Paper Reference(s)

6679/01

Edexcel GCE

Mechanics M3

Advanced Level

Wednesday 13 May 2015 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle P of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 1.2 m and modulus of elasticity λ newtons. The other end of the spring is attached to a fixed point A on a ceiling. The particle is hanging freely in equilibrium at a distance 1.5 m vertically below A.

(*a*) Find the value of λ.

(3)

The particle is now raised to the point B, where B is vertically below A and $AB = 0.8$ m. The spring remains straight. The particle is released from rest and first comes to instantaneous rest at the point C.

(*b*) Find the distance AC.

(4)

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2. The finite region bounded by the x-axis, the curve with equation $y = 2e^x$, the y-axis and the line $x = 1$ is rotated through one complete revolution about the x-axis to form a uniform solid.

Use algebraic integration to

(*a*) show that the volume of the solid is $2\pi(e^2 - 1)$,

(4)

(*b*) find, in terms of e, the x-coordinate of the centre of mass of the solid.

(6)

3.

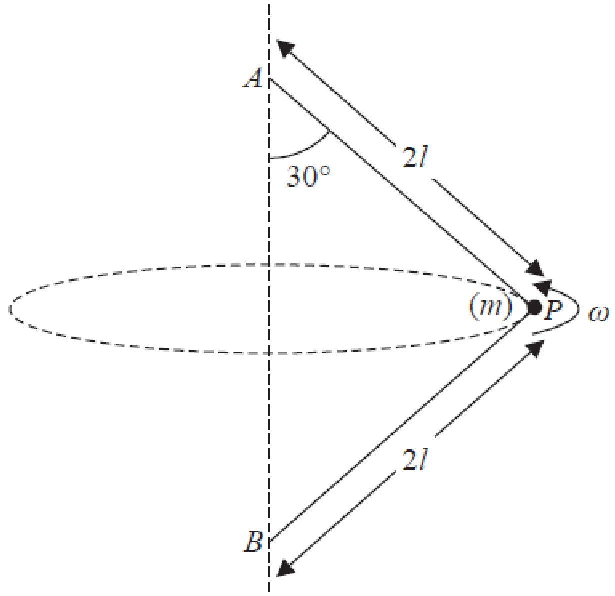

Figure 1

A small ball P of mass m is attached to the midpoint of a light inextensible string of length $4l$. The ends of the string are attached to fixed points A and B, where A is vertically above B. Both strings are taut and AP makes an angle of 30° with AB, as shown in Figure 1. The ball is moving in a horizontal circle with constant angular speed ω.

(*a*) Find, in terms of m, g, l and ω,

(i) the tension in AP,

(ii) the tension in BP.

(8)

(*b*) Show that $\omega^2 \geq \dfrac{g\sqrt{3}}{3l}$.

(2)

4. A vehicle of mass 900 kg moves along a straight horizontal road. At time t seconds the resultant force acting on the vehicle has magnitude $\frac{63000}{kt^2}$ N, where k is a positive constant. The force acts in the direction of motion of the vehicle. At time t seconds, $t \geq 1$, the speed of the vehicle is v m s^{-1} and the vehicle is a distance x metres from a fixed point O on the road. When $t = 1$ the vehicle is at rest at O and when $t = 4$ the speed of the vehicle is 10.5 m s^{-1}.

(*a*) Show that $v = 14 - \frac{14}{t}$.

(7)

(*b*) Hence deduce that the speed of the vehicle never reaches 14 m s^{-1}.

(1)

(*c*) Use the trapezium rule, with 4 equal intervals, to estimate the value of x when $v = 7$.

(4)

5.

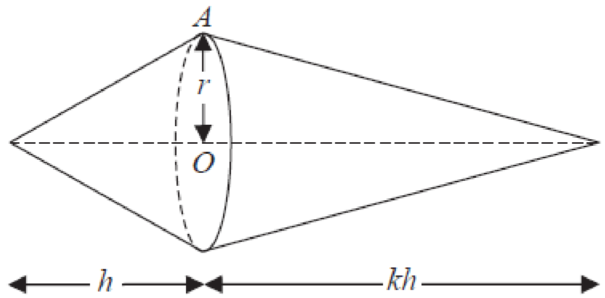

Figure 2

Figure 2 shows a uniform solid spindle which is made by joining together the circular faces of two right circular cones. The common circular face has radius r and centre O. The smaller cone has height h and the larger cone has height kh. The point A lies on the circumference of the common circular face. The spindle is suspended from A and hangs freely in equilibrium with AO at an angle of 30° to the vertical.

