OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2607

Mechanics 1

Wednesday **15 JANUARY 2003** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

[Turn over

1 A car starts from rest and travels along a straight road. Its speed, v m s^{-1}, at time t seconds is modelled by

$$v = 4t - 0.2t^2, \qquad 0 \leqslant t \leqslant 10,$$

$$v = \text{constant}, \qquad 10 \leqslant t \leqslant 15,$$

$$v = 8 + 0.8t, \qquad t \geqslant 15.$$

(i) Calculate the speed of the car at $t = 0$, $t = 10$, $t = 15$ and $t = 20$. [3]

(ii) Find the values of the acceleration at

(A) $t = 7$,

(B) $t = 12$,

(C) $t = 16$. [4]

(iii) Calculate the distance the car travels in the interval $10 \leqslant t \leqslant 20$. [5]

(iv) Calculate the distance the car travels in the interval $0 \leqslant t \leqslant 10$. [4]

[Total 16]

2 A sprinkler on horizontal ground is projecting drops of water from ground level in all directions. The water droplets all have an initial speed of $15\,\mathrm{m\,s^{-1}}$.

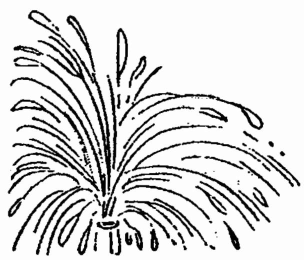


Fig. 2.1

Take $g = 10\,\mathrm{m\,s^{-2}}$ in this question. Air resistance should be neglected.

(i) Calculate the height reached by a water droplet projected vertically upwards.

What can you say about the height reached by water droplets projected at other angles? Explain your answer briefly. [4]

Fig. 2.2 shows x- and y-axes drawn through O, the point of projection. The units of the axes are metres.

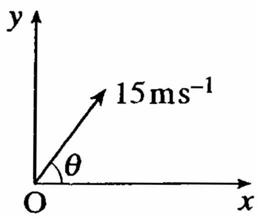


Fig. 2.2

One water droplet is projected at time $t = 0$ at an angle θ to the horizontal, where $\cos\theta = 0.6$ and $\sin\theta = 0.8$.

(ii) Show that, after time t seconds, the position of the water droplet is given by

$$x = 9t,\ \ y = 12t - 5t^2.$$ [3]

(iii) Show that the equation of the trajectory of this water droplet is

$$y = \tfrac{4}{3}x - \tfrac{5}{81}x^2.$$ [3]

(iv) Hence calculate how far from the sprinkler this water droplet lands. [3]

Another water droplet on the same trajectory hits a bird at a horizontal distance of 18 m from O.

(v) How far above the ground does the water droplet hit the bird? [2]

[Total 15]

[Turn over

3 **(a)** What force gives a mass of 48 kg an acceleration of $2\ \mathrm{m\,s^{-2}}$? [2]

(b) A girl of mass 48 kg stands in a lift that is going upwards.

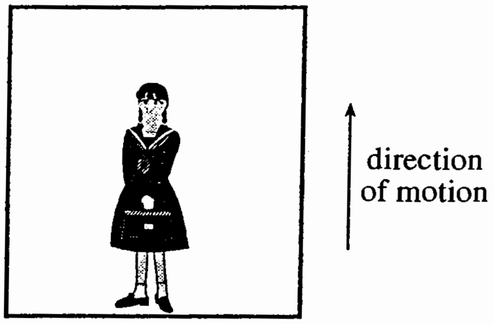


Fig. 3

The lift initially accelerates at $2\ \mathrm{m\,s^{-2}}$ and then travels at a constant speed of $1.5\ \mathrm{m\,s^{-1}}$. Finally, the lift decelerates at $3\ \mathrm{m\,s^{-2}}$.

The normal reaction of the floor of the lift on the girl is R N.

(i) Draw a diagram showing the weight of the girl and the normal reaction of the floor on her.

Write down the value of R while the lift is travelling at constant speed. [2]

(ii) Calculate the value of R when the lift is accelerating at $2\ \mathrm{m\,s^{-2}}$. [4]

(iii) Calculate the value of R when the lift is decelerating at $3\ \mathrm{m\,s^{-2}}$. [3]

The girl travels up in the lift on another occasion when she is holding a parcel of mass 5 kg by means of a light inextensible string. The lift moves as before.

(iv) The string does not break during the upwards motion. What force must the string be able to sustain?

Calculate also the value of the normal reaction of the floor of the lift on the girl (holding the parcel) while the lift is accelerating. [4]

[Total 15]

4 **(a)** Three forces act on a small object: its weight of 20 N and forces P N and Q N. These forces are shown in the **force diagram** in Fig. 4.1. They are represented as vectors in the **triangle of forces** in Fig. 4.2.

