

Paper Reference(s)

6679/01

Edexcel GCE

Mechanics M3

Advanced Level

Monday 19 May 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 7 questions in this question paper.

The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

1.

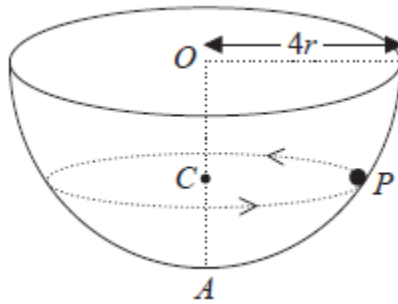


Figure 1

A hemispherical bowl of internal radius $4r$ is fixed with its circular rim horizontal. The centre of the circular rim is O and the point A on the surface of the bowl is vertically below O . A particle P moves in a horizontal circle, with centre C , on the smooth inner surface of the bowl. The particle moves with constant angular speed $\sqrt{\frac{3g}{8r}}$.

The point C lies on OA , as shown in Figure 1.

Find, in terms of r , the distance OC .

(9)

2. A particle P of mass m is fired vertically upwards from a point on the surface of the Earth and initially moves in a straight line directly away from the centre of the Earth. When P is at a distance x from the centre of the Earth, the gravitational force exerted by the Earth on P is directed towards the centre of the Earth and has magnitude $\frac{k}{x^2}$, where k is a constant.

At the surface of the Earth the acceleration due to gravity is g . The Earth is modelled as a fixed sphere of radius R .

(a) Show that $k = mgR^2$.

(2)

When P is at a height $\frac{R}{4}$ above the surface of the Earth, the speed of P is $\sqrt{\frac{gR}{2}}$.

Given that air resistance can be ignored,

(b) find, in terms of R , the greatest distance from the centre of the Earth reached by P .

(7)

3.

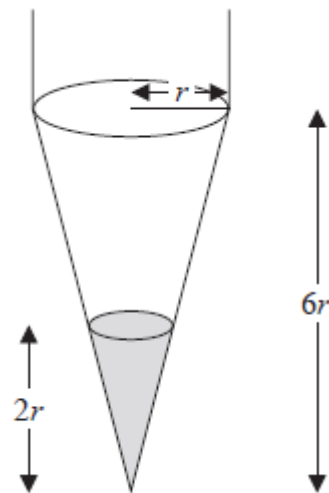


Figure 2

Figure 2 shows a container in the shape of a uniform right circular conical shell of height $6r$. The radius of the open circular face is r . The container is suspended by two vertical strings attached to two points at opposite ends of a diameter of the open circular face. It hangs with the open circular face uppermost and axis vertical.

Molten wax is poured into the container. The wax solidifies and adheres to the container, forming a uniform solid right circular cone. The depth of the wax in the container is $2r$. The container together with the wax forms a solid S .

The mass of the container when empty is m and the mass of the wax in the container is $3m$.

(a) Find the distance of the centre of mass of the solid S from the vertex of the container. (4)

One of the strings is now removed and the solid S hangs freely in equilibrium suspended by the remaining vertical string.

(b) Find the size of the angle between the axis of the container and the downward vertical. (3)

4.

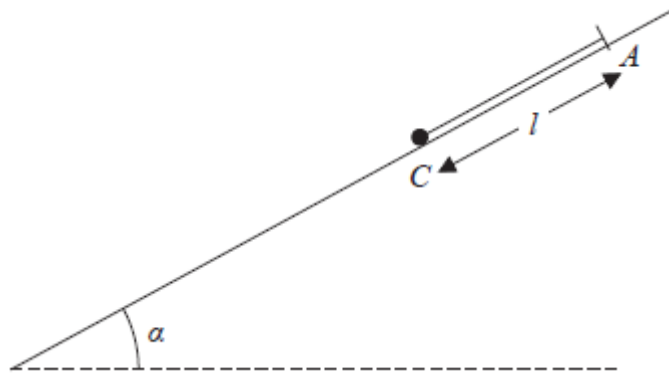


Figure 3

One end of a light elastic string, of natural length l and modulus of elasticity $3mg$, is fixed to a point A on a fixed plane inclined at an angle α to the horizontal, where $\sin \alpha = \frac{3}{5}$.