Show that $k = \frac{4r}{h\sqrt{3}} + 1$.

(6)

6.

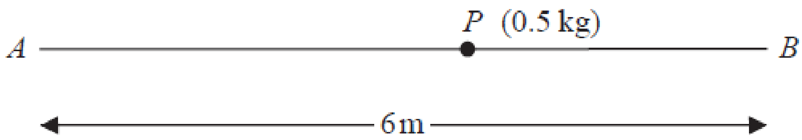

Figure 3

Two points A and B are 6 m apart on a smooth horizontal floor. A particle P of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 2.5 m and modulus of elasticity 20 N. The other end of the spring is attached to A. A second light elastic spring, of natural length 1.5 m and modulus of elasticity 18 N, has one end attached to P and the other end attached to B, as shown in Figure 3. Initially P rests in equilibrium at the point O, where AOB is a straight line.

(*a*) Find the length of AO.

(4)

The particle P now receives an impulse of magnitude 6 N s acting in the direction OB and P starts to move towards B.

(*b*) Show that P moves with simple harmonic motion about O.

(4)

(*c*) Find the amplitude of the motion.

(4)

(*d*) Find the time taken by P to travel 1.2 m from O.

(3)

7. A solid smooth sphere, with centre O and radius r, is fixed to a point A on a horizontal floor. A particle P is placed on the surface of the sphere at the point B, where B is vertically above A. The particle is projected horizontally from B with speed $\frac{\sqrt{(gr)}}{2}$ and starts to move on the surface of the sphere. When OP makes an angle θ with the upward vertical and P remains in contact with the sphere, the speed of P is v.

(*a*) Show that $v^2 = \frac{gr}{4}(9 - 8\cos\theta)$.

(4)

The particle leaves the surface of the sphere when $\theta = \alpha$.

(*b*) Find the value of $\cos\alpha$.

(4)

After leaving the surface of the sphere, P moves freely under gravity and hits the floor at the point C.

Given that $r = 0.5$ m,

(*c*) find, to 2 significant figures, the distance AC.

(7)

TOTAL FOR PAPER: 75 MARKS

END

June 2015
6679 M3
Mark Scheme

Question Number	Scheme	Marks
1		
(a)	$0.5g = T = \dfrac{\lambda \times 0.3}{1.2}$	M1A1
	$\lambda = 2g = 19.6$	A1 (3)
(b)	$\dfrac{1}{2}\times\dfrac{19.6\times x^2}{1.2} - \dfrac{1}{2}\times\dfrac{19.6\times 0.4^2}{1.2} = 0.5\times g\times(x+0.4)$	M1A1ftA1
	$5x^2 - 3x - 2 = 0$	
	$(5x+2)(x-1)=0$ or use of diff of 2 squares to obtain and then solve a linear equation	
	$x = 1$ $(x = -0.4$ need not be seen$)$	
	$AC = 2.2$ m	A1 (4) [7]

(a) M1 Use Hooke's law to obtain the tension and equate to the weight
A1 Correct equation
A1 Solve to get $\lambda = 19.6$ Accept 20 or $2g$

(b) M1 Attempt an energy equation with the difference of 2 EPE terms and a loss of GPE
EPE formula must be of the form $k\dfrac{\lambda x^2}{l}$
A1ft EPE terms correct follow through their λ
A1 GPE term correct, including all signs in the equation correct If x used for EPE and GPE A0 here
A1 Correct length AC If $\lambda = 20$ is used, this is p.a. and so scores A0

ALT: Find BC first: $\dfrac{1}{2}\times\dfrac{19.6\times(h-0.4)^2}{1.2} - \dfrac{1}{2}\times\dfrac{19.6\times 0.4^2}{1.2} = 0.5gh$ M1A1A1
$BC = 1.4$ $AC = 2.2$ A1

Methods depending on SHM must prove SHM first, but if correct answer only is given award B1 (M1 on e-PEN)
By integration: Integrating and substituting yields an equation equivalent to the one shown - mark from here M1A1A1ft -1 each error ft on λ

