Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M3

Advanced Level

Thursday 14 June 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle P is moving along the positive x-axis. At time $t = 0$, P is at the origin O. At time t seconds, P is x metres from O and has velocity $v = 2e^{-x}$ m s^{-1} in the direction of x increasing.

(*a*) Find the acceleration of P in terms of x.

(3)

(*b*) Find x in terms of t.

(6)

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2. A particle P moves in a straight line with simple harmonic motion about a fixed centre O. The period of the motion is $\frac{\pi}{2}$ seconds. At time t seconds the speed of P is v m s^{-1}. When $t = 0$, P is at O and $v = 6$. Find

(*a*) the greatest distance of P from O during the motion,

(3)

(*b*) the greatest magnitude of the acceleration of P during the motion,

(2)

(*c*) the smallest positive value of t for which P is 1 m from O.

(3)

3.

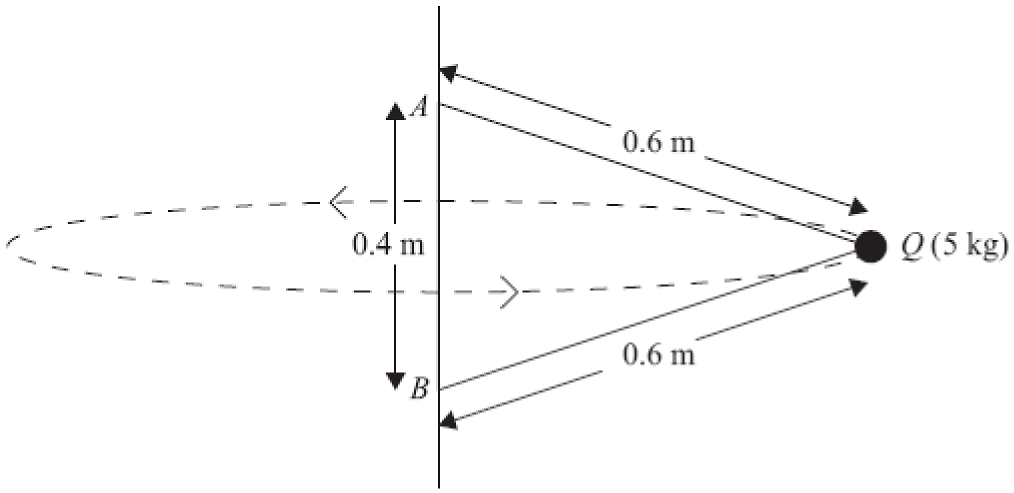

Figure 1

A particle Q of mass 5 kg is attached by two light inextensible strings to two fixed points A and B on a vertical pole. Each string has length 0.6 m and A is 0.4 m vertically above B, as shown in Figure 1.

Both strings are taut and Q is moving in a horizontal circle with constant angular speed 10 rad s^{-1}.

Find the tension in

(i) AQ,

(ii) BQ.

(10)

4.

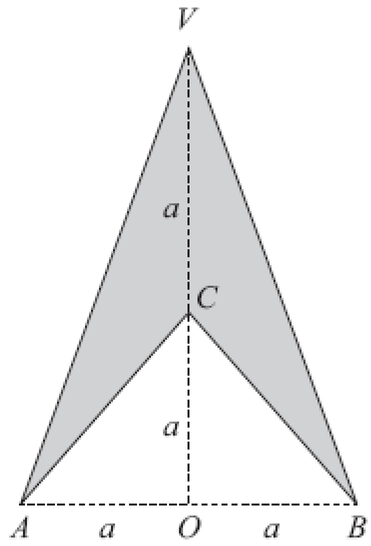

Figure 2

Figure 2 shows the cross-section *AVBC* of the solid *S* formed when a uniform right circular cone of base radius *a* and height *a*, is removed from a uniform right circular cone of base radius *a* and height 2*a*. Both cones have the same axis *VCO*, where *O* is the centre of the base of each cone.

