Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2607

Mechanics 1

Friday **19 JANUARY 2001** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

Answer **all** questions.

You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

Take $g = 9.8$ m s^{-2} unless otherwise instructed.

The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

HN/2

1

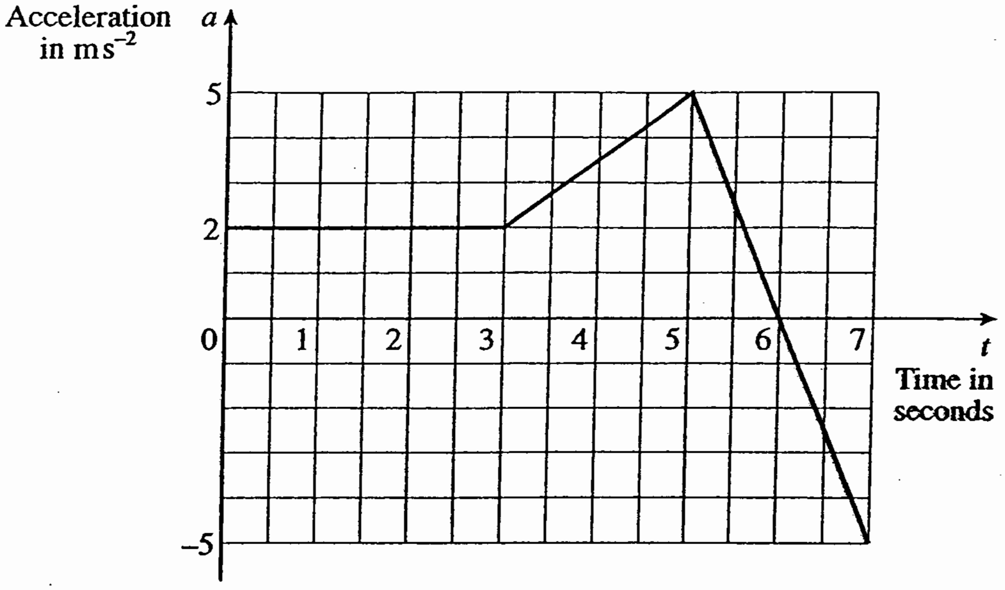


Fig. 1

A car is travelling due east along a straight road when it passes a point P. The *acceleration* of the car during the next 7 seconds is modelled in the acceleration-time graph, Fig. 1, where a m s^{-2} is the acceleration of the car due east and t seconds is the time after passing the point P.

(i) Explain why the *speed* of the car is greatest when $t = 6$. [1]

The speed of the car when it passes P is 12 m s^{-1}.

(ii) Calculate the speed of the car when $t = 3$ and the distance of the car from P at this time. [4]

(iii) Show that, when $t = 5$, the speed of the car is 25 m s^{-1}. [3]

(iv) Show that, for $5 \leqslant t \leqslant 7$, the acceleration is given by

$$a = -5t + 30.$$ [2]

(v) Hence find an expression in terms of t for the velocity of the car for $5 \leqslant t \leqslant 7$. [4]

(vi) Explain how the graph in Fig. 1 may be used to show that the speeds at $t = 5$ and $t = 7$ are equal. [2]

[Total:16]

2 A golf ball is hit over horizontal ground from a point O on the ground. The velocity of projection is 30 m s^{-1} at 40° to the horizontal. The effects of air resistance should be neglected.

(i) The ball is y m above the ground t seconds after projection. Write down an expression for y in terms of t and hence determine the time at which the ball first hits the ground. [5]

The ball passes directly over a tree which is at a horizontal distance of 34 m from O.

(ii) Determine the speed of the ball as it passes over the tree. Calculate also the angle between the direction of motion of the ball and the horizontal at that time, making it clear whether the ball is rising or falling. [11]

[Total: 16]

3 A child is pulling a toy lorry and trailer along a horizontal garden path by means of a light horizontal string. The lorry and trailer have masses 3.5 kg and 1.5 kg and are subject to resistances to motion of 6 N and 4 N respectively. The coupling between the lorry and the trailer is light, rigid and horizontal. This situation is shown in Fig. 3.