A small ball of mass $2m$ is attached to the free end of the string. The ball is held at a point C on the plane, where C is below A and $AC = l$ as shown in Figure 3. The string is parallel to a line of greatest slope of the plane. The ball is released from rest. In an initial model the plane is assumed to be smooth.

(a) Find the distance that the ball moves before first coming to instantaneous rest. **(5)**

In a refined model the plane is assumed to be rough. The coefficient of friction between the ball and the plane is μ . The ball first comes to instantaneous rest after moving a distance $\frac{2}{5}l$.

(b) Find the value of μ . **(6)**

5.

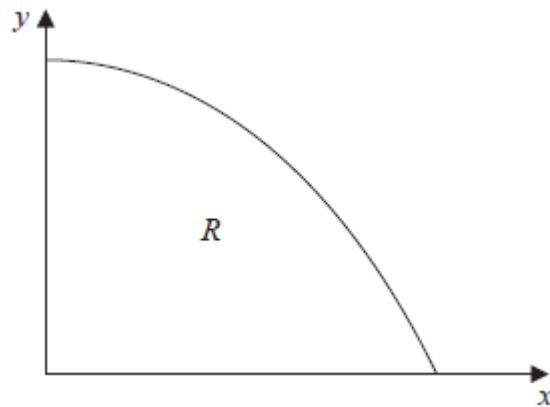


Figure 4

Figure 4 shows the region R bounded by part of the curve with equation $y = \cos x$, the x -axis and the y -axis. A uniform solid S is formed by rotating R through 2π radians about the x -axis.

(a) Show that the volume of S is $\frac{\pi^2}{4}$. (4)

(b) Find, using algebraic integration, the x -coordinate of the centre of mass of S . (7)

6. A particle P is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point. The particle is hanging freely at rest, with the string vertical, when it is projected horizontally with speed U . The particle moves in a complete vertical circle.

(a) Show that $U \geq \sqrt{5ag}$. (8)

As P moves in the circle the least tension in the string is T and the greatest tension is kT .

Given that $U = 3\sqrt{ag}$,

(b) find the value of k . (5)

7. A particle P of mass m is attached to one end of a light elastic spring of natural length l . The other end of the spring is attached to a fixed point A . The particle is hanging freely in equilibrium at the point B , where $AB = 1.5l$.

(a) Show that the modulus of elasticity of the spring is $2mg$. (3)

The particle is pulled vertically downwards from B to the point C , where $AC = 1.8l$, and released from rest.

(b) Show that P moves in simple harmonic motion with centre B . (6)

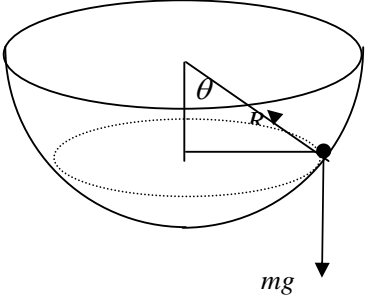
(c) Find the greatest magnitude of the acceleration of P . (2)

The midpoint of BC is D . The point E lies vertically below A and $AE = 1.2l$.

