Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

Statistics S4

Advanced/Advanced Subsidiary

Friday 24 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6686/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. A new diet has been designed. Its designers claim that following the diet for a month will result in a mean weight loss of more than 2 kg. In a trial, a random sample of 10 people followed the new diet for a month. Their weights, in kg, before starting the diet and their weights after following the diet for a month were recorded. The results are given in the table below.

Person	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Weight before diet (kg)	96	110	116	98	121	91	98	106	110	116
Weight after diet (kg)	91	101	111	96	121	91	90	101	104	110

(*a*) Using a suitable *t*-test, at the 5% level of significance, state whether or not the trial supports the designers' claim. State your hypotheses and show your working clearly.

(8)

(*b*) State an assumption necessary for the test in part (a).

(1)

(Total 9 marks)

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2. The weights of piglets at birth, M kg, are normally distributed $N(\mu, \sigma^2)$

A random sample of 9 piglets is taken and their weights at birth, m kg, are recorded. The results are summarised as

$$\sum m = 11.6 \quad \sum m^2 = 15.2$$

Stating your hypotheses clearly, test at the 5% level of significance

(*a*) whether or not the mean weight of piglets at birth is greater than 1.2 kg,

(7)

(*b*) whether or not the standard deviation of the weights of piglets at birth is different from 0.3 kg.

(6)

(Total 13 marks)

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3. A jar contains a large number of sweets which have either soft centres or hard centres.

The jar is thought to contain equal proportions of sweets with soft centres and sweets with hard centres. A random sample of 20 sweets is taken from the jar and the number of sweets with hard centres is recorded.

(*a*) Using a 5% level of significance, find the critical region for a two-tailed test of the hypothesis that there are equal proportions of sweets with soft centres and sweets with hard centres in the jar.

(2)

(*b*) Calculate the probability of a Type I error for this test.

(2)

Given that there are 3 times as many sweets with soft centres as there are sweets with hard centres,

(*c*) calculate the probability of a Type II error for this test.

(2)

(Total 6 marks)

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4. A manufacturer produces boxes of screws containing short screws and long screws. The manufacturer claims that the probability, p, of a randomly selected screw being long, is 0.5.

A shopkeeper does not believe the manufacturer's claim. He designs two tests, A and B, to test the hypotheses $H_0 : p = 0.5$ and $H_1 : p < 0.5$.

In test A, a random sample of 10 screws is taken from a box of screws and H_0 is rejected if there are fewer than 3 long screws.

In test B, a random sample of 5 screws is taken from a box of screws and H_0 is rejected if there are no long screws, otherwise a second random sample of 5 screws is taken from a box of screws. If there are no long screws in this second sample H_0 is rejected, otherwise it is accepted.

(*a*) Find the size of test A.

(1)

(*b*) Find the size of test B.

(3)

(*c*) Find an expression for the power function of test B in terms of p.

(2)

Some values, to 2 decimal places, of the power function for test A and the power function for test B are given in the table below.

p	0.1	0.2	0.3	0.4
Power test A	0.93	r	0.38	0.17
Power test B	0.83	0.55	0.31	0.15

(*d*) Find the value of r.

(1)

The shopkeeper believes that the value of p is less than 0.4

(*e*) Suggest which of the tests the shopkeeper should use. Give a reason for your answer.

(2)

(Total 9 marks)

5. Fire brigades in cities X and Y are in similar locations. The response times, in minutes, during a particular month, for randomly selected calls are summarised in the table below.

	Sample size	Sample mean	Standard deviation s
X	9	14.8	6.76
Y	6	7.2	5.42

You may assume that the response times are from independent normal distributions.

Stating your hypotheses and showing your working clearly

(*a*) test, at the 10% level of significance, whether or not the variances of the populations from which the response times are drawn are the same,

(5)

(*b*) test, at the 5% level of significance, whether or not the mean response time for the fire brigade in city X is more than 5 minutes longer than the mean response time for the fire brigade in city Y.

(8)

(*c*) Explain why your result in part (*a*) enables you to carry out the test in part (*b*).