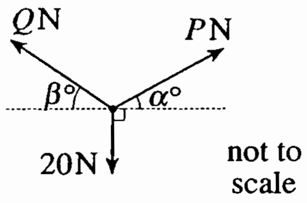


Fig. 4.1

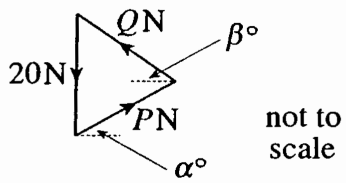


Fig. 4.2

(i) How does Fig. 4.2 show that the object is in equilibrium? [2]

A block of weight 20 N is in equilibrium on a plane inclined at 20° to the horizontal. The frictional force is F N and the normal reaction of the plane on the block is R N.

(ii) Sketch a **force diagram** for the block, labelling each of the forces. [1]

(iii) Sketch also a **triangle of forces** for this situation. Mark in the angles and again label each of the forces. [2]

(iv) Calculate the values of R and F. [2]

(b) Fig. 4.3 shows an object of weight 20 N. It is supported by a light string AB that passes through a small *smooth* ring attached to the object at P. A further horizontal force of 15 N acts at P. The string section PB is inclined at $\alpha°$ to the horizontal and at 90° to PA.

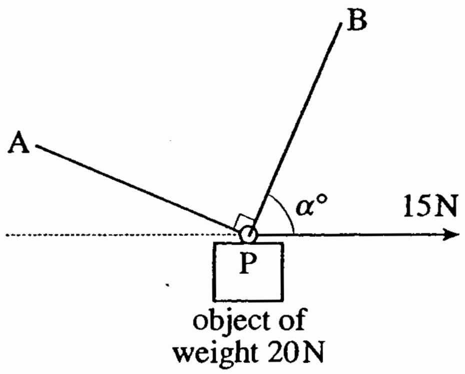


Fig. 4.3

(i) Draw a labelled diagram of all the forces acting at P. [1]

(ii) Why is the tension the same in the string sections PA and PB?

Explain briefly how you know that $\alpha \neq 45$. [3]

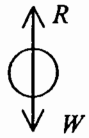
(iii) Verify that, correct to four significant figures, $\alpha = 81.87$ and the tension in the string is 17.68 N. [3]

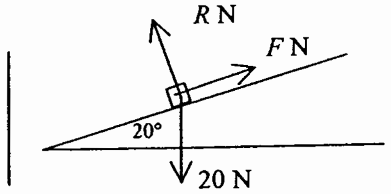
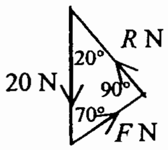
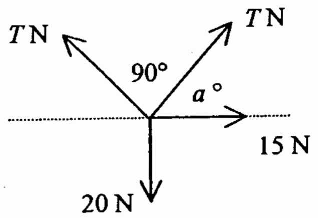
[Total 14]

Mark Scheme

Q1		Mark				
(i)	t 0 10 15 20 v 0 20 20 24	B1 B1 B1	$t = 10$ $t = 0$ and $t = 15$ FT on their $t = 10$ $t = 20$	3		
(ii) (A)	$a = 4 - 0.4t$ so $4 - 0.4 \times 7 = 1.2 \text{ m s}^{-2}$	M1 A1	Differentiating with one term correct			
(B)	0 m s^{-2}	B1				
(C)	0.8 m s^{-2}	B1		4		
(iii)	$10 \le t \le 15$ $5 \times 20 = 100$ m $15 \le t \le 20$ $20 \times 5 + 0.5 \times 0.8 \times 25$	B1 M1 A1	'*uvast*' or integrate from $t = 15$ to $t = 20$. Correct subst into *uvast* or correct integration(neglect limits). If *uvast* FT only $v(15)$, $v(20)$ from (i) and $a(16)$ from (ii)(c)			
	$= 110$ m	A1	cao			
	Total is 210 m	A1	FT dep on both B1and M1 awarded	5		
			[If single rule applied from $t = 10$ to 20: Using *uvast*. FT u, v and $a \neq 0$. Allow sign errors. SC1 If integration of $v = 8 + 0.8t$ attempted and integration correct SC1]			
(iv)	$\int_0^{10} (4t - 0.2t^2)\,dt$	M1	Integration; must see evidence. Neglect limits M0 for use of const accn			
	$= \left[2t^2 - \frac{2}{30}t^3\right]_0^{10}$	A1	At least one term correct. Neglect limits.			
	$= 200 - \frac{2000}{30}$	M1	Dependent on 1st M1. Subst correct limits in definite integral or correct subst for arb constant. Need $\int_0^{10}$ or $[\ ]_0^{10}$ or evidence of $t = 0$ substituted			
	$= 133\frac{1}{3}$ m or 133 m (3 s. f.)	B1 Tot 16	At least 3 s. f. accuracy. Award if seen [SC M1 for correct attempt at numerical integration (i.e. find area under curve) M1 for attempt at trapezia with strips $\le$ 1s. A2 only if accurate to 3. s. f.]	4		