(d) Find the time taken by P to move directly from D to E . (4)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	<div style="text-align: center;">  </div> $R \sin \theta = m \times 4r \sin \theta \times \frac{3g}{8r}$ $R = \frac{3}{2} mg$ $R \cos \theta = mg$ $\frac{3}{2} mg \cos \theta = mg$ $\cos \theta = \frac{2}{3}$ $OC = 4r \cos \theta = 4r \times \frac{2}{3} = \frac{8}{3} r \text{ oe}$	<p>M1A1A1</p> <p>M1A1</p> <p>M1(dep)</p> <p>A1</p> <p>M1A1</p> <p>[9]</p>
Notes for Question 1		
	<p>M1 for NL2 towards C - Accept use of $v = \sqrt{\frac{3g}{8r}}$ and $a = \frac{v^2}{r}$ as a mis-read</p> <p>A1 for LHS fully correct</p> <p>A1 for RHS fully correct</p> <p>ALT: Work in the direction of R and obtain the same equation with $\sin \theta$ "cancelled". Give M1A1A1 if fully correct, M0 otherwise.</p> <p>M1 for resolving vertically</p> <p>A1 for the equation fully correct</p> <p>M1 dep for eliminating R between the two equations Dependent on both above M marks</p> <p>A1 for $\cos \theta = \frac{2}{3}$</p> <p>M1 for attempting to use trig or Pythagoras to obtain OC</p> <p>A1 cso for $OC = \frac{8}{3} r$</p>	

Alternative for Question 1

M1A1A1	$R \sin \theta = m \times a \times \frac{3g}{8r}$
M1 A1	$R \cos \theta = mg$
M1 A1	$\tan \theta = \frac{3a}{8r}$
M1	$\frac{a}{OC} = \frac{3a}{8r}$
A1	$OC = \frac{8r}{3}$

Question Number	Scheme	Marks
<p>2.</p> <p>(a)</p> <p>(b)</p>	<p>(At surface) $\frac{k}{R^2} = mg \Rightarrow k = mgR^2$</p> $m\ddot{x} = -\frac{mgR^2}{x^2}$ $v \frac{dv}{dx} = -\frac{gR^2}{x^2}$ $\int v \frac{dv}{dx} dx = -gR^2 \int \frac{1}{x^2} dx \quad \text{or} \quad \int \frac{d(\frac{1}{2}v^2)}{dx} dx$ $\frac{1}{2}v^2 = \frac{gR^2}{x} (+c)$ $x = \frac{5R}{4}, v = \sqrt{\frac{gR}{2}} \Rightarrow c = -\frac{11gR}{20}$ $v = 0 \quad 0 = \frac{gR^2}{x} - \frac{11gR}{20}$ $x = \frac{20R}{11}$	<p>M1A1 (2)</p> <p>M1</p> <p>DM1A1</p> <p>DM1A1</p> <p>DM1</p> <p>A1 (7)</p> <p>[9]</p>

Notes for Question 2

(a)

for $\frac{k}{R^2} = mg$. If not made clear that this applies at the surface of the Earth award M0 **or**

M1 $\frac{k}{x^2} = mg$ **and** $x = R$.

A1 cso for $k = mgR^2$ *

(b)

M1 for using accel = $v \frac{dv}{dx}$ oe in NL2 with or w/o m Minus sign not required.

M1 dep for attempting to integrate both sides - minus not needed

A1 for fully correct integration, with or w/o the constant. Must have included the minus sign from the start.

M1 dep for using $x = \frac{5R}{4}$, $v = \sqrt{\frac{gR}{2}}$ to obtain a value for the constant. Use of $x = \frac{R}{4}$ scores M0 Depends on both previous M marks

A1 for $c = -\frac{11gR}{20}$

M1 dep for setting $v = 0$ and solving for x Depends on 1st and 2nd M marks, but not 3rd

A1 cso for $x = \frac{20R}{11}$

ALT: By definite integration
First 3 marks as above, then

DM1 Using limits $x = \frac{5R}{4}$, $v = \sqrt{\frac{gR}{2}}$

DM1 Using limit $v = 0$

A1 Correct substitution

A1 cso for $x = \frac{20R}{11}$

NB: The penultimate A mark has changed position, but must be entered on e-pen in its original position.