(1)

(Total 14 marks)

6. A random sample of size n is taken from the random variable X, which has a continuous uniform distribution over the interval $[0, a]$, $a > 0$.

The sample mean is denoted by $\overline{X}$.

(a) Show that $Y = 2\overline{X}$ is an unbiased estimator of a.

(2)

The maximum value, M, in the sample has probability density function

$$\mathrm{f}(m) = \begin{cases} \dfrac{nm^{n-1}}{a^n} & 0 \leq m \leq a \\ 0 & \text{otherwise} \end{cases}$$

(b) Find E(M).

(2)

(c) Show that $\operatorname{Var}(M) = \dfrac{na^2}{(n+2)(n+1)^2}$.

(4)

The estimator S is defined by $S = \dfrac{n+1}{n}M$.

Given that $n > 1$,

(d) state which of Y or S is the better estimator for a. Give a reason for your answer.

(7)

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

June 2016
6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
1(a)	d: 5 9 5 2 0 0 8 5 6 6	M1
	$\bar{d} = \frac{\sum d}{n} = 4.6$	M1
	$s^2 = \frac{296 - 10\times 4.6^2}{9} = 9.378$	M1
	$H_0: \mu_d = 2 \quad H_1: \mu_d > 2$	B1
	$t = \pm\frac{4.6-2}{\sqrt{\frac{9.378}{10}}} = \pm 2.6848$	M1 A1
	$t_9(5\%) = \pm\ 1.833\ldots$	B1
	There is evidence to reject H_0. There is sufficient evidence to support the designers claim.	A1ft
		(8)
(b)	The **differences** in weights are **normally** distributed.	B1
		(1)
	Notes	**Total 9**
(a)	M1 for attempting the ds	
	M1 for attempting $\bar{d}$	
	M1 for s_d or $s_d{}^2$	
	B1 for both hypotheses correct in terms of μ or μ_d .(allow a defined symbol)	
	M1 for attempting the correct test statistic $\frac{\bar{d}}{s_d/\sqrt{10}}$	
	A1 awrt 2.68	
	B1 awrt 1.83	
	A1ft for a correct comment in context	
(b)	B1 for a comment that mentions "differences" and "normal" distribution	

Question Number	Scheme	Marks
2. (a)	$H_0 : \mu = 1.2 \quad H_1 : \mu > 1.2$	B1
	$t_8(5\%) = 1.860$	B1
	$\bar{m} = 1.28888..$	B1
	$t = \dfrac{1.28... - 1.2}{\sqrt{\dfrac{0.031111}{9}}} = 1.511$ awrt 1.51	M1 A1ft A1
	Not significant. There is not sufficient evidence that the mean weight of piglets is greater than 1.2 kg	A1
		(7)
(b)	$H_0 : \sigma^2 = 0.09 \quad H_1 : \sigma^2 \neq 0.09 \quad [H_0 : \sigma = 0.3 \; H_1 : \sigma \neq 0.3]$	B1
	$s^2 = \dfrac{15.2 - 9\times\left(\dfrac{11.6}{9}\right)^2}{8} = 0.031111$	B1
	$[\chi_8^2(0.25) = 17.535] \quad \chi_8^2(0.975) = 2.18$	B1
	Critical region $\dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2_8$ test statistic = 2.7654… awrt 2.77	M1A1
	2.77 is not in the critical region. There is no evidence that the standard deviation of the weights of **piglets** is different to 0.3	A1
		(6)
	Notes	**Total 13**
(a)	B1 both hypotheses M1 for attempting the correct statistic A1ft follow through their s^2 A1 awrt 1.51	
(b)	B1 both hypotheses, must be two tail B1 awrt 0.0311 B1 NB allow 2.733 for one tail hypotheses. (no hypotheses gains B0) M1 for a correct test statistic NB one tail test can get B0 B1 B1 (2.733)B0 M1 A1 A1	