Question Number	Scheme	Marks
2 (a)	$\text{Vol} = \pi\int_0^1 4e^{2x}dx$	M1
	$= \pi\left[2e^{2x}\right]_0^1$	DM1A1
	$= 2\pi\left(e^2 - 1\right)$ *	A1cso (4)
(b)	$\pi\int_0^1 4xe^{2x}\,dx$	M1
	$= 4\pi\left\{\left[x\times\frac{1}{2}e^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}e^{2x}dx\right\}$	DM1
	$= 4\pi\left[\frac{1}{2}e^2 - 0\right] - 4\pi\left[\frac{1}{4}e^{2x}\right]_0^1$	A1
	$= \pi\left(e^2 + 1\right)$	A1
	x coord $= \frac{\pi\left(e^2+1\right)}{2\pi\left(e^2-1\right)}, \quad = \frac{e^2+1}{2\left(e^2-1\right)}$ oe	M1A1 (6)
		[10]

(a) M1 Using $\pi\int y^2dx$ with the equation of the curve, no limits needed

DM1 Integrating their expression for the volume

A1 Correct integration inc limits now

A1 Substituting the limits to obtain the GIVEN answer

(b) M1 Using $(\pi)\int xy^2dx$ with the equation of the curve, no limits needed, π can be omitted

DM1 Attempting to use integration by parts; allow $\pm$ between the two parts. No limits needed

A1 Correct integration, including limits; no substitution needed for this mark

A1 Correct after limits substituted

M1 Use of $\frac{\pi\int xy^2dx}{\pi\int y^2dx}$ with their $\pi\int xy^2dx$. π must be seen in both numerator and denominator or in neither. This mark is not dependent on the previous M marks

A1cao Correct answer.

Question Number	Scheme	Marks
3(a)	R$(\uparrow)$ $T_A\cos 30 = mg + T_B\cos 30$	M1A1
	NL2 $T_A\cos 60 + T_B\cos 60 = mr\omega^2$	M1A1
	$= m\times 2l\cos 60\omega^2$ or $ml\omega^2$	A1
	$T_A + T_B = 2ml\omega^2$	
	$T_A - T_B = \frac{2mg}{\sqrt{3}}$	
(i)	$T_A = \frac{m}{3}\left(3l\omega^2 + g\sqrt{3}\right)$ oe	DM1A1
(ii)	$T_B = \frac{m}{3}\left(3l\omega^2 - g\sqrt{3}\right)$ oe	A1 (8)
(b)	$T_B \geqslant 0 \Rightarrow 3l\omega^2 \geqslant g\sqrt{3}$	M1
	$\omega^2 \geqslant \frac{g\sqrt{3}}{3l}$ *	A1cso (2)
		[10]

(a) M1	Resolving vertically
A1	Correct equation
M1	NL2 along radius, acceleration in either form
A1	LHS correct
A1	Correct radius substituted and accel in $r\omega^2$. Can be awarded later by implication if work implies correct radius used.
DM1	Solving the two equations to obtain an expression for either tension. Depenent on both previous M marks
A1	Tension in *AP* correct – simplified to two terms
A1	Tension in *BP* correct – simplified to two terms
(b) M1	Using their tension in *BP* $\geqslant 0$ **must be** $\geqslant$ for this mark
A1cso	Obtaining the GIVEN answer. Only error allowed is the expression for the tension in *AP*

Question Number	Scheme	Marks
4(a)	$\dfrac{63000}{kt^2} = 900\dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1
	$-\dfrac{70}{kt} \ (+c) = v$	DM1A1ft
	$t = 1 \ \ v = 0 \ \Rightarrow c = \dfrac{70}{k}$	M1(either)
	$t = 4 \ \ v = 10.5 \ \Rightarrow -\dfrac{70}{4k} + c = 10.5$	A1(both)
	$-\dfrac{70}{4k} + \dfrac{70}{k} = 10.5$	
	$k = 5, \quad c = 14$	A1
	$v = 14 - \dfrac{14}{t}$ *	A1 cso (7)
(b)	$\dfrac{14}{t} > 0 \ \ \Rightarrow v < 14$ or v never reaches 14	B1 (1)
(c)	$7 = 14 - \dfrac{14}{t}$	
	$\dfrac{14}{t} = 7 \quad t = 2$	B1
	t 1 1.25 1.5 1.75 (2) v 0 2.8 4.666.. 6 7	
	$x = \dfrac{0.25}{2}\left(0 + 2\times2.8 + 2\times4.666... + 2\times6 + 7\right)$	M1A1
	X = 4.24175 Accept 4.2 or 4.24	A1 (4) [12]

