OCR

RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2607

Mechanics 1

Tuesday **7 JUNE 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8\text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

 [Turn over

1 A car of mass 1000 kg is travelling along a straight, level road.

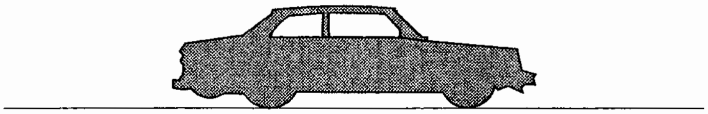


Fig. 1.1

(i) Calculate the acceleration of the car when a resultant force of 2000 N acts on it in the direction of its motion. [1]

The car has an acceleration of $1.4\,\mathrm{m\,s^{-2}}$ when there is a driving force of 2000 N.

(ii) Show that the resistance to motion of the car is 600 N. [2]

A trailer is now attached to the car, as shown in Fig. 1.2. The car still has a driving force of 2000 N and resistance to motion of 600 N. The trailer has a mass of 800 kg. The tow-bar connecting the car and the trailer is light and horizontal. The car and trailer are accelerating at $0.7\,\mathrm{m\,s^{-2}}$.

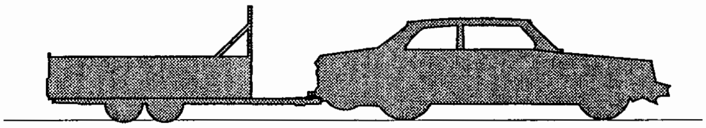


Fig. 1.2

(iii) Show that the resistance to the motion of the trailer is 140 N. [3]

(iv) Calculate the force in the tow-bar. [3]

The driving force is now removed and a braking force of 610 N is applied to the car. All the resistances to motion remain as before. The trailer has no brakes.

(v) Calculate the new acceleration. Calculate also the force in the tow-bar, stating whether it is a tension or a thrust (compression). [6]

[Total 15]

2 In this question take the value of g to be $10\,\mathrm{m\,s^{-2}}$.

A particle A is projected over horizontal ground from a point P which is 9 m above a point O on the ground. The initial velocity has horizontal and vertical components of $10\,\mathrm{m\,s^{-1}}$ and $12\,\mathrm{m\,s^{-1}}$ respectively, as shown in Fig. 2. Air resistance may be neglected.

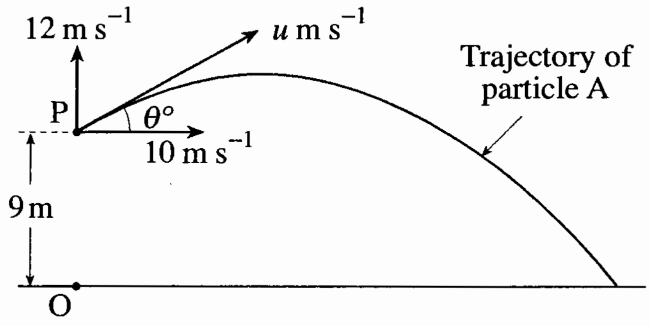


Fig. 2

(i) Calculate the speed of projection $u\,\mathrm{m\,s^{-1}}$ and the angle of projection $\theta°$. [3]

(ii) Show that, t seconds after projection, the height of particle A above the ground is $9 + 12t - 5t^2$. Write down an expression in terms of t for the horizontal distance of the particle from O at this time. [4]

(iii) Calculate the maximum height of particle A above the point of projection. [2]

A second particle, B, is projected from O with speed $20\,\mathrm{m\,s^{-1}}$ at $60°$ to the horizontal. The trajectories of A and B are in the same vertical plane. Particles A and B are projected at the same time.

(iv) Show that the horizontal displacements of A and B are always equal. [2]

(v) Show that, t seconds after projection, the height of particle B above the ground is $10\sqrt{3}t - 5t^2$. [1]

(vi) Show that the particles collide 1.7 seconds after projection (correct to two significant figures). [3]

[Total 15]

[Turn over

3 **(a)** Fig. 3 shows a block of mass 20 kg held in equilibrium on a horizontal plane. A light string at 35° to the horizontal connects one end of the block D to a wall at E. A second, horizontal, light string is attached to the other end of the block at A. This string passes over a smooth pulley at B and is attached to a freely hanging object of mass 5 kg at C.