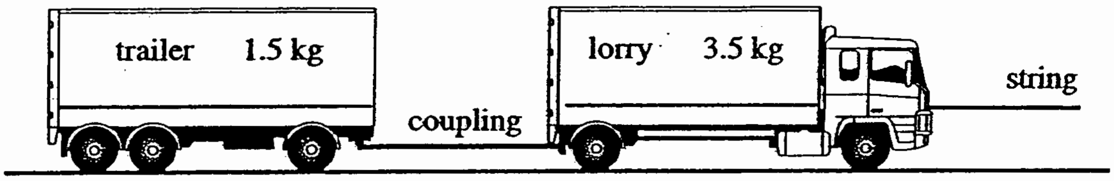


Fig. 3

The tension in the string is 20 N.

(i) Draw a diagram showing all the horizontal forces acting on the lorry and on the trailer, including the force in the coupling. Calculate the acceleration of the lorry and trailer. [4]

(ii) Calculate the force in the coupling. [2]

The child's father decides to join in the game by pushing the trailer forwards with a horizontal force. The child pulls the string with the same force as before and the resistances to motion are unchanged. The force in the coupling is now a thrust (compression) of 1.75 N.

(iii) Calculate the force with which the father is pushing. [5]

The father soon tires of this game and stops pushing. The child now pulls the lorry and trailer up a uniform slope at a steady speed. The child pulls on the string with a force of 20 N parallel to the slope. The resistances to motion are unchanged.

(iv) Calculate the angle of the slope. [3]

[Total: 14]

[Turn over

4

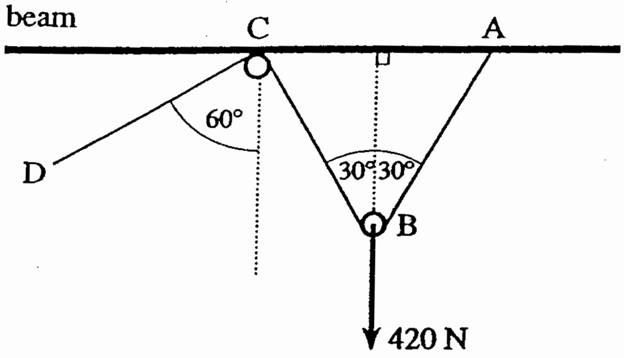


Fig. 4.1

A load of 420 N is raised using a smooth, light rope and two small, light, smooth pulleys. The rope is attached to a horizontal beam at A, passes round pulleys at B and C and is held at D. The load is attached to the pulley at B and hangs freely. The pulley at C is fixed. The system is in equilibrium with BA and BC at 30° to the upward vertical and CD at 60° to the downward vertical. This situation is shown in Fig. 4.1.

(i) Explain briefly why the tensions in the rope sections AB, BC and CD are the same. [1]

(ii) Draw a diagram showing all the forces acting on the pulley at B, and calculate the tension in the rope ABCD. [3]

(iii) Calculate the vertical component of the force on the pulley at C due to the tension in the rope. [3]

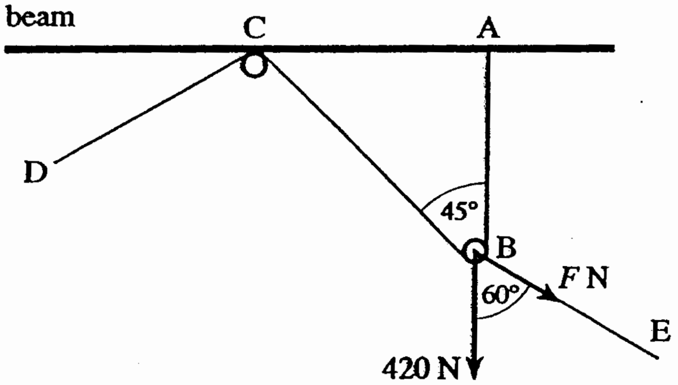


Fig. 4.2

A further light rope BE is attached to the pulley at B and held at E. BE is at 60° to the downward vertical and the system is now in equilibrium with the rope section AB vertical and with angle ABC = 45°. This situation is shown in Fig. 4.2. The tension in BE is F N.