(a) M1 Forming an equation of motion with acceleration as $\dfrac{\mathrm{d}v}{\mathrm{d}t}$ 900 or m

DM1 Attempting the integration

A1 Correct equation. Constant of integration not needed

M1 Substituting either pair of given values

A1 Obtaining correct equations using each pair of values

A1 Obtaining correct values for c **and** k or use $k = 5, \quad c = \dfrac{70}{k}$

A1 Substituting these values to obtain the GIVEN answer
Misread eg 6300 for 63000: M1DM1A1M1A0A0A0

(b) B1 Must be clear that $v < 14$ not just never = 14 $\dfrac{14}{t} > 0$ essential

(c) B1 Showing that $t = 2$ when $v = 7$ Award if seen as upper limit for t in trapezium rule or values 1.25, 1.5, 1.75 seen for t

M1 Using the trapezium rule. Must have 4 intervals and values of t shown in the table.

A1 Correct numbers in the trapezium rule statement.
Values of v can be in the form $14 - \dfrac{14}{1.25}$ etc

A1 Correct final answer. It is an estimate, so 2 or 3 sf only.

Question Number	Scheme	Marks
5	Dist of c of m from $O = r\tan 30 = \frac{r}{\sqrt{3}}$	M1A1
	Ratio of masses $\quad M \quad kM \quad (1+k)M$ $\quad\quad\quad\quad 1 \quad k \quad 1+k$	
	Dist from $O \quad -\frac{1}{4}h \quad \frac{kh}{4} \quad \frac{r}{\sqrt{3}}$	
	M(O) $\quad -\frac{1}{4}h+\frac{k^2h}{4}=\left(1+k\right)\frac{r}{\sqrt{3}}$	M1A1A1ft
	$\frac{h}{4}\left(k^2-1\right)=\left(k+1\right)\frac{r}{\sqrt{3}}$	
	$k=\frac{4r}{h\sqrt{3}}+1 \quad *$	A1 [6]
Alt 1	By moments about A	
	$kMg\left(\frac{1}{4}kh\cos 30-r\sin 30\right),\quad Mg\left(\frac{1}{4}h\cos 30+r\sin 30\right)$	M1A1,M1A1
	$h\cos 30\left(k^2-1\right)=4r\sin 30\left(k+1\right)$	A1ft
	$\left(k-1\right)=\frac{4r}{h}\tan 30$	
	$k=\frac{4r}{h\sqrt{3}}+1 \quad *$	A1

1

Question Number	Scheme	Marks
Alt 2	Find $\bar{x}$ first	
	M(0) $-\frac{1}{4}h+\frac{k^2h}{4}=(1+k)\bar{x}$	M1 A1
	$\bar{x}=\frac{h(k-1)}{4}$ oe	A1
	Then suspend: $\frac{\bar{x}}{r}=\tan 30$	M1
	$\frac{h(k-1)}{4r}=\frac{1}{\sqrt{3}}$ $(or \tan 30)$	A1ft
	$k=\frac{4r}{h\sqrt{3}}+1$ *	A1

M1 Finding the distance of the c of m from O by using the angle given. Must use tan.

A1 Obtaining $\frac{r}{\sqrt{3}}$ (no approx allowed)

M1 Forming a moments equation using the three known distances; mass ratio only needed – do not penalise use of incorrect formulae

A1 LHS correct

A1ft RHS correct for their distance

A1cao Obtaining the GIVEN answer

ALT 1 Taking moments about *A*

M1 Attempting the LHS – must have two appropriate terms inc the necessary resolution

A1 Correct LHS

M1 Attempting the RHS – must have two appropriate terms inc the necessary resolution

A1 Correct RHS

A1ft Collecting the terms and cancelling Mg

A1cao Completing to the GIVEN answer

ALT 2 Find $\bar{x}$ first

M1 First M mark on e-PEN: Attempting an equation to find $\bar{x}$ in terms of h and k - mass ratio as above