Question Number	Scheme	Marks
3. (a)	X = No of soft centres. $X \sim B(20, 0.5)$ Critical region $X \le 5$ or $X \ge$ 15	B1B1 (2)
(b)	P(Type I error) = $P(X \le 5 \mid p = 0.5) + P(X \ge 15 \mid p = 0.5)$ $= 0.0207 + 0.0207 \quad = 0.0414$	M1 A1 (2)
(c)	P(Type II error) = $P(X < 15 \mid p = 0.25) - P(X < 6 \mid p = 0.25)$ $= 1 - 0.6172 \quad = 0.3828$	M1 A1 (2)
	Notes	**Total 6**
(a)	B1 $X \le 5$ B1 $X \ge 15$	
(b)	M1 Adding their two CR together or a correct answer A1 awrt 0.0414	
(c)	M1 FT their CR A1 awrt 0.383	

Question Number	Scheme	Marks
4. (a)	Size of test $A = \mathrm{P}(Y \le 2)$	
	$= 0.0547$	B1
		(1)
(b)	Size of test $B = \mathrm{P}(\text{Rejecting } \mathrm{H}_0 \mid p = 0.5)$	
	$= \mathrm{P}(X = 0) + (1 - \mathrm{P}(X = 0)) \times \mathrm{P}(X = 0)$	M1
	$= 0.5^5 + (1 - 0.5^5)(0.5^5)$	A1
	$= 0.03125 + (0.96875)(0.03125)$	
	$= 0.0615/0.0614$	A1
(c)	Power function of test B = P(0 long screws in first 5) + P(0 long screws in second 5| > 0 long screws in first 5)	(3)
	$= \mathrm{P}(X = 0 \mid p) + [1 - \mathrm{P}(X = 0 \mid p)]\, \mathrm{P}(X = 0 \mid p)$	M1
	$= (1-p)^5 + [1 - (1-p)^5](1-p)^5$	A1
	$= 2(1-p)^5 - (1-p)^{10}$	
		(2)
(d)	$r = 0.68$	B1
		(1)
(e)	Test A as it is more powerful for values of $p < 0.4$	M1 A1
		(2)
	Notes	**Total 9**
(b)	M1 for a correct expression/selection of probabilities	
	A1 for a correct expression in terms of probabilities. Allow 0.0312 + (0.9688)(0.0312)	
(c)	M1 for a correct expression	
	A1 for a correct expression in terms of p	
(e)	M1 for reason based on the power function	
	A1 test A	

Question Number	Scheme	Marks
5. (a)	H$_0$: $\sigma^2_X = \sigma^2_Y$ H$_1$: $\sigma^2_X \neq \sigma^2_Y$	B1
	$F_{8,5} = \frac{6.76^2}{5.42^2} = 1.556$	M1A1
	$F_{8,5}$ is 4.82	B1
	There is evidence that the variances are the same.	A1
		(5)
(b)	$\text{H}_0 : \mu_X = \mu_Y + 5 \quad \text{H}_1 : \mu_X > \mu_Y + 5$	B1
	${s_p}^2 = \frac{8 \times 6.76^2 + 5 \times 5.42^2}{13}, = 39.42... \quad \text{or} \quad s_p = 6.278...$	M1 A1
	$(t_{13} =)(\pm)\frac{14.8 - 7.2 - 5}{s_p\sqrt{\frac{1}{9}+\frac{1}{6}}} = (\pm)0.78578...$ awrt 0.786	M1 M1dA1
	Critical value t_{13} (2.5%) = 1.771	B1
	There is no evidence to Reject H$_0$	
	There is evidence that the **fire brigade** in *X* does not take more than 5 minutes longer than those in *Y*.	A1cso
		(8)
(c)	Test in part (b) requires the variances to be equal. The test in part (a) showed that the variances could be assumed to be equal.	B1
		(1)
	notes	**Total 14**
(a)	B1 both hypotheses M1 Allow use of 6.76 and 5.42 instead of 6.76^2 and 5.42^2 A1 awrt 1.56	
(b)	B1 both hypotheses M1 allow use of 6.76 and 5.42 instead of 6.76^2 and 5.42^2 A1 awrt 39.4 or 6.28 B1 allow p value 0.650 instead of critical value M1 use of correct formula with their S_p – condone missing 5 M1 use of correct formula with their S_p	