(*a*) Show that the distance of the centre of mass of *S* from the vertex *V* is $\frac{5}{4}a$.

(5)

The mass of *S* is *M*. A particle of mass *kM* is attached to *S* at *B*. The system is suspended by a string attached to the vertex *V*, and hangs freely in equilibrium. Given that *VA* is at an angle 45° to the vertical through *V*,

(*b*) find the value of *k*.

(5)

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5. A fixed smooth sphere has centre *O* and radius *a*. A particle *P* is placed on the surface of the sphere at the point *A*, where *OA* makes an angle α with the upward vertical through *O*. The particle is released from rest at *A*. When *OP* makes an angle θ to the upward vertical through *O*, *P* is on the surface of the sphere and the speed of *P* is *v*.

Given that $\cos\alpha = \frac{3}{5}$,

(*a*) show that

$$v^2 = \frac{2ga}{5}(3 - 5\cos\theta),$$

(4)

(*b*) find the speed of *P* at the instant when it loses contact with the sphere.

(8)

6.

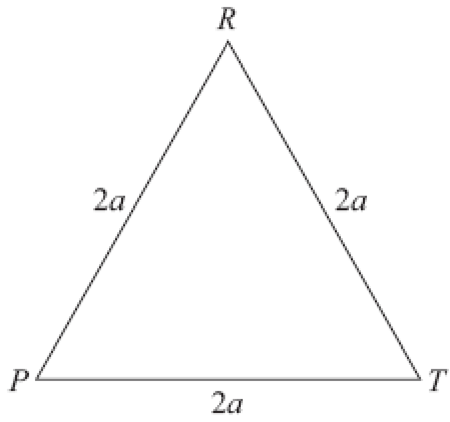


Figure 3

Figure 3 shows a uniform equilateral triangular lamina *PRT* with sides of length $2a$.

(*a*) Using calculus, prove that the centre of mass of *PRT* is at a distance $\frac{2\sqrt{3}}{3}a$ from *R*.

(6)

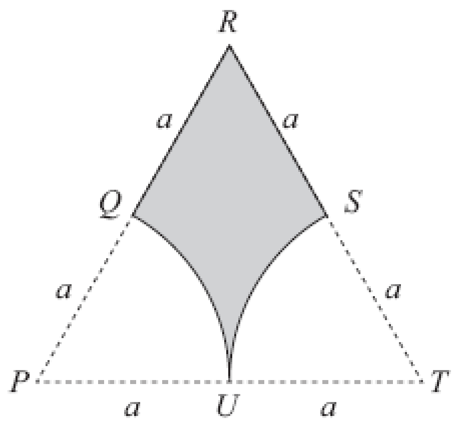


Figure 4

The circular sector *PQU*, of radius a and centre *P*, and the circular sector *TUS*, of radius a and centre *T*, are removed from *PRT* to form the uniform lamina *QRSU* shown in Figure 4.

(*b*) Show that the distance of the centre of mass of *QRSU* from *U* is $\frac{2a}{3\sqrt{3}-\pi}$.

(6)

7. A particle B of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.75 m and modulus of elasticity 24.5 N. The other end of the string is attached to a fixed point A. The particle is hanging in equilibrium at the point E, vertically below A.

(*a*) Show that AE = 0 9 m.

(3)

The particle is held at A and released from rest. The particle first comes to instantaneous rest at the point C.

(*b*) Find the distance AC.

(5)

(c) Show that while the string is taut, B is moving with simple harmonic motion with centre E.

(4)

(*d*) Calculate the maximum speed of B.