Alternative for Question 2

Qu 2 (a):

Using $F = \frac{GM_1M_2}{x^2}$ with $x = R$ and one mass as mass of Earth:

$$mg = \frac{GmM_E}{R^2}$$

$$GM_E = gR^2 \Rightarrow F = \frac{mgR^2}{x^2} \Rightarrow F = \frac{k}{x^2} \text{ with } k = mgR^2 *$$

M1 Complete method A1 Correct answer

Qu 2 (b):

By conservation of energy:

$$\text{Work done against gravity} = \int_{\frac{5r}{4}}^z \frac{mgR^2}{x^2} dx = \int_{\frac{5r}{4}}^z mgR^2 x^{-2} dx$$

M1

$$= \frac{4mgR}{5} - \frac{mgR^2}{z}$$

DM1(integration)A1(correct)

$$\text{Work-energy equation: } \frac{mgR}{4} = \frac{4mgR}{5} - \frac{mgR^2}{z}$$

DM1A1

$$z = \frac{20R}{11}$$

DM1A1

Question Number	Scheme	Marks																																								
<p>3. (a)</p>	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 30%;"></td> <td style="width: 15%; text-align: center;">Shell</td> <td style="width: 15%; text-align: center;">wax</td> <td style="width: 15%; text-align: center;">filled shell</td> <td style="width: 25%;"></td> </tr> <tr> <td>Mass ratio</td> <td style="text-align: center;">m</td> <td style="text-align: center;">$3m$</td> <td style="text-align: center;">$4m$</td> <td></td> </tr> <tr> <td>Dist. above vertex</td> <td style="text-align: center;">$\frac{2}{3} \times 6r$</td> <td style="text-align: center;">$\frac{3}{4} \times 2r$</td> <td style="text-align: center;">\bar{x}</td> <td style="text-align: center;">B1</td> </tr> <tr> <td></td> <td colspan="3" style="text-align: center;">$4mr + \frac{9}{2}mr = 4m\bar{x}$</td> <td style="text-align: center;">M1A1ft</td> </tr> <tr> <td></td> <td colspan="3" style="text-align: center;">$\bar{x} = \frac{17}{8}r$</td> <td style="text-align: center;">A1 (4)</td> </tr> <tr> <td>(b)</td> <td colspan="3" style="text-align: center;">$\tan \theta = \frac{r}{6r - \bar{x}} = \frac{r}{31r/8}$</td> <td style="text-align: center;">M1A1ft</td> </tr> <tr> <td></td> <td colspan="3" style="text-align: center;">$\tan \theta = \frac{8}{31}$</td> <td></td> </tr> <tr> <td></td> <td colspan="3" style="text-align: center;">$\theta = 14.47...^\circ$</td> <td style="text-align: center;">A1 (3) [7]</td> </tr> </table>		Shell	wax	filled shell		Mass ratio	m	$3m$	$4m$		Dist. above vertex	$\frac{2}{3} \times 6r$	$\frac{3}{4} \times 2r$	\bar{x}	B1		$4mr + \frac{9}{2}mr = 4m\bar{x}$			M1A1ft		$\bar{x} = \frac{17}{8}r$			A1 (4)	(b)	$\tan \theta = \frac{r}{6r - \bar{x}} = \frac{r}{31r/8}$			M1A1ft		$\tan \theta = \frac{8}{31}$					$\theta = 14.47...^\circ$			A1 (3) [7]	
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	Notes for Question 3																																									
<p>(a) B1 M1 A1 ft A1 cso</p>	<p>for correct distances from the vertex or any other point for a dimensionally correct moments equation with their distances and masses for a correct moments equation, follow through their distances but must have correct masses for $\bar{x} = \frac{17}{8}r$</p> <p>NB: If $\frac{2}{3}$ and $\frac{3}{4}$ are interchanged in the distances, the correct answer is obtained but the solution is incorrect. Score: B0M1A1A0</p>																																									
<p>(b) M1 A1 ft A1 cso</p>	<p>for $\tan \theta = \frac{r}{6r - \bar{x}}$. Can be either way up, but must include $6r - \bar{x}$. Substitution for \bar{x} not required for $\tan \theta = \frac{r}{31r/8}$ oe ft their \bar{x} for $\theta = 14.47...^\circ$ Accept 14°, 14.5° or better or $\theta = 0.2525...rad$ Accept 0.25 or better Obtuse angle accepted.</p>																																									