The plane is smooth.

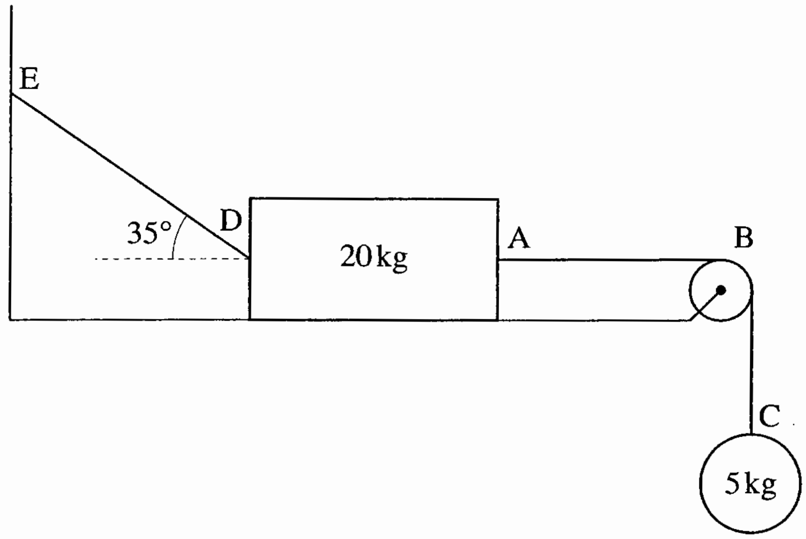


Fig. 3

(i) State the tension in the string ABC. [1]

(ii) Draw a diagram showing all the forces acting on the block. [2]

(iii) Calculate the tension in the string DE. [2]

(iv) Calculate the normal reaction of the plane on the block. [3]

(b) A particle rests on a smooth, horizontal plane. Horizontal unit vectors **i** and **j** lie in this plane. The particle is in equilibrium under the action of the three forces $(-3\mathbf{i} + 4\mathbf{j})$ N, $(21\mathbf{i} - 7\mathbf{j})$ N and $\boldsymbol{R}$ N.

(i) Write down an expression for $\boldsymbol{R}$ in terms of **i** and **j**. [2]

(ii) Find the magnitude of $\boldsymbol{R}$ and the angle between $\boldsymbol{R}$ and the **i** direction. [4]

[Total 14]

4 A particle travels along a straight line. Its *acceleration* during the time interval $0 \leqslant t \leqslant 8$ is given by the acceleration-time graph in Fig. 4. The particle starts from rest when $t = 0$.