(iv) By considering the equation for the horizontal equilibrium of the pulley at B, or otherwise, show that the tension in the rope ABCD is $\sqrt{\frac{3}{2}}F$. [3]

(v) Calculate the value of F. [4]

[Total: 14]

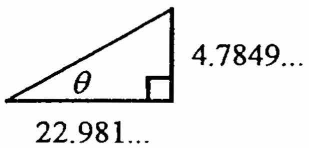
Mark Scheme

1. (i)	Car accelerates in + ve direction up to this time.	E1 $a = 0$ is not enough. Need reference to sign of a to justify max [1]
(ii)		[NB v or s may be found first] [Condone $u \leftrightarrow v$ in this part]
For v		
Either	$a = 2$	B1
	$v = 12 + 2 \times 3 = 18$ so 18 m s^{-1}	F1 (FT wrong a)
Or		
	$\triangle v = 2 \times 3$	B1
	$v = 12 + 2 \times 3 = 18$ so 18 m s^{-1}	F1 12 + **their** $\triangle v$. Must add.
For s		
Either	$s = 12 \times 3 + \frac{1}{2} \times 2 \times 9$	M1 Appropriate '*uvast*'. Allow $u = 0$
	$= 45$ so 45 m	F1 (FT wrong a only)
Or	$18^2 = 12^2 + 2 \times 2 \times s$	M1 . Allow $u = 0$
	$s = 45$ so 45 m	F1 (FT wrong a only) [4]
(iii)		
Either	$v(5) = v(3)$ + area under curve from $t = 3$ to $t = 5$.	M1 May be implied by working
	so $18 + \frac{1}{2}(2 + 5) \times 2$	B1 use of **their** 18
	$= 25$ m s^{-1}	E1 Clearly shown
Or	Establish $a = 1.5t - 2.5$	B1
	Integrate **and** consider arb constant	M1
	25 m s^{-1}	E1 [3]
(iv)	Grad –5 thro' (5, 5)	B1 Accept equivalent. Require statement about grad or calculation.
	so $a - 5 = -5(t - 5)$ and $a = -5t + 30$	E1 Clearly shown [2] [SC $a = ct + d$; form 2 correct equns B1 Solve simultaneously E1] [SC Two points shown to fit equation. Award SC2 even if no comment]

(v)	$v = \int(-5t + 30)dt$	M1 Expression for a must be correct
	$v = -\frac{5t^2}{2} + 30t + C$	A1 (Neglect arb const)
	$v = 25$ when $t = 5$	B1 $v = 25$ when $t = 5$ used not **their** '25'
	so $v = -\frac{5t^2}{2} + 30t - 62.5$	A1 [4]
Or	Use area under curve $\triangle v = \frac{1}{2}(35 - 5t)(t - 5)$	M1 consider appropriate trapezia A1
	$v(t) = 25 + \triangle v$ $v = 25 + \frac{1}{2}(35 - 5t)(t - 5)$	M1 **their** $\triangle v$ but must use 25 A1 Any form
(vi)	Areas are equal Areas are 'signed'	E1 E1 Depends on 1st E1 [2] [Award E1 for comment linking the form of the a-t graph to the speed. E2 for clear correct argument]

[Total 16]

2. (i) $y = 30\sin 40t - 4.9t^2$	M1 Appropriate '*uvast*' with $g = \pm 10$ or ± 9.8. A1 Correct
$y = 0$	M1
solving	M1 Solving a quadratic
$t = 0$ or $\frac{30\sin 40}{4.9} = 3.9354...$ so 3.94 s (3 s.f.)	A1 Allow any reasonable accuracy. Allow no mention of $t = 0$. [5]
(ii) Time of flight is given by	
$30\cos 40T = 34$	M1 Use of horizontal component
$T = 1.47946....$	A1 Need not be evaluated until later.
$\dot{y} = 30\sin 40 - 9.8t$	M1 Allow $\pm 9.8, \pm 10$ A1
$= 4.7849....$	F1 FT **their** T. Need not be evaluated here.
$\dot{x} = 30\cos 40 = 22.981...$	
speed is $\sqrt{\dot{x}^2 + \dot{y}^2}$	M1
$= \sqrt{22.981...^2 + 4.7849..^2}$	A1 **Their** $\dot{x}$ and $\dot{y}$ substituted. Award if correct answer seen.
$= 23.474...$ so 23.5 m s^{-1} (3 s.f.)	A1 cao. Any reasonable accuracy.