Q 2		Mark				
(i)	$0^2 = 15^2 - 20s$	M1	Appropriate use of '*uvast*' with $u = 0$ or $v = 0$. Condone sign errors			
	$s = 11.25$ so 11.25 m	A1	Accept signs not clear but not – 11.25 m			
	It is less than 11.25 m	E1	Accept any intimation			
	(Height depends on vert cpt of velocity) All other angles give smaller vert cpt of velocity	E1	Accept no direct mention of cpt. e.g. 'some of vel/momentum/energy is horiz' Not enough to say height depends on θ	4		
(ii)	$\uparrow \quad y = 15\sin\theta t - 0.5\times 10t^2$	M1	Use of $s = ut + 0.5at^2$ for vert cpt. g must be used; accept $\pm 9.8(1), \pm 10$ [$s = vt - 0.5at^2$, M0 unless reasoning clear]			
	$y = 12t - 5t^2$	E1	Subst of $\sin\theta$ must be shown. Accept 15×0.8 seen.			
	$\rightarrow \quad x = 15\cos\theta t = 9t$	E1	Either $15\cos\theta$ or 15×0.6 seen.	3		
(iii)	$t = \frac{x}{9}$	M1	Attempt to eliminate t			
	$y = \frac{12x}{9} - 5\times\left(\frac{x}{9}\right)^2$	A1	Substitution shown clearly ($\frac{12x}{9}$ must be seen)			
	$y = \frac{4x}{3} - \frac{5x^2}{81}$	E1	Clearly shown	3		
(iv)	$y = 0$ so $x\left(\frac{4}{3} - \frac{5x}{81}\right) = 0$	M1 M1	Consider $y = 0$ Attempt to solve for x			
	$x = 0, 21.6$ so 21.6 m	A1	Accept $x = 0$ not considered	3		
(v)	$y = \frac{4\times 18}{3} - \frac{5\times 18^2}{81}$	M1	Subst $x = 18$ or $t = 2$ subst into $y = 12t - 5t^2$			
	$= 24 - 20 = 4$ so 4 m	A1		2		
		Tot 15				

Q 3		Mark				
(a)	$F = 48 \times 2 = 96$ so 96 N	M1 A1	Use of N2L. Accept $F = mga$ Must have units	2		
(b) (i)		B1	Accept W, $48g$, 470.4 N for weight but **not** for R. d.l.s.'s need not touch girl.			
	$R = W = 470.4$ so 470.4 N	B1	Accept $48g$	2		
(ii)	N2L ↑ $R - 470.4 = 48 \times 2$ $R = 566.4$	M1 A1 A1 A1	Use of N2L with R **and** W. Accept $F = mga$ but not extra forces. LHS (accept $W - R$ but **not** $R + W$) FT **their** 470.4 RHS cao	4		
(iii)	N2L ↑ $R - 470.4 = -48 \times 3$ $R = 326.4$	M1 A1 A1	Use of N2L **with** deceleration. $F = ma$ only. No extra forces. $W \pm R$ on LHS Signs correct. FT **their** 470.4 cao	3		
(iv)	Greatest T when accelerating $T - 5g = 5 \times 2$ $T = 59$ so 59 N	M1 A1	May be implied. Do not award for statement without equation or solution Accept only sign errors. [Award B2 for 59 N WW]			
	(ii) + 59 = 625.4 N	M1 F1	(ii) + **their** 59 or start again with $m = 53$ consistently used. $F = ma$. No extra forces. Accept 3 s. f. or better. [Award B2 for 59 N WW. FT]	4		
		Tot 15				

Q 4		Mark				
(a) (i)	The sum of the forces is shown to be zero because the 3 forces add to form a triangle	E1 E1	The three forces form a triangle Recognised vector sum is zero or equivalent	2		
(ii)		B1	Accept no angle given. Labels and arrows required. No extra forces. Allow *F* down the plane. [Do not FT to (iii)]	1		
(iii)		B1 B1	Angles correct. Must show 90° and either 20° in diag or as angle of *F* with horiz Arrows and labels correct	2		
(iv)	$F = 20\cos 70 = 6.840..$ so 6.84 (3 s.f.) $R = 20\sin 70 = 18.79...$ so 18.8 (3 s.f.)	M1 A1	Solving triangle or equiv method leading to one answer. May use force diag. Allow $\sin \leftrightarrow \cos$ Both correct. Accept 2 s.f. or better. cao [Award B1 if just one value correct] [Only FT $F \leftrightarrow R$ from (ii)]	2		
(b) (i)		B1	All correct with arrows and labels. Angles not required. B0 for quadrilateral of forces.	1		
(ii)	Ring is smooth Resolve horizontally. Get impossible equation	E1 M1 E1	Accept pulley instead of ring Any attempt to resolve or some argument involving horiz cpts. Complete argument.	3		
(iii)	Resolve along PB $T + 15\cos\alpha = 20\sin\alpha$ Resolve along PA $T = 15\sin\alpha + 20\cos\alpha$ Both equations are satisfied by the given values	B1 B1 E1 Tot 14	Resolve correctly in any direction Resolve correctly in a second direction Substitute given values in **both** equations or solve simultaneously and obtain both values. [20*g* used throughout. Lose in (a) (ii) or first occurrence and last mark in (b)(iii) giving 12/14]	3		