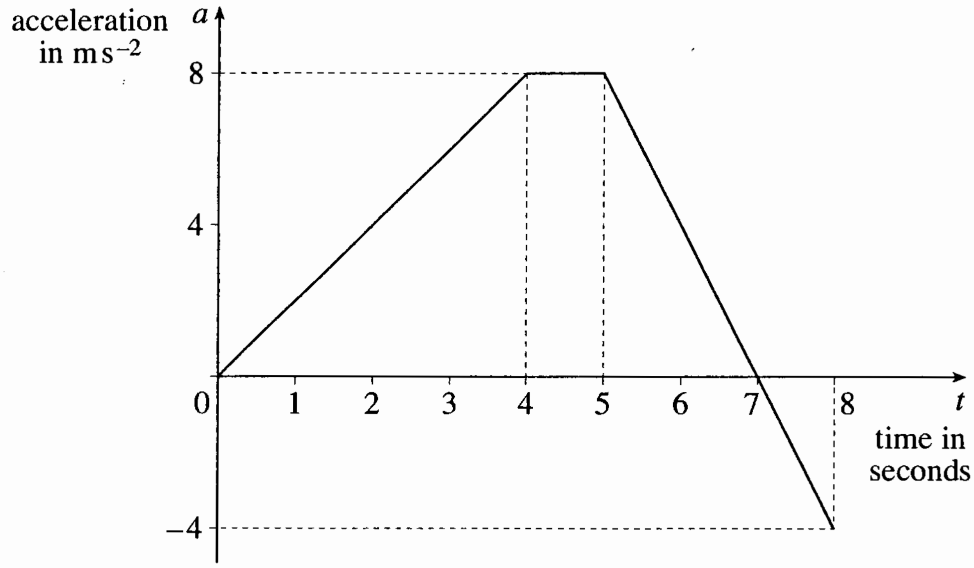


Fig. 4

(i) Write down the acceleration of the particle when $t = 4$. Also find its speed at this time. [2]

(ii) Without calculation, state the time at which the *speed* of the particle is greatest. Give a reason for your answer. [2]

(iii) Calculate the change in speed of the particle from $t = 5$ to $t = 8$, indicating whether this is an increase or a decrease. [3]

(iv) Write down an expression in terms of t for the acceleration, $a\,\mathrm{m\,s^{-2}}$, of the particle in the time interval $0 \leqslant t \leqslant 4$. Hence show that, in this interval, the speed of the particle, $v\,\mathrm{m\,s^{-1}}$, is given by $v = t^2$ at time t. [3]

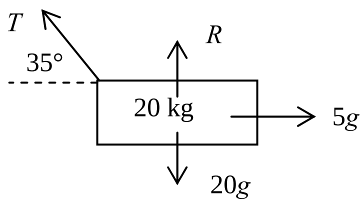
(v) Find how far the particle travels in the interval from $t = 1$ to $t = 5$. [6]

[Total 16]

Mark Scheme 2607
June 2005

Q 1		mark		sub
(i)	$2000 = 1000a$ so $a = 2$ so 2 m s^{-2}	B1		1
(ii)	$2000 - R = 1000 \times 1.4$ $R = 600$ so 600 N (AG)	M1 E1	N2L. Accept $F = mga$. Accept sign errors. Both forces present. Must use $a = 1.4$	2
(iii)	$2000 - 600 - S = 1800 \times 0.7$ $S = 140$ so 140 N (AG)	M1 A1 E1	N2L overall or 2 paired equations. $F = ma$ and use 0.7. Mass must be correct. Allow sign errors and 600 omitted. All correct Clearly shown	3
(iv)	$T - 140 = 800 \times 0.7$ $T = 700$ so 700 N	M1 B1 A1	N2L on trailer (or car). $F = 800a$ (or $1000a$). Condone missing resistance otherwise all forces present. Condone sign errors Use of 140 (or 2000 – 600) and 0.7	3
(v)	N2L in direction of motion car and trailer $-600 - 140 - 610 = 1800a$ $a = -0.75$ For trailer $T - 140 = -0.75 \times 800$ so $T = -460$ so 460 thrust	 M1 A1 A1 M1 A1 F1	 Use of $F = 1800a$ to find new accn. Condone 2000 included but not T. Allow missing forces. All forces present; no extra ones. Allow sign errors. Accept $\pm$. cao. N2Lwith their a ($\neq 0.7$) on trailer or car. Must have correct mass and forces. Accept sign errors cao. Accept ± 460 Dep on M1. Take tension as +ve unless clear other convention	6
				15