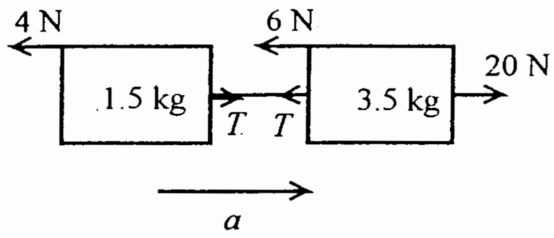


$\tan\theta = \frac{4.7849...}{22.981...}$	M1 Use of components and correct trig ratio. FT wrong $\dot{x}$ and $\dot{y}$.
$\theta = 11.761...$ so 11.8° (3 s.f.) upwards	A1 cao. Value. F1 Upwards. Accept direction implied by diagram.

[11]

[Total: 16]

3 (i)



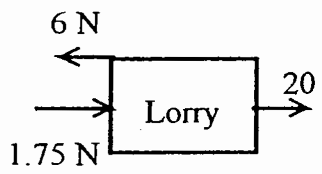
B1 All forces. With arrows and distinct labels. Two separate diagrams Must have consistent T. Ignore non-horizontal forces. No 'extra' forces.

N2L $\rightarrow 20-(6+4)=(3.5+1.5)a$	M1 N2L. Condone $F=mga$, omitted forces, wrong m and sign errors.
	B1 LHS correct with $5a$ or $5ga$ on RHS
$a=2$ so 2 m s^{-2}	A1 [4]
(ii) For trailer	
$T-4=1.5\times 2$	M1 Use of N2L and **their** a. No sign errors. May consider lorry or trailer. Condone $1.5ga$ **or** inclusion of 20 N driving force but not both.
$T=7$ so 7 N	A1 FT **their** a from (i) [2]

Bodies considered separately in (i) and (ii)

$20-6-T=3.5a$	M1 N2L. Condone $F=mga$, omitted forces, wrong m and sign errors
	B1 LHS correct with $3.5a$ or $3.5ga$ on RHS
$T-4=1.5a$	M1 Condone $1.5ga$ **or** inclusion of 20 N driving force but not both Must try to eliminate a variable. No sign errors.
$T=7, a=2$	A1 cao one answer
	F1 second answer

(iii)



B1 Diagram. 1.75 N in correct direction. (Award this mark if a correct)

For the lorry	
N2L $\rightarrow 20+1.75-6=3.5a$	M1 (± 1.75 and 3.5 present. No other sign errors. Condone omission of 20 N or 6 N but not both)
$a=4.5$	A1

Either	Overall N2L $\rightarrow 20 + P - 10 = 5 \times a$	M1 Allow $a = 2$ only if all forces present with correct signs.
	$P = 12.5$ so 12.5 N	A1
Or	Apply N2L to trailer $P - 4 - 1.75 = 1.5a$	M1 Allow $a = 2$ only if all forces present with ± 1.75 otherwise correct. A1 [5]

[SC Form two equations and solve simultaneously

Diagram	B1 award if solution correct
Form an equation	M1
Form second equation	M1 Must also attempt to eliminate one of the variables
Solve for P	A2. (Award A1 if a found but not P)

(iv)	$20 - 10 = 5 \times 9.8 \times \sin\theta$	M1 Equating forces up slope to zero Weight component attempted, signs correct. Condone sin $\leftrightarrow$ cos; condone 10 N omitted.
		B1 for $mg \sin\theta$
	angle is 11.775... so about 11.8°	A1 Allow $g = 10$ giving 11.5° [3]

[Total 14]

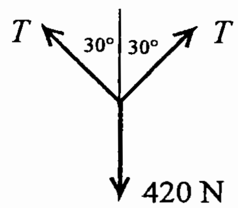
4.

[NB If $420g$ used consistently, treat as mis-read. Allow all M marks, penalise first error (A or B mark) and then FT.

Consistent sin ↔ cos gains all M marks (including (iii) and (iv)) but no A or E marks.

(i) Pulleys are smooth — B1 [1]

(ii)



B1 Arrows and correct labels. Labels must be the same (unless their equivalence is implicit later). Angles not required. All forces present. No extra forces.

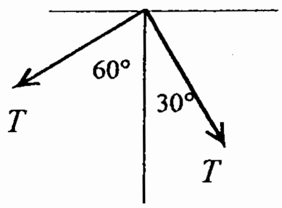
$\uparrow\ 2T\cos 30 = 420$

M1 resolving vertically and give an equation. Condone one tension omitted. Condone 'extra' forces.
Or force triangle (arrows not required) **used** with at least one correct angle shown or implied.
Or Lami's Theorem with at most one angle error. No sin ⟷ cos.