Question Number	Scheme	Marks
<p>4</p> <p>(a)</p>	$\frac{3mgx^2}{2l} = 2mgx \sin \alpha$ $3x^2 = 4xl \times \frac{3}{5}$ $5x^2 = 4xl$ $x = \frac{4}{5}l$ <p>(b)</p> $R = 2mg \cos \alpha \left(= \frac{8}{5}mg \right)$ $\frac{3mg}{2l} \times \frac{4}{25}l^2 = 2mg \times \frac{2}{5}l \times \frac{3}{5}, \quad \mu \frac{8}{5}mg \times \frac{2}{5}l$ $6 = 12 - 16\mu$ $16\mu = 6 \quad \mu = \frac{3}{8}$	<p>M1A1 B1(A1 on e- pen)</p> <p>DM1A1 (5)</p> <p>B1</p> <p>M1A1ft, B1ft (A1 on e- pen)</p> <p>DM1A1 (6)</p> <p>[11]</p>

Notes for Question 4

	(a)
M1	for an energy equation with an EPE term of the form $\frac{kmgx^2}{l}$ and a GPE term. If a KE term is included it must become 0 later.
A1	for a correct EPE term
B1	for a correct GPE term. This can be in terms of the distance moved down the plane or the vertical distance fallen
M1 dep	for solving their equation to obtain the distance moved or using the vertical distance and obtaining the distance moved along the plane.
A1	for $x = \frac{4}{5}l$ oe eg $x = \frac{12}{15}l$
	(b)
B1	for resolving perpendicular to the plane to obtain $R = 2mg \cos \alpha$. May only be seen in an equation.
M1	for an work-energy equation with an EPE term of the form $\frac{kmgx^2}{l}$, a GPE term and the work done against friction. The work term must include a distance along the plane.
A1	for EPE and GPE terms correct and work subtracted from the GPE
B1 ft	for the work term ft their R
M1 dep	for solving to obtain a value for μ
A1 cso	for $\mu = \frac{3}{8}$ oe inc 0.375 but not 0.38
	If m used instead of $2m$, assuming correct otherwise:
	(a) M1A1B0M1A0 (so 2 penalties for mis-read)
	(b)
B1	$R = mg \cos \alpha$
M1, A1	Equation, with EPE correct and $mg \times \frac{2}{5}l \times \frac{3}{5}$
B1 ft	$\mu \frac{4mg}{5} \times \frac{2}{5}l$
DM1, A1	$\mu = 0$

Alternative for Question 4

Qu 4: Using NL2:

(a)

$$2ma = 2mg \sin \alpha - \frac{3mgx}{l}$$

$$2v \frac{dv}{dx} = \frac{6g}{5} - \frac{3gx}{l}$$

M1 (equation and attempt integration)

$$v^2 = \frac{6gx}{5} - \frac{3gx^2}{2l}, +c$$

A1, A1 (show $c = 0$)

$$v = 0 \quad 3gx \left(\frac{2}{5} - \frac{x}{2l} \right) = 0$$

M1 (set $v = 0$ and solve)

$$x = \frac{4l}{5}$$

A1

(b)

$$R = 2mg \cos \alpha$$

B1

$$2v \frac{dv}{dx} = \frac{6g}{5} - \frac{3gx}{l} - \mu \frac{8g}{5}$$

$$v^2 = \frac{6gx}{5} - \frac{3gx^2}{2l} - \mu \frac{8gx}{5}, +c$$

M1 (eqn and int) A1, A1 (show $c = 0$)

$$v = 0 \quad x = \frac{2l}{5} \quad \mu \frac{8}{5} = \frac{6}{5} - \frac{3}{2l} \times \frac{2l}{5}$$

M1 (set $v = 0$ and solve)

$$\mu = \frac{3}{8}$$

A1

If SHM methods are used, SHM must be proved first.