Q 2		mark		sub
(i)	$u = \sqrt{10^2 + 12^2} = 15.62..$	B1	Accept any accuracy 2 s. f. or better	
	$\theta = \arctan\left(\frac{12}{10}\right) = 50.1944...$ so 50.2 (3 s. f.)	M1	Accept $\arctan\left(\frac{10}{12}\right)$ (Or **their** 15.62cosθ = 10 or **their** 15.62sinθ = 12)	
		A1	[FT **their** 15.62 if used] [If θ found first M1 A1 for θ F1 for u] [If B0 M0 SC1 for both $u\cos\theta = 10$ and $u\sin\theta = 12$ seen]	3
(ii)	vert $12t - 0.5\times10t^2 + 9$	M1	Use of $s = ut + 0.5at^2$, $a = \pm9.8$ or ±10 and u = 12 or 15.62.. Condone $-9 = 12t - 0.5\times10t^2$, condone $y = 9 + 12t - 0.5\times10t^2$. Condone g.	
		A1	All correct with origin of u = 12 clear; accept 9 omitted	
	$= 12t - 5t^2 + 9$ (AG)	E1	Reason for 9 given. Must be clear unless $y = s_0 + ...$ used.	
	horiz $10t$	B1		4
(iii)	$0 = 12^2 - 20s$	M1	Use of $v^2 = u^2 + 2as$ or equiv with u = 12, v = 0. Condone $u \leftrightarrow v$	
	s = 7.2 so 7.2 m	A1	From CWO. Accept 16.2.	2
(iv)	Horiz displacement of B: $20\cos 60t = 10t$	B1	Condone unsimplified expression. Award for 20cos60 = 10	
	Comparison with Horiz displacement of A	E1	Comparison clear, must show 10t for each or explain.	2
(v)	vertical height is $20\sin 60t - 0.5\times10t^2 = 10\sqrt{3}t - 5t^2$ (AG)	A1	Clearly shown. Accept decimal equivalence for $10\sqrt{3}$ (at least 3 s. f.). Accept $-5t^2$ and 20sin60 = $10\sqrt{3}$ not explained.	1
(vi)	Need $10\sqrt{3}t - 5t^2 = 12t - 5t^2 + 9$	M1	Equating the **given** expressions	
	$\Rightarrow t = \frac{9}{10\sqrt{3} - 12}$	A1	Expression for t obtained in any form	
	t = 1.6915… so 1.7 s (2 s. f.) (AG)	E1	Clearly shown. Accept 3 s. f. or better as evidence Award M1 A1 E0 for 1.7 sub in each ht	3
				15

Q 3		**mark**		sub
(a) (i)	$5g$ (=49) N	B1	[If MR of 5N B0 then FT for remainder of (a)]	1
(ii)		B1 B1	All forces present with labels. No extras. Accept 49 N, *mg*, *T* and *w* without duplication. Angle not required. All forces on diagram with correct arrows	2
(iii)	$T\cos 35 = 49$ T = 59.81795… so 59.8 N (3 s. f.)	M1 A1	Resolve horizontally. Condone *T*sin35 used. No extra forces. Any reasonable accuracy	2
(iv)	$R + T\sin 35 = 20g$ R = 161.6898… so 162 N (3 s. f.)	M1 B1 A1	Resolve vertically. All forces present. Condone *T*cos35 used and sign errors. No extra forces. *T* sin35 (FT their *T*) in an equation Any reasonable accuracy. FT their *T*.	3
(b) (i)	$\mathbf{R} + \begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 21 \\ -7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\mathbf{R} = \begin{pmatrix} -18 \\ 3 \end{pmatrix}$	M1 A1	Sum to zero Award if seen here or in (ii) or used in (ii). [SC 1 for $\begin{pmatrix} 18 \\ -3 \end{pmatrix}$]	2
(ii)	$\lvert\ \mathbf{R}\ \rvert = \sqrt{18^2 + 3^2}$ = 18.248… so 18.2 N (3 s. f.) angle is $180 - \arctan\left(\frac{3}{18}\right)$ = 170.53…° so 171° (3 s. f.)	M1 A1 M1 A1	Use of Pythagoras Any reasonable accuracy. FT **R** (with 2 non-zero cpts). Allow $\arctan\left(\frac{\pm 3}{\pm 18}\right)$ or $\arctan\left(\frac{\pm 18}{\pm 3}\right)$ Any reasonable accuracy. FT **R** provided their angle is obtuse but not 180°	4
				14