$T = 140\sqrt{3}$ so 242.487… , say 242 N

A1 Accept either form. Accept answer to 2 s.f. or better. [3]

(iii)



B1 Diagram. Award if method correct. Labels must be the same (unless their equivalence is implicit later). Angles not required.

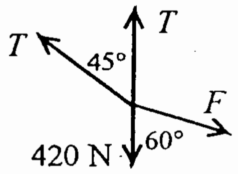
$\downarrow\ T\cos 60 + T\cos 30$

M1 resolving vertically. At least one of the two terms correct. Components must be added.

$= 70(3 + \sqrt{3}) = 331.243..$,say 331 N

A1 Accept either form. Accept answer to 2 sf or better. FT **their** T. [3]
[SC Use of Pythagoras
B1 Diagram
M1 Pythagoras + attempt at vertical component of resultant
A1]

(iv)



B1 Diagram. Award if method correct. Labels must be the same (unless their equivalence is implicit later). Angles not required.
(Award if seen in part (v))

$\rightarrow T\sin 45 = F\sin 60$ — M1 No 'extra' forces. At least one term correct

Subst $T = \sqrt{\frac{3}{2}}F$. — E1 Accept decimal equivalence. [3]

(v)

Either Resolve perp to BE
$420\cos 30 = T\cos 30 + T\cos 75$ — M1 Allow $\sin \leftrightarrow \cos$. No forces omitted. Correct signs.
A1

$T = 323.3608...$ — A1
$F = 264.02...$ so about 264 N — A1 cao. Accept 2 s.f. or better.

Or $\uparrow\ T + T\cos 45 = 420 + F\cos 60$ — M1 Allow $\sin \leftrightarrow \cos$. Condone vertical tension term omitted. Correct signs.

A1 All correct

Subst for T — M1 Dependent on 1st M1. Allow $T = \frac{\sqrt{3}}{2}F$

$F = 264.02...$ so about 264 N — A1 **cao** Accept 2 s.f. or better. [4]

[Total 14]

Examiner’s Report

Report on 2607/1; MEI Mechanics 1, January 2001

General comments

There were a number of really excellent scripts and also many that indicated that the candidates possessed a very limited knowledge of the syllabus. Some of the candidates made a good attempt at one question but seemed unfamiliar with the basic techniques required to answer the others.

It was recognised that strong mathematicians taking Mechanics 1 early in the first year of their course would have entered 2607 and consideration was given to this at the award.

Many candidates failed to give due consideration to the accuracy of their final answer. Unless otherwise specified, two or three significant figure accuracy for the answer will be accepted provided that the accuracy of the working supports this. Candidates often work to, say, two figures and then quote their answers to two or even more figures; this type of error may be penalised.

Some centres commented on the comparable difficulty of this paper to the 5507 Mechanics 1 and also on the inclusion of an unstructured part in Q 2 worth 11 marks. The specification for 2607 does not lead to an easing of the average standard expected in 5507 in the past but some of the hardest parts set in earlier papers would no longer be appropriate. In particular, the paper will still expect candidates to be able to solve simultaneous linear equations and quadratic equations with relative ease. The inclusion of one part of a question that is essentially unstructured and is worth at least 10 marks is part of the specification.

Q1 (Kinematics using an acceleration-time graph)

Part (i) was generally answered correctly.

Part (ii) was answered correctly by most of the candidates but some thought the car was at rest at $t = 0$ and quite a few thought that the distance travelled could be found using *distance* = *speed* × *time*, despite having used a non-zero acceleration to find the speed of the car at $t = 3$.

In part (iii), many candidates used an *average acceleration* method which was condoned. A large number fallaciously argued, $a = 5\text{ m s}^{-2}$, $t = 5$ so $v = 5 \times 5 = 25\text{ m s}^{-1}$. However, there were many solutions based on the correct methods of calculating areas or using integration.

In part (iii), there were many attempts to fudge the given result by applying $v = u + at$. In many other attempts the argument was not clear. Candidates would do well to remember that a full argument is expected whenever they are asked to *show* an answer or result displayed in the question.