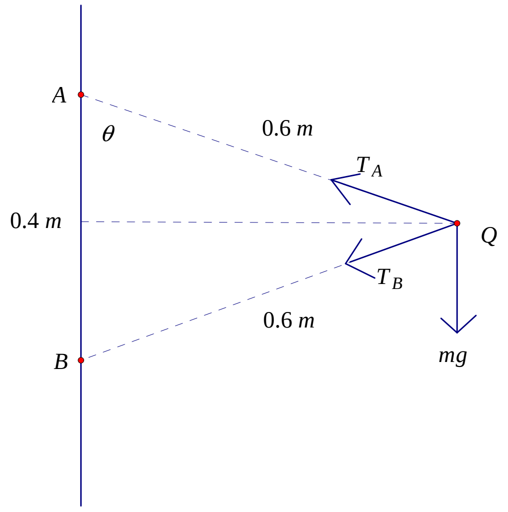
(2)

TOTAL FOR PAPER: 75 MARKS

END

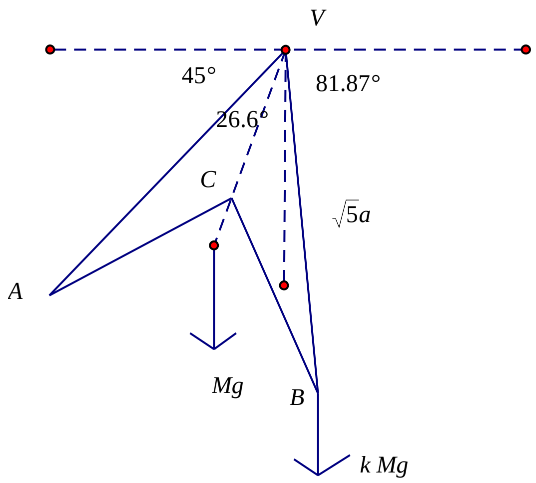
Summer 2012
6679 Mechanics M3
Mark Scheme

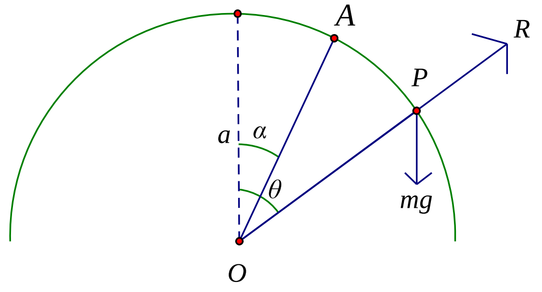
Question Number	Scheme	Marks
1(a)	Use of $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$ or $a = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right)$	M1
	$a = 2e^{-x}.-2e^{-x}$ or $v^2 = 4e^{-2x}$	A1
	$a = -4e^{-2x}$	A1
		(3)
(b)	Separate the variables and attempt to integrate:	M1
	$\int 2\mathrm{d}t = \int e^x \mathrm{d}x$	
	$2t = e^x + C$	A1A1
	t=0, x=0 $\Rightarrow C$=-1, $2t = e^x - 1$	M1A1
	$x = \ln(2t+1)$	A1
		(6)
		9
2(a)	$T = \frac{2\pi}{\omega} \Rightarrow \omega = 4$	B1
	Use of $v^2 = \omega^2(v^2 - x^2)$, or $v = a\omega$	M1
	a = 1.5 (m)	A1
		(3)
(b)	Use of max. accn. = $\omega^2 a$	M1
	24 ms^{-2}	A1
		(2)
(c)	$x = a\sin\omega t$ with their values for a & ω	B1
	$1 = 1.5\sin 4t$ (with their 1.5 & 4) and attempt to solve for t	M1
	t = 0.18 (or awrt)	A1
		(3)
		8