A1 First A mark on e-PEN: Correct equation

A1 Second A mark on e-PEN: Correct expression for $\bar{x}$ (as shown or equivalent)

M1 Second M mark on e-PEN: Using $\frac{\bar{x}}{r}=\tan 30$ (LHS either way up)

A1ft Third A mark on e-PEN: Substitute their $\bar{x}$; LHS must be the correct way up

A1cao Final A mark on e-PEN: Obtaining the GIVEN answer

Question Number	Scheme	Marks
6 (a)	$T_A = \frac{20x}{2.5}\ (=8x) \qquad T_B = \frac{18(2-x)}{1.5}\ (=12(2-x))$	
	$\frac{20x}{2.5} = \frac{18(2-x)}{1.5}$	M1A1
	$x = \frac{12}{10} = 1.2$	A1
	$AO = 3.7$ m	A1ft (4)
(b)	$\frac{18(0.8-y)}{1.5} - \frac{20(1.2+y)}{2.5} = 0.5\ddot{y}$	M1A1A1
	$-40y = \ddot{y}\ \therefore$ SHM (or $\ddot{y} = (-20/m)y$	A1cso (4)
(c)	(Max) speed $= \frac{6}{0.5} = 12$ m s^{-1}	B1
	$\omega = \sqrt{40} = 2\sqrt{10}$	B1ft
	$12 = a \times 2\sqrt{10}$	M1
	$a = \frac{6}{\sqrt{10}}$ or $\frac{3\sqrt{10}}{5}$ m (accept 1.897... ie 1.9, 1.90 or better)	A1ft (4)
(d)	$1.2 = a\sin\omega t$	M1(their a, ω)
	$t = \frac{1}{2\sqrt{10}}\sin^{-1}\left(\frac{1.2\sqrt{10}}{6}\right)$	M1(must use radians)
	$t = 0.1082...$ s (Accept 0.11 or better)	A1cso (3) [15]

(a) M1 Using Hooke's law to find **both** tensions and equating them. The extension in *BP* can be used instead of the extension in *AP*. ALT: Use both extensions and use $e_a + e_b = 2$ later

A1 Correct equation

A1 Correct value found for either extension

A1ft Correct length for *AO*; follow through their extension

(b) M1 Forming an equation of motion at a general point. Difference of 2 tensions, both including. the variable. Use of *a* instead of $\ddot{x}$ can score M1A1A0A0 max (ie an A error)

A1 A1 A1A1 fully correct; A1A0 one error May have *m* instead of 0.5 Extensions measured from *O*

A1cso A correct simplified equation. Any equivalent form, including having *m* instead of 0.5. There must be a concluding statement.

(c) B1 Correct speed following impulse Can be awarded if seen in (b) or (d)

B1ft Correct value of ω; must be numerical. FT from (b) Can be awarded if seen in (b) or (d)

M1 Using $v_{\max} = a\omega$ (their values). By energy – equation must have all terms

A1ft Correct value of *a* any equivalent form including decimals. Follow through their ω

(d) M1 Using $y = a\sin\omega t$ with their *a* and ω If $y = a\cos\omega t$ is used there must be some indication of moving from the time obtained to the required time.

M1 Solving their equation to find a time. **Must** use radians

A1cso Correct time, min 2 sf. ω and *a* must have been obtained from correct work.

Question Number	Scheme	Marks
7 (a)	$\frac{1}{2}mv^2 - \frac{1}{2}m\frac{rg}{4} = mgr\left(1-\cos\theta\right)$	M1A1A1
	$v^2 = \frac{rg}{4}\left(9-8\cos\theta\right)$ *	A1 (4)
(b)	$\left(R\right) + mg\cos\theta = \frac{mv^2}{r}$	M1A1
	$R=0 \quad mg\cos\alpha = \frac{mg}{4}\left(9-8\cos\alpha\right)$	DM1
	$12\cos\alpha = 9$	
	$\cos\alpha = \frac{3}{4}$ or 0.75	A1 (4)
(c)	Initial vert comp of speed $= \sqrt{\frac{3g}{8}}\sin\alpha = \sqrt{\frac{3g}{8}} \times \frac{\sqrt{7}}{4} \quad \left(=1.2679...\right)$	M1A1
	$\frac{7}{8} = 1.2679...t + \frac{1}{2}gt^2$	M1
	$7 = 10.143...t + 39.2t^2$	
	$39.2t^2 + 10.143...t - 7 = 0$	
	$t = \frac{-10.143 \pm \sqrt{10.143^2 + 4\times7\times39.2}}{2\times39.2}$	DM1
	$t = 0.3125...$	A1
	Horiz speed $= \sqrt{\frac{3g}{8}}\cos\alpha = \frac{1}{4}\sqrt{\frac{27g}{8}}$	
	$AC = \frac{1}{4}\sqrt{\frac{27g}{8}} \times 0.3125 + r\sin\alpha = 0.4493 + 0.3307 = 0.78$ m	M1A1cso (7) [15]