Q 4		mark		sub
(i)	Acceleration is 8 m s $^{-2}$ speed is $0+0.5\times4\times8=16$ m s $^{-1}$	B1 B1		2
(ii)	$t = 7$ $a > 0$ for $t < 7$ and $a < 0$ for $t > 7$	B1 E1	 Full reason required	2
(iii)	Area under graph $0.5\times2\times8-0.5\times1\times4=6$ so 6 m s $^{-1}$ Increase	M1 B1 E1	Both areas under graph attempted. Accept both positive areas. If 2×3 seen accept ONLY IF reference to average accn has been made. Award for $v=-2t^2+28t+c$ seen or 24 and 30 seen Award if 6 seen. Accept '24 to 30'. This must be clear. Mark dept. on award of M1	3
(iv)	$a = 2t$ $v=\int 2t\,\mathrm{d}t$ $=t^2+C$ $v = 0$ when $t = 0$ so $C = 0$ (AG)	B1 M1 E1	 Integration. No arb const required Must be explicit	3
(v)	1st part $s=\int_1^4 t^2\mathrm{d}t=\left[\frac{t^3}{3}\right]_1^4$ $=\frac{64}{3}-\frac{1}{3}=21$ 2nd part **either** $16\times1+0.5\times8\times1^2$ =20 **or** $s=\int_0^1 8t+16\,\mathrm{d}t$ = 20 $s=\int_1^4 t^2\,\mathrm{d}t$ + distance (t = 4 to t = 5) total 21 + 20 = 41 m	 M1 F1 M1 A1 M1 A1 M1 A1	 Integrate. No arb const or limits required FT limits only if there has been integration Use of constant accn results with u = 16 and a = 8. $v = 8t + c$ (c non-zero) and integrate (ignore limits) Both parts of motion considered and results added cao	6
				16

2607 - Mechanics 1

General Comments

This paper was found to be far more accessible by the majority of candidates than those of previous sessions. Questions 1, 2 and 3 were done completely correctly (or very nearly so) by many of the candidates. Question 4, however, caused problems due to lack of knowledge of the properties of an acceleration-time graph.

Generally the quality of mathematics offered was high, however many candidates did not know how to *show* displayed results properly. It was quite clear at times that their knowledge of mechanics was not deficient but their skill in demonstrating a given answer or result was. In these situations many candidates would probably have fared better had the result not been displayed at all.

The proportion of candidates who were seemingly totally unprepared for this examination was far lower than in previous years.

Comments on Individual Questions

1) **The motion of a car and a trailer and the force in the tow-bar**

This question was generally answered very well. Many completely correct solutions were in evidence.

(i) Almost always correct.

(ii) Many demonstrated the given result effectively and sufficiently. However it is a concern that there are a significant number of candidates who treat a numerical "show" as an invitation to play a numbers game. Their "solution" merely consists of a sequence of arithmetical operations involving the given values and it seems reaching the "target number" is their only concern. It was not unusual to see such arithmetical listing without any indication whatsoever of the mechanical principles involved, or even an indication of which physical quantities were being considered.

(iii) As in (ii), much good work. Many candidates were able to demonstrate the given result – others "played" with the given values until the target value was reached. Some candidates (usually successfully) found the force in the tow-bar first.

(iv) Well answered although some of the number players had their bluff called for the first time here as the value was not displayed. Errors common with similar past questions were made: missing forces, extra forces and sign errors.

(v) Once again, very well done. There were many completely correct solutions given. Common errors mentioned in (iv) were also evident here. A number of candidates used the wrong mass when attempting to find the force in the tow-bar, many thinking this could be found by applying Newton's second law to the entire system.

2 **A projectile problem**

Generally extremely well done. The majority of candidates found this question very accessible and, with the possible exception of (ii), allowed them to display their knowledge effectively.

(i) Usually correct. Incorrect methods were normally due to equations of the form $10\cos\theta = u$ and $12\sin\theta = u$ being used.

(ii) The displayed result (vertical displacement of particle A above the ground) was perhaps too helpful and worked to the candidates' own detriment as this trivial result was almost always written straight down by candidates whose answers were generally good. It seemed as though they were unable to decide what exactly needed to be written down to convince the examiner of their knowledge. The short answer to this is to absolutely "spell it out" to the Examiner; in that way the Examiner – and candidate - cannot be left in any doubt about the completeness of the solution. Omission of the reason for the first term (9) of the given expression was very common.