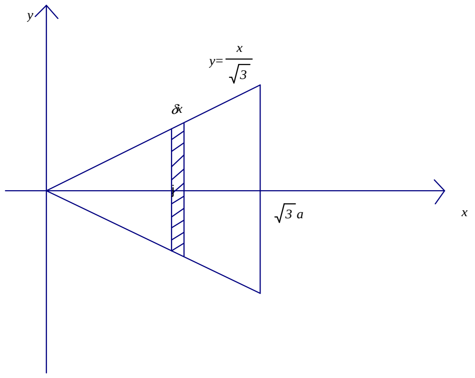
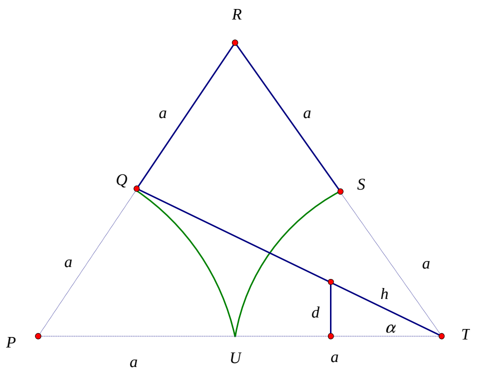
Question Number	Scheme	Marks
3		
	$\cos\theta = \frac{0.2}{0.6}\left(=\frac{1}{3}\right)$	B1
	Resolve vertically:	M1
	$T_A\cos\theta = T_B\cos\theta + mg \qquad \left(T_A = T_B + 3mg\right)$	A2,1,0
	Acceleration towards the centre:	M1
	$T_A\sin\theta + T_B\sin\theta = m\times 0.6\sin\theta\times\omega^2 \quad \left(T_A + T_B = 5\times\frac{3}{5}\times 100 = 300\right)$	A2,1,0
	Substitute values for ω and trig functions and solve to find T_A or T_B	M1
	$T_B + 147 + T_B = 300, \quad 2T_B = 300 - 147 = 153$	A1,A1
	$T_A = 223.5\text{(N)} \quad , \quad T_B = 76.5\text{(N)}$	
	$T_A = 224$ or $220 \quad T_B = 76$	
	$T_B = 76.5$ or $77 \quad T_A = 223$	
		(10) **10**

Question Number	Scheme	Marks
4 (a)		

	volume	Mass ratio	C of M from V
Large cone	$\frac{1}{3}\pi a^2 .2a = \frac{2}{3}\pi a^3$	2	$\frac{3}{4}\times 2a = \frac{3}{2}a$
Small cone	$\frac{1}{3}\pi a^2 .a = \frac{1}{3}\pi a^3$	1	$a + \frac{3}{4}a = \frac{7}{4}a$
S	$\frac{1}{3}\pi a^2 .a = \frac{1}{3}\pi a^3$	1	D

Question Number	Scheme	Marks
	(table above)	B1, B1
	$1\times D = 2\times\frac{3}{2}a - 1\times\frac{7}{4}a$	M1A1
	$= \frac{12-7}{4}a = \frac{5}{4}a$ **	A1
		(5)
(b)		
	$45° + 26.6° (= 71.6°)$, $(81.8698...... =) 81.9°$	
	Take moments about V:	M1
	$Mg\times\frac{5}{4}a\times\cos 71.6 = kMg\times\sqrt{5}a\times\cos 81.9$	A2
	$k = \frac{5\cos 71.6}{4\sqrt{5}\cos 81.9} = 1.25$	M1A1
		(5)
		10

Question Number	Scheme	Marks
5(a)		
	Conservation of energy : Loss in GPE = gain in KE	M1
	$mga(\cos\alpha - \cos\theta) = \frac{1}{2}mv^2$	A2,1,0
	Substitute for $\cos\alpha$ and rearrange to **given answer**:	A1
	$v^2 = \frac{2mga}{m}\left(\frac{3}{5} - \cos\theta\right) = \frac{2ga}{5}(3 - 5\cos\theta)$ *	
		(4)
(b)	Considering the acceleration towards the centre of the hemisphere:	M1
	$mg\cos\theta - R = \frac{mv^2}{a}$	A2,1,0
	Substitute for v^2 to form expression for R:	DM1
	$R = mg\cos\theta - \frac{mv^2}{a} = mg(3\cos\theta - 2\cos\alpha)\left(= mg\left(3\cos\theta - \frac{6}{5}\right)\right)$	A1
	Loses contact with the surface when $R = 0$	M1
	$\cos\theta = \frac{2}{5}$	A1
	$v^2 = \frac{2ga}{5}, \quad v = \sqrt{\frac{2ga}{5}}$	A1
		(8)
		12
Alt:	$R = 0 \Rightarrow mg\cos\theta = \frac{mv^2}{a}$	DM1
	$\cos\theta = \frac{v^2}{ga}$	A1
	Substitute in given (a) $v^2 = \frac{2ga}{5}\left(3 - 5\frac{v^2}{ga}\right)$	M1
	$v^2 = \frac{6ga}{5} - 2v^2, \quad 3v^2 = \frac{6ga}{5}$	A1
	$v = \sqrt{\frac{2ga}{5}}$	A1