Question Number	Scheme	Marks
6.(a)	$\mathrm{E}(Y) = 2\mathrm{E}\left(\bar{X}\right)$	
	$= 2\times\dfrac{a}{2}$	M1
	$= a$	A1cso
		(2)
(b)	$\mathrm{E}(M) = \int_0^a \dfrac{nm^n}{a^n}\,\mathrm{d}m$	M1
	$= \left[\dfrac{nm^{n+1}}{a^n(n+1)}\right]_0^a$	
	$= \dfrac{na}{n+1}$	A1
		(2)
(c)	$\mathrm{Var}\,(M) = \int_0^a \dfrac{nm^{n+1}}{a^n}\,\mathrm{d}m - \left(\dfrac{na}{n+1}\right)^2$	M1A1
	$= \left[\dfrac{nm^{n+2}}{a^n(n+2)}\right]_0^a - \dfrac{n^2a^2}{(n+1)^2}$	M1d
	$= na^2\left(\dfrac{(n+1)^2 - n(n+2)}{(n+1)^2(n+2)}\right)$	
	$= \dfrac{na^2}{(n+2)(n+1)^2}$	A1cso
		(4)
(d)	$\mathrm{E}(S) = \dfrac{n+1}{n}\mathrm{E}(M) = \dfrac{n+1}{n}\times\dfrac{na}{n+1} = a$	B1
	$\mathrm{Var}\,(S) = \left(\dfrac{n+1}{n}\right)^2 \dfrac{na^2}{(n+2)(n+1)^2} = \dfrac{a^2}{n(n+2)}$	B1
	$\mathrm{Var}\,(Y) = 4\,\mathrm{Var}\left(\bar{X}\right)$	M1
	$= 4\times\dfrac{a^2}{12n}$	
	$= \dfrac{a^2}{3n}$	A1
	As $n > 1$ $n(n+2) > 3n$; therefore $\mathrm{Var}(S) < \mathrm{Var}(Y)$	M1;M1
	$\therefore$ S is the better estimator	A1cso
		(7)
		Total 15

	notes	
(a)	M1 for $2\mathrm{E}\left(\bar{X}\right)$ A1 For $2\times\frac{a}{2}$ leading to a	
(b)	M1 attempting to integrate correct expression	
(c)	M1 for attempting to integrate a correct expression for $\mathrm{E}(X^2)$ A1 correct $\mathrm{E}(X^2)$ M1d dependent on previous M mark, using correct formula for $\mathrm{Var}(M)$	
(d)	B1 for $\frac{n+1}{n}\mathrm{E}(M)=a$ or $\frac{n+1}{n}\times\frac{na}{n+1}=a$ M1 using 4 Var $\left(\bar{X}\right)$ NB Failure to show S is unbiased gains a maximum of 5/7 lose first B1 and final A1	

Question Number	Scheme	Marks
7	$\bar{x} - 2.262\frac{s}{\sqrt{10}} = 28.5$	B1 M1
	$\bar{x} + 2.262\frac{s}{\sqrt{10}} = 48.7$	A1
	$2\bar{x} = 48.7 + 28.5$ or $2.262\frac{s}{\sqrt{10}} = \frac{1}{2}(48.7 - 28.5)$	M1
	$s = 14.1198...$ ($s^2 = 199.36$)	A1
	$\left\{\frac{9(14.1198^2)}{23.589}, \frac{9(14.1198^2)}{1.735}\right\}$	M1 B1 B1
	= (76.0659…, 1034.19…)	A1 (9)
	notes	**Total 9**

B1 awrt 2.262

M1 $\bar{x} - t\ value\frac{s}{\sqrt{10}} = 28.5$

A1 both equations correct

M1 solving simultaneous leading to a value for $\bar{x}$ or s

A1 awrt 14.1 or awrt 199

M1 $\frac{9(s^2)}{\chi^2 value}$

B1 23.589

B1 1.735

A1 awrt 76.1 and awrt 1030