Question Number	Scheme	Marks
5.	<p>(a) $\text{Vol} = \pi \int_0^{\frac{\pi}{2}} y^2 dx = \pi \int_0^{\frac{\pi}{2}} \cos^2 x dx$</p> $= \pi \int_0^{\frac{\pi}{2}} \frac{1}{2} (\cos 2x + 1) dx$ $= \frac{\pi}{2} \left[\frac{1}{2} \sin 2x + x \right]_0^{\frac{\pi}{2}} = \frac{\pi^2}{4}$ <p>(b) $\pi \int_0^{\frac{\pi}{2}} y^2 x dx = \pi \int_0^{\frac{\pi}{2}} x \cos^2 x dx$</p> $= \pi \int_0^{\frac{\pi}{2}} \frac{1}{2} x (\cos 2x + 1) dx$ $= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} x \cos 2x dx + \frac{\pi}{2} \left[\frac{x^2}{2} \right]_0^{\frac{\pi}{2}}$ $\frac{\pi}{2} \left[x \times \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{2}} - \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{1}{2} \sin 2x dx, + \frac{\pi^3}{16}$ $= 0 + \frac{\pi}{2} \left[\frac{1}{4} \cos 2x \right]_0^{\frac{\pi}{2}} + \frac{\pi^3}{16}$ $= \frac{\pi}{8} [-1 - 1] + \frac{\pi^3}{16} = \frac{\pi^3}{16} - \frac{\pi}{4}$ $\bar{x} = \frac{\pi^3 - 4\pi}{16} \div \frac{\pi^2}{4} = \frac{\pi^2 - 4}{4\pi} \text{ or } 0.467088\dots$	<p>M1</p> <p>M1</p> <p>DM1A1 (4)</p> <p>M1</p> <p>M1,B1</p> <p>DM1</p> <p>A1ft</p> <p>M1A1 (7)</p> <p>[11]</p>

Notes for Question 5

(a)

M1 for using $\text{Vol} = \pi \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$. If π is missing here it must be included later to earn this mark.

Limits not needed

M1 for using the double angle formula (correct) to prepare for integration. Formula must be correct. π and limits not needed for this mark.

M1 dep for attempting to integrate and substitute the correct limits (only sub of non-zero limit needed be to seen) dependent on both M marks.

A1 cso for $\frac{\pi^2}{4}$ * (check integration is correct, answer can be obtained by luck due to the limits)

(b) NB: The first 5 marks can be earned with or without π

M1 for using $\pi \int_0^{\frac{\pi}{2}} x \cos^2 x \, dx$ π not needed; limits not needed.

M1 for using the double angle formula (correct) and attempting the first stage of integration by parts

B1 for $\frac{\pi^3}{16}$ or $\frac{\pi^2}{16}$ if π not included. NB integration by parts not needed for this mark

M1 dep for completing the integration by parts, limits not needed yet

A1 ft for $= \frac{\pi}{8}[-1-1] + \frac{\pi^3}{16} = \frac{\pi^3}{16} - \frac{\pi}{4}$ or $= \frac{1}{8}[-1-1] + \frac{\pi^2}{16} = \frac{\pi^2}{16} - \frac{1}{4}$ ft on $\frac{\pi^3}{16}$

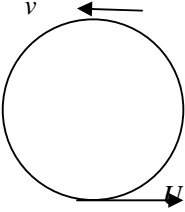
M1 for using $\bar{x} = \frac{\int \pi y^2 x \, dx}{\int \pi y^2 \, dx}$ The numerator integral need not be correct.

π should be seen in both or neither integral

for $\bar{x} = \frac{\pi^2 - 4}{4\pi}$ oe eg $\frac{\pi}{4} - \frac{1}{\pi}$ or 0.467088....