The expression for the horizontal displacement was usually correct although a number seemingly overlooked the request for it.

(iii) Often correct although a common oversight was to find the maximum height of particle A above ground level (rather than the point of projection). Some candidates took an indirect route by finding the time taken to the greatest height first. Nevertheless this was well done and sign errors that have occurred in past projectiles questions were far less evident in this session.

(iv) Many demonstrated that the horizontal displacements were equal. However a large number thought it was sufficient to show the horizontal components of projection speeds were the same. Unless the other initial conditions (position, time) were mentioned this was deemed insufficient.

(v) Usually correct.

(vi) Again, very well done. Many equated the given expressions and then solved successfully. Some did not give evidence that their equation led to a solution of 1.7 seconds correct to two significant figures - they merely wrote 1.7 seconds as their answer without mention of a more accurate value. A significant number of candidates simply substituted the given time into the two given equations for vertical displacement; this, of course, did not show that the solution was correct to the stated degree of accuracy.

3 **A block in equilibrium & a vector statics problem**

This was perhaps not as well done as the first two questions. Nevertheless the majority of candidates scored high marks.

(a)(i) Almost always correct. 5 kg was seldom misread as 5 N. Some candidates, clearly unprepared for even the simplest calculation, took the tension to be a combination of 5(g) with 20(g).

(ii) Diagrams were normally clear and correctly labelled. Usual errors involved duplication of labels (e.g. T and T), extra forces (for example, friction) and arrows missing.

(iii) Many candidates were able to find the tension correctly. sin/ cos muddles were pleasingly uncommon and very few candidates included extra forces.

(iv) This was handled correctly by a large number of candidates. Few seemed to have any real difficulties although there remain a few who maintain that the normal reaction is equal to the weight of the block. As with (iii) it was pleasing to see that this candidature seemed to experience few problems with resolution.

(b)(i) About half of the candidates correctly found the missing force vector; the majority of the remainder thought it was the sum of the two given forces i.e. the force equal and opposite to the correct one. The remainder combined the given forces in a variety of astounding ways.

(ii) Almost all were able to follow through correctly to gain full marks for the magnitude of their vector found in (i). Many knew how to find the direction of a vector but overlooked the instruction to find the angle between $\boldsymbol{R}$ and the $\mathbf{i}$ direction (an obtuse angle) – many gave the acute angle between $\boldsymbol{R}$ and the $-\mathbf{i}$ direction for which part credit was given.

4 **The use of an acceleration-time graph**

This was by far the least well done question on the paper. A highly significant number of candidates did not appreciate that the area beneath an acceleration-time graph represented the change in velocity; because of this most of the marks in parts (i), (ii) and (iii) were not awarded. The common misconception was that the gradient represented change in speed. Also many thought the constant acceleration formulae applied when, of course, they didn't.

(i) Almost all were able to read off the acceleration from the graph. Only those who knew about the area beneath the graph were able to find the speed at $t = 4$. The common errors/ misconceptions were to find the gradient of the line segment or to use constant acceleration formulae.

(ii) A number of candidates identified the correct time as $t = 7$ but were unable to explain fully why it was the time at which the speed was greatest. A highly popular incorrect response was t = 5 (or t = 4- 5) presumably because of the maximum acceleration there.

(iii) Those who knew how to obtain change in speed had few problems. The majority, however, did not and thus scored zero with the same mistakes/ misconceptions discussed earlier.

(iv) The majority of candidates were able to write the correct expression for a (some, no doubt, by differentiation of the given expression for v). Integration was then normally and successfully used to prove the given result. The vast majority however forgot to include a constant of integration and show this was zero which deprived them of the final mark.

(v) Despite some excellent solutions, performance on this part was quite disappointing. Many candidates applied the constant acceleration formulae throughout irrespective of the strong hint given in (iv). Others integrated correctly but used limits from t = 0 to t = 4 or, even worse, from t = 1 to t = 5 (this was indeed quite common). The majority who knew how sensibly to tackle the problem made silly mistakes; for example, using wrong values when applying the constant acceleration formulae over the 1 second interval.