Question Number	Scheme	Marks
6(a)		
	Mass of lamina = $\rho\frac{1}{2}\times 2a\times\sqrt{3}a=\sqrt{3}\rho a^2$	B1
	$\sum \rho x\times\frac{2x}{\sqrt3}\times\delta x=\rho\int_0^{\sqrt3 a}\frac{2x^2}{\sqrt3}\mathrm{d}x$	M1
	$=\rho\left[\frac{2x^3}{3\sqrt3}\right]_0^{\sqrt3 a}$	A1
	$=\rho\frac{2\times3\sqrt3 a^3}{3\sqrt3}=2\rho a^3$	A1
	Distance from vertex = $\frac{2\rho a^3}{\sqrt3\rho a^2}=\frac{2}{3}a\sqrt3$ **	M1A1
		(6)
(b)		
	Area of each sector $=\frac{1}{6}\pi a^2$	B1
	Using sector formula, $d=h\sin\alpha=\frac{2a\sin\alpha}{3\alpha}\sin\alpha=\frac{a}{3\frac{\pi}{6}}\times\frac{1}{2}=\frac{a}{\pi}$	B2,1,0
	Taking moments: $\left(\sqrt3 a^2-2\times\frac{\pi a^2}{6}\right)D=\sqrt3 a^2\times\frac{\sqrt3 a}{3}-2\times\frac{\pi a^2}{6}\times\frac{a}{\pi}$	M1A1

Question Number	Scheme	Marks
	$D=\dfrac{\frac{2a^3}{3}}{\left(\sqrt{3}-\frac{\pi}{3}\right)a^2}=\dfrac{2a}{3\sqrt{3}-\pi}$ **	A1 (6) **12**

Question Number	Scheme	Marks
7(a)	Use of $T=\frac{\lambda x}{a}=mg$	M1
	$T=\frac{24.5x}{0.75}=0.5g$	A1
	$x=\frac{0.75\times 0.5g}{24.5}=0.15$, $AE=0.75+0.15=0.9$ (m) **(**)**	A1
		(3)
(b)	Using gain in EPE = loss in GPE	M1
	$\frac{\lambda x^2}{2a}=\frac{24.5x^2}{1.5}=.....$	A1
	….. = 0.5g(0.75 + x)	A1
	Form quadratic in x and attempt to solve for x :	DM1
	$24.5x^2=5.5125+7.35x$, $24.5x^2-7.35x-5.5125=0$, $x=\frac{7.35\pm\sqrt{7.35^2+4\times 24.5\times 5.5125}}{49}$ (or $40x^2-12x-9=0$, $x=\frac{12\pm\sqrt{144+3600}}{80}$)	
	$x=0.647...$ (m) $AC\approx 1.4$ (m)	A1
		(5)
(c)	Using $F = ma$ and displacement x from E:	M1
	$0.5g-\frac{24.5(x+0.15)}{0.75}=0.5\ddot{x}$	A2,1,0
	$\ddot{x}=-\frac{196}{3}x$, so SHM	A1
		(4)
(d)	Max speed = their a x their ω	M1
	$=(0.647-0.15)\times\sqrt{\frac{196}{3}}$	
	≈ 4.0 ms^{-1} (4.02)	A1
		(2)
		14