(a) M1 Attempting an energy equation. 2KE terms needed and a PE term.
Award if mass missing throughout, but **not** for use of $v^2 = u^2 + 2as$

A1 KE terms correct (and subtracted) Mass not needed if M mark earned

A1 PE correct Again, mass not needed if M mark earned

A1cso Obtaining the GIVEN answer

(b) M1 Attempting an equation of motion along the radius. Accel in either form, $\left(\pm\right)R$ may be included.

A1 Correct equation, with or without $\left(\pm\right)R$

DM1 Set $R=0$ and substitute for v

A1 $\cos\alpha = 3/4$ obtained

(c) M1 Attempting the initial vertical component of the speed

A1 Correct vertical component - decimal or exact

M1 Using $s = ut + \frac{1}{2}at^2$ to form a quadratic in *t*, with *their* vertical speed and attempt at the vertical distance **Must** satisfy $0.5 < \text{distance} < 1$

DM1 Solving their quadratic; formula must be shown (and correct) if answer is incorrect, but allow with $+\sqrt{....}$ instead of $\pm\sqrt{....}$

A1 Correct *t*. Give by implication if stored on a calculator and final answer correct
Second solution need not be shown; ignore any shown

M1 Using the horizontal speed and completing to obtain the required distance.

A1 $AC = 0.78$ **must** be 2 sf.

ALT for (c):

M1A1 As main method above

M1 Use the horizontal speed and distance travelled as a projectile to get an expression for *t* and substitute in $s = ut + \frac{1}{2}at^2$ Vertical distance must be between 0.5 and 1

DM1 Solve their quadratic - see above

A1 Correct (projectile) distance

M1A1 As main method above

7(c) Using energy etc:

M1	Using energy to get the speed at the floor. Can be from the top or the point of leaving the surface
A1	Correct speed at floor
M1	Using the horizontal component of the speed and Pythagoras to obtain the vertical component at the floor
M1	Using $v = u + at$ vertically to get t
A1	Correct t
M1A1	Complete as main method

Other alternative Methods

Question 4(a) **by definite integration**

$900\frac{dv}{dt}=\frac{63000}{kt^2}$	M1
$\int_0^{10.5} dv=\int_1^4 \frac{70}{kt^2}dt$	
$[v]_0^{10.5}=\left[-\frac{70}{kt}\right]_1^4$	DM1A1 Integration, limits not needed
$10.5\ (-0)=-\frac{70}{4k}+\frac{70}{k}$	M1 Substitute limits
$k=5$	A1 Correct value
$\int_0^v dv=\int_1^t \frac{14}{t^2}dt$	A1 Integrate again with limits as shown
$v=14-\frac{14}{t}$ *	A1 Obtain GIVEN answer

OR:

$900\frac{dv}{dt}=\frac{63000}{kt^2}$	M1
$\int_0^v dv=\int_1^t \frac{70}{kt^2}dt$	
$[v]_0^v=\left[-\frac{70}{kt}\right]_1^t$	DM1A1 Integration, limits not needed
$v=\frac{70}{k}\left[-\frac{1}{t}\right]_1^t=\frac{70}{k}\left(1-\frac{1}{t}\right)$	M1 Substitute limits and $v=10.5$, $t=4$
$k=5$	A1 Correct value
$v=\frac{70}{5}\left(1-\frac{1}{t}\right)$	A1 substitute
$v=14-\frac{14}{t}$ *	A1 Obtain GIVEN answer

Question 6(c) **by reference circle**

M1 Finding the required angle in radians.

M1 Using the period $\left(\frac{2\pi}{\omega}\right)$ and their angle to find the required time.

A1 Correct time.