A1 cso Accept 0.47 or better but no fractions within fractions

(a) has a given answer, so the cso applies to the solution of (b) only.

Question Number	Scheme	Marks
6.	<div style="text-align: center;">  </div> <p data-bbox="224 527 256 569">(a)</p> $\frac{1}{2}mU^2 - \frac{1}{2}mv^2 = 2mga$ $T + mg = m\frac{v^2}{a}$ $T = \frac{(mU^2 - 4mga)}{a} - mg$ $T = \frac{mU^2 - 5mga}{a}$ $T \geq 0 \Rightarrow U^2 \geq 5ga$ $U \geq \sqrt{5ag} \quad *$ <p data-bbox="224 1178 256 1220">(b)</p> <p data-bbox="321 1157 727 1230">At top: $T = \frac{9mga - 5mga}{a} = 4mg$</p> <p data-bbox="321 1272 667 1346">At bottom: $T' - mg = \frac{mU^2}{a}$</p> $kT = mg + \frac{9mag}{a} = 10mg$ $k = \frac{10mg}{4mg} = \frac{5}{2}$	<p data-bbox="1255 541 1328 573">M1A1</p> <p data-bbox="1255 657 1328 688">M1A1</p> <p data-bbox="1255 779 1312 810">DM1</p> <p data-bbox="1255 894 1287 926">A1</p> <p data-bbox="1255 993 1312 1024">DM1</p> <p data-bbox="1255 1077 1409 1108">A1 (8)</p> <p data-bbox="1255 1167 1377 1230">M1(either tension)A1</p> <p data-bbox="1255 1293 1287 1325">A1</p> <p data-bbox="1255 1409 1312 1440">DM1</p> <p data-bbox="1255 1524 1409 1556">A1 (5)</p> <p data-bbox="1255 1577 1304 1608">[13]</p>

Notes for Question 6

(a)	for an energy equation, from the bottom to the top. A difference of KE terms and a PE term needed.
M1	From bottom to a general point gets M0 until a value for θ at the top is used. $v^2 = u^2 + 2as$ scores M0
A1	for all terms correct (inc signs)
M1	for NL2 along the radius at the top. Two forces and mass x acceleration needed. Accel can be in either form here. But see NB at end of (a)
A1	for a fully correct equation. Acceleration should be $\frac{v^2}{a}$ now.
M1 dep	for eliminating v (vel at top) between the two equations. Dependent on both previous M marks. If v is set = 0, award M0
A1	for a correct expression for T
M1 dep	for using $T \geq 0$ to obtain an inequality for U^2 or U . Allow with $>$ Dependent on all previous M marks.
A1 cso	for $U \geq \sqrt{5ag}$ * Watch square root! Give A0 if $>$ seen on previous line.
	NB: The second and fourth M marks (and their As if earned) can be given together if $mg \leq m \frac{v^2}{a}$ is seen
(b)	
M1	for obtaining an expression for the tension at the top or at the bottom, no need to substitute for U yet.
A1	Substitute for U and obtain one correct tension ($4mg$ at top or $10mg$ at bottom)
A1	for the other tension correct
M1 dep	for using tension at bottom = k x tension at the top and solving for k
A1 cso	for $k = \frac{5}{2}$ oe