Examiner’s Report

2607 Mechanics 1 report January 2003

There were many good solutions to the first three questions but Q4 was not answered so well. Most candidates were able to make a substantial attempt at every question and there were few who could do very little. Many of the difficulties found by the candidates came from the parts of the questions that required explanations; although some explanations were clear and correct and used mathematical language appropriately, these were the exception. Many candidates revealed how little they understood and others struggled to express their ideas.

Q1 (The kinematics of a car)

Most of the candidates could work out the speeds for part (i) but quite a few omitted the value for $t = 15$, thereby losing a mark.

In part (ii), many candidates tried to calculate the acceleration by dividing a change of speed by time. This method was appropriate (but not efficient) for (B) and (C) but there were many errors in calculation; it was not appropriate but widely used for (A).

Although many candidates found the distance travelled from $t = 10$ to $t = 20$ correctly by dividing their calculation into two parts, the velocity function changing at $t = 15$ threw very many candidates who applied a constant acceleration formula to the whole interval or integrated using the velocity function for $t > 15$ for the whole interval.

Many candidates realised that they had to integrate in part (iv) and many clearly showed that they had considered the lower limit or the constant of integration. There were many others who incorrectly applied the constant acceleration formulae.

(i) 0, 20, 20, 24 m s^{-1}; (ii) 1.2, 0, 0.8 m s^{-2}; (iii) 210 m; (v) $133\frac{1}{3}$ m.

Q2 (A water droplet as a projectile)

In part (i), most candidates obtained the correct height but not all by the direct use of $v^2 = u^2 + 2as$. The explanations were less competent and few could produce a complete argument using technical terms correctly.

Most candidates knew what they had to do in part (ii) but quite a few did not include enough working to receive full marks for *showing* a given result.

The majority of candidates knew how to eliminate t in part (iii) and did so efficiently and very many obtained full marks on parts (iv) and (v). Some candidates struggled to solve the quadratic with common factor x found in part (iv), using the quadratic formula and often not coping with '$c = 0$'.

[(i) 11.25 m; (iv) 21.6 m; (v) 4 m;]

Q3 (Newton's second law applied to a lift)

Part (a) was answered correctly by almost every candidate.

Part (b)

Not all candidates produced an adequate diagram for part (i). The most common errors were to attach the normal reaction to the roof of the lift or to mark in this force with the numerical value of the weight. Some candidates could not calculate the weight.

Most candidates knew that they should use Newton's second law in parts (ii) and (iii) but some forgot the weight and others made sign errors, especially in part (iii) when the lift was decelerating. However, there were a pleasing number of accurate and efficient solutions.

The most common mistake in part (iv) was to forget the weight when considering the strength of the string. Many candidates worked the problem from first principles when calculating the normal reaction and were often correct; few solved the problem by adding the tension in the string to their answer to part (ii).

[(a) 96 N; (b) (i) 470.4 N; (ii) 566.4; (iii) 326.4; (iv) 59 N, 625.4 N;]

Q(4) (Statics problems including the use of a triangle of forces)

Part (a)

In part (i), many candidates had the right idea but failed properly to communicate that the triangle of forces showed a vector sum of zero. There were also many incorrect ideas based, for instance, on the supposition that the triangle is equilateral.

In parts (ii) and (iii), many candidates obtained a correct force diagram (although many had the friction force acting down the plane) but could not produce the correct triangle of forces; common mistakes were to omit angles, labels or arrows.

Most candidates answered part (iv) correctly, many working from their force diagram instead of their triangle of forces.

Part (b)

Part (i) was usually answered well.

In part (ii), many of the explanations were wrong or incomplete. Many candidates thought that APB being a single string ensured equal tensions in sections AP and PB and did not mention that the ring must be smooth.

The instruction to *verify* the given figures in part (iii) was intended to prompt the candidates to find two equations for the equilibrium that were satisfied by the given values. Many candidates found only one equation for equilibrium and substituted in that. Others found two equations and solved them, not always correctly and not always efficiently.

[(a) (iv) $R = 18.8$, $F = 6.84$ (3 s. f.);]