1	may be on diagram or distance formula or $\sin^2\theta + \cos^2\theta = 1$	3
6	f(−3) seen or used = 81 − 75 − 6 [= 0] f(−2) seen or used = 16 − 50 − 6 or −40	M1 A1 M1 A1	or M1 for long division as far as $x^3 - 3x^2$ A1 for $x^3 - 3x^2 + 9x - 2$ or M1 for long division as far as $x^3 - 2x^2$ A1 for $x^3 - 2x^2 + 4x + 17$ and remainder	4
7	0.34 to 0.35 eg $(\sqrt{2.01} - \sqrt{2})/(2.01 - 2)$ 0.35 to 0.357	B2 M1 A1	M1 for attempt at $(\sqrt{2.1} - \sqrt{2})/(2.1 - 2)$ allow one pt in [1.9,2] and one in [2, 2.1] or 2 and pt closer than 2.1 do not allow for 0.35 in both parts	4
8	1.5 to 1.501 3.69 to 3.7	B2 B2	M1 for 5 × 0.3 o.e. M1 for $0.5 \times 5^2 \times \sin 0.3$ or complete longer method	4
9	attempt at $\pi\int y^2 dx$ used with $y = x^2$ subst <u>or</u> limits 1 to 3 $[\pi]\frac{x^5}{5}$ $[\pi]$(value at 3 – value at 1) $\frac{242\pi}{5}$ o.e. eg 152 to 152.1	M1 A1 M1 A1	condone omission of π attempt seen ft their integral; condone no π cao	4

		Section B			
10	(i)	A 8 [litres] B straight line through origin with positive gradient $V = 0.2t$ o.e.	B1 G1 B1	 need not be ruled must be V and t, not y and x. V must be subject	1 2
	(ii)	8.8	B3	M2 for $0.5 \times 10 \times [0.2 + 0.18 + 2(0.3 + 0.24 + 0.15)]$ o.e. or M1 if h wrong or brackets missing or for 3 separate traps results correct [2.5, 2.7, 1.95, 1.65] [NB may have earned M2]	3
	(iii) A	$[0.001](100t + 40t^2/2 - t^3/3)$ 14.6 to 14.7	M2 A1	M1 if one error, condone x's	3
	(iii) B	$b = 20$ $a = 500$	B1 B2	allow for $(t - 20)^2$ seen M1 for $100 - [(t - 20)^2 - 400]$ or B1 for $a = -500$, 300 or -300	3
	(iii) C	eg 'symmetrical about $t = 20$' 'max at $r = 0.5$' <u>or</u> 'max at $t = 20$' 'correct value at $t = 20$' 'correct value at $t = 0$' <u>or</u> 'correct value at $t = 40$'	B1 + 1	one each for two comments also allow: 'it is an upside-down parabola'	2
	(iii) D	'r will be negative' or 'hose will be sucking water'	B1	or 'we don't know what happens for hose C outside 0 to 40'	1
11	(i)	(2,0) (6,0) (0, 12)	B1+1 B1	or M1 for $(x - 2)(x - 6)$ or formula condone $x = 2$ and 6, and $y = 12$	3
	(ii)	$y' = 2x - 8$ $= 2$ when $x = 5$ www when $x = 5$ [on curve], $y = -3$ $y + 3 = 2(x - 5)$ or $-3 = 2 \times 5 + c$ or showing $(5, -3)$ is on $y = 2x-13$	M1 A1 B1 B1	or M1 for $2x - 13 = x^2 - 8x + 12$ M1 for $0 = x^2 - 10x + 25$ cao M1 for $0 = (x - 5)^2$ A1 for equal roots $x = 5$, imply tangent at $x = 5$	4
	(iii)	$y' = -4x + 23.5$ At $(6, -1)$, $y' = -0.5$ Grad of normal $= -1$ / their y' $y + 1 =$ their normal grad $\times (x - 6)$	M1 A1 M1 ft M1 ft	 must be numerical; or $2 \times -1/2 = -1$ or showing $(6, -1)$ is on $y = 2x - 13$	4
	(iv)	$3x^2 - 31.5x + 82$ [= 0] attempted use of quad. formula or completing square 4.77 and 5.73	M1 M1 A1 + A1	at least 2 terms correct. or $x^2 - 8x + 12 = -2x^2 + 23.5x - 70$ ft if previous M gained A1 for both of 4.771.. and 5.7287.. to wrong dp	4

Examiner’s Report

2601 Pure Mathematics 1

General Comments

In general, candidates were well-prepared for this examination, with relatively few weak scripts being seen.