Question Number	Scheme	Marks
7.	<p>(a) $T = \frac{\lambda x}{l} = \frac{\lambda \times 0.5l}{l}$</p> <p>$\lambda = 2mg$ *</p> <p>(b) $mg - T = m\ddot{x}$</p> <p>$mg - \frac{2mg(0.5l + x)}{l} = m\ddot{x}$</p> <p>$\ddot{x} = -\frac{2gx}{l}$</p> <p>$\therefore$ SHM</p> <p>(c) $a = 0.3l$</p> <p>$\ddot{x} _{\max} = 2g \times \frac{0.3l}{l} = 0.6g$ (= 5.88 or 5.9 m s⁻²)</p> <p>(d) $x = a \cos \omega t = 0.3l \cos \left(\sqrt{\frac{2g}{l}} t \right)$</p> <p>Time C to D: $0.15 = 0.3 \cos \left(\sqrt{\frac{2g}{l}} t \right)$</p> <p>$t = \sqrt{\frac{l}{2g}} \cos^{-1} 0.5$</p> <p>Time C to E: $t' = \text{half period} = \pi \sqrt{\frac{l}{2g}}$</p> <p>Time D to E: $= (\pi - \cos^{-1} 0.5) \sqrt{\frac{l}{2g}} = \frac{2\pi}{3} \sqrt{\frac{l}{2g}}$</p>	<p>M1A1</p> <p>A1 (3)</p> <p>M1</p> <p>DM1A1A1</p> <p>A1</p> <p>A1cso(B1 on e-pen) (6)</p> <p>M1A1ft (2)</p> <p>M1</p> <p>B1</p> <p>M1A1 (4)</p> <p>[15]</p>

Notes for Question 7

(a)

- M1 for using Hooke's Law
- A1 for a correct equation
- A1 for solving to get $\lambda = 2mg$ *

(b)

- M1 for using NL2. Weight and tension must be seen. Acceleration can be a here, but must be an equation at a general position
- M1 dep for using Hooke's Law for the tension. Acceleration can be a
- A1 A1 for a fully correct equation inc acceleration as \ddot{x} (-1 ee)
- A1 for simplifying to $\ddot{x} = -\frac{2gx}{l}$ oe
- A1 cso for the conclusion

(c)

- M1 for using $|\dot{x}|_{\max} = \omega^2 a$ with their ω and $a = 0.3l$. ω must be dimensionally correct
- A1 ft for obtaining the max magnitude of the accel, accept 0.6g, 5.9 or 5.88 only. ft their ω

(d)

- M1 for using $x = a \cos \omega t$ with $x = \pm 0.15l$, $a = 0.3l$ and their ω to obtain an expression for the time from C to D
- B1 for time C to E = half period = $\pi \sqrt{\frac{l}{2g}}$
- M1 For any correct method for obtaining the time from D to E
- A1 cao for $\frac{2\pi}{3} \sqrt{\frac{l}{2g}}$ oe inc $0.473\sqrt{l}$ $0.47\sqrt{l}$

ALT for

(d):

(i)

- M1 Use $x = a \sin \omega t$ with $x = 0.15l$, $a = 0.3l$ and their ω to obtain an expression for the time from B to D
- M1, A1 as above

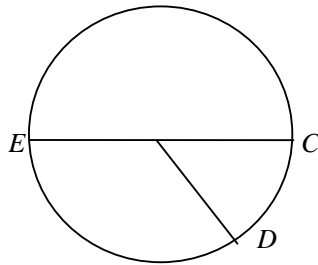
(ii)

- Using $x = a \cos \omega t$ with $x = \pm 0.15l$, $a = 0.3l$ and their ω
- This gives the required time in one step.
- Award M2 A1 for correct substitution
- A1 correct answer
- However do not isw if further work shown. Mark according to mark scheme method and give max M1B1M0A0.**

Alternative for Question 7

Qu 7 (d)

By reference circle:



Centre of circle is O

Angle $COD = \theta$ Angle $EOD = \alpha$

$$\cos \theta = \frac{0.15l}{0.3l} \quad \theta = \frac{\pi}{3} \quad \text{M1}$$

$$\alpha = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \quad \text{B1}$$

$$\omega = \sqrt{\frac{2g}{l}}$$

$$\text{time} = \frac{\alpha}{\omega} = \frac{2\pi/3}{\sqrt{\frac{2g}{l}}} = \frac{2\pi}{3} \sqrt{\frac{l}{2g}} \quad \text{M1A1}$$