Most candidates realised that the acceleration was non-uniform and hence integration was required in part (iv). The integration was usually accurate but many omitted the constant of integration or failed to find its value using $v = 25$ when $t = 5$.

In part (v), many candidates realised that the relevant areas were equal but failed to mention the significance of an area below the t axis. Others either gave spurious reasons or failed to communicate their ideas properly.

[(ii) 18 m s^{-1}, 45 m; (v) $v = -\frac{5}{2}t^2 + 30t - 62.5$;]

Q2 (Motion of a projectile)

This question was answered well by many of the candidates. The unstructured part (ii) caused no problems at all for a large number of candidates and they produced short efficient solutions. However, some of the candidates did not recognise this standard problem and produced working that was irrelevant to the solution.

Most candidates answered part (i) correctly. A common, but indirect, method was to double the time taken to reach the maximum height.

There were many complete answers to this part. A common mistake was to fail to answer part of the question, omitting either the angle or the speed or not to state whether the ball was rising or falling. Some candidates clearly thought that the speed of the ball was the vertical component of its velocity. Other candidates took the direction of motion of the ball to be its angle of elevation as viewed from the origin. Many candidates realised that a positive vertical component of velocity meant that the ball was rising, others did unnecessary calculations to establish this; false arguments such as *since it is not at its maximum height it is rising/falling* were seen.

[(i) 3.94 s (3 s. f.); 23.5 m s^{-1} (3 s. f.), 11.8° (3 s. f.), rising;]

Q3 (Newton's second law applied to a lorry pulling a trailer)

In part (i), many candidates failed to produce a correct diagram. Quite common errors were to fail to label the force in the coupling and not to give arrows correctly showing the direction of the forces. The common acceleration was calculated correctly by most of the candidates.

Part (ii) was generally done well but some candidates included the weight of one or both of the bodies or gave incorrect signs. Some did not appreciate that Newton's second law was needed and merely considered the difference in the resistances.

Part (iii) was very poorly done by the majority of candidates as they failed to realise that the new system moved with an acceleration different from that calculated in part (i). Some did not seem to know how to take account of the coupling being in compression. Again, a number of candidates did not apply Newton's second law and just combined some or all of the forces given in the question. Some of those who realised that a new acceleration was needed made sign errors, usually with the sense of the tension or the compression, or omitted forces.

Part (iv) was often done well. Many candidates realised that resolution along the slope was required, that the weight has a component in this direction and that the resultant force is zero. Some candidates made errors with the resolution of the weight and others omitted the total resistance. Some candidates who seemed weaker overall drew incorrect right-angled force triangles which gave values of sine or cosine greater than 1.
[(i) 2 m s^{-2}; (ii) 7 N; (iii) 12.5 N; (iv) 11.8° (3 s. f.)]

Q4 (A load held in equilibrium by a rope passing over smooth pulleys)

This was the least well done question on the paper.

In part (i), many candidates correctly mentioned that the pulleys were smooth but others seemed to think that the tension was the same throughout because the rope was continuous.

The diagrams in part (ii) were generally good although some candidates failed to show or imply that the tensions in the sections of the string were equal. Some candidates showed the string in compression.

In part (ii), many candidates seemed to think that a tension could be found by simply resolving the weight alone in the direction of the tension. Some other candidates drew right-angled force triangles, omitting one of the tensions, and then solved the triangle. It seemed as if they were adding a weight to a technique they had learned to assist them in the calculation of the components of a tension. These candidates rarely did well on the rest of the question. However, many candidates encountered few difficulties with this routine calculation.

In part (iii), a large number of candidates only found the vertical component of *one* of the forces, which trivialised the solution to such an extent that no credit was given (unless a correct diagram showing two forces acting at C was shown). Some candidates worked the problem with one or both of the sections of the string in compression.

In part (iv), many candidates were able to find $T\sin 45 = F\sin 60$, or an equivalent expression but then simply wrote down the given result without attempting to establish it. Demonstrating decimal equivalence is, of course, acceptable in this paper and many candidates used this method.

Many of the stronger candidates clearly knew how to deal with part (v) although it was not uncommon to see the tension in the vertical section of the string omitted from the equation for the vertical equilibrium.
[(ii) 242 N (3 s.f.); (iii) 331 N (3 s. f.); (v) 264 N (3 s. f.)]