The examiners wish to discourage the use of graph paper for sketch graphs. Many candidates wasted time in drawing graphs on graph paper. This and other long methods used in question 6 and 8, for instance, meant that a few candidates ran out of time on question 11 and so penalised themselves. For the majority of candidates, time was not a problem.

Comments on Individual Questions

Q.1. This was very well answered, with nearly all candidates obtaining both marks.

$18x^5 + 2$

Q.2. Candidates have difficulty in handling the modulus function, although most knew that something had to be done to the ordinary graph. The correct graph was given by about a half of the candidates.

Q.3. Relatively few candidates coped correctly with the powers of 2 and 3, often failing to square the 3, even when $(3x)^2$ was seen. Most used the correct binominal coefficient, although some candidates tried to multiply out the brackets longhand – and usually failed in the process.

720

Q.4. Errors in the graph were mostly through drawing too much of a zigzag instead of a correctly shaped curve or not marking the amplitude on the y axis, although a few used a sine graph. Few found more than two values in the second part, but 336.4 instead of 293.6 was the common error.

66.4 and 293.6

Q.5. Despite the hint in the question, many did not use the diagram they drew and did not show the lengths $x - a$ and $y - b$ in the correct places, with some transposing them and many omitting them. The majority did realise that Pythagoras' theorem was involved and gained 1 mark.

Proving $(x - a)^2 + (y - b)^2 = r^2$

Q.6. Many candidates successfully used the factor and remainder theorems. Those who attempted long division often made errors – and took far longer, of course.

Q.7. The majority knew what was expected and gained both method marks, although premature approximation often spoilt the solution. Some differentiated, particularly in the second part, gaining no credit for this.

0.34 to 0.35

Q.8. Most candidates knew what was needed for the arc, although some attempted conversion to degrees, with 0.3 x 180 = 54° being a common error. Some found the area of the sector rather than the triangle. Relatively few used the ½ *ab* sin *C* formula effectively – those who did and could cope with radian mode on their calculator gained a quick 4 marks in this question. However many candidates used long methods to find the base and height of the triangle – for instance use of the cosine rule in spite of the fact that the triangle is isosceles. Such candidates often muddled themselves and failed to obtain the correct answer.

1.5, 3.7

Q.9. Better candidates had few problems, although some omitted π on the way. Integrating πy instead of πy^2 was common. Most correctly converted their function to x then integrated their function and substituted correctly.

$\frac{242}{5}\pi$

Q.10. This question discriminated well. Parts (iii)*(B)* and *(C)* in particular proved stretching even for able candidates.

(i) Nearly all candidates picked up marks in this part, although some drew a curve or failed to give V in terms of t.

(ii) The trapezium rule was applied very well by many, although with the usual errors in arithmetic or handling of the brackets, etc.

(iii)*(A)* was often correct, though some made errors in integration or with the arithmetic, for instance forgetting 0.001.

(iii)*(B)* Many had great difficulty in completing the square and handling the minus signs correctly, so that the 100 and 400 were often combined to 300 or –300 or –500. Weaker candidates often made little attempt, not knowing what to do.

(iii)*(C)* Many candidates missed the point of this question and gave reasons why the model was useful rather than appropriate, or only gave vague generalities – saying that r was positive was not deemed to be sufficient. Many candidates also think that a model is more accurate than the actual data.

(iii)*(D)* Many who had not scored in part *(C)* recovered here to realise that a negative answer was not likely to be appropriate.

(i) *(A)* 8 litres
(B) straight line through origin
(ii) 8.8 litres
(iii) *(A)* 14 ⅔
(B) $500 - (t - 20)^2$
(C) features such as correct value at $t = 0$, symmetrical about $t = 20$
(D) unsuitable since r will be negative, or comment re beyond range of information

Q.11. Many competent solutions were seen, although some did not realise which methods were necessary in each part.

(i) Most correctly found the intersections, although some made heavy weather of solving the quadratic equation, and some omitted (0, 12).

(ii) There were many correct responses to find the equation of the tangent.

(iii) Most correctly differentiated to find the gradient of the tangent and then used the relationship between the gradients of perpendicular lines. However, many thought they had completed the argument at this point and did not realise that a further step was required to show that (6, -1) was on the given line or to obtain the equation of the normal passing through this point. Some thought that showing both the line and the curve passed through (6, -1) was sufficient, and did not differentiate.

(iv) Many had a correct method for eliminating *y* and then attempted use of the quadratic formula. However in this process many slips were made, in rearranging the equation or in using a wrong formula. Only good and careful candidates scored full marks. Many spent time working out the *y*-coordinates, which were not required.

(i) (2, 0) (6, 0) and (0, 12)   (iv) 4.77 and 5.73