Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Thursday 14 May 2015 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 4 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P44829RA

1. $$f(x) = 9x^3 - 33x^2 - 55x - 25.$$

Given that $x = 5$ is a solution of the equation $f(x) = 0$, use an algebraic method to solve $f(x) = 0$ completely.

(5)

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2. In the interval $13 < x < 14$, the equation

$$3 + x \sin \frac{x}{4} = 0, \qquad \text{where } x \text{ is measured in radians,}$$

has exactly one root, α.

(*a*) Starting with the interval [13, 14], use interval bisection twice to find an interval of width 0.25 which contains α.

(3)

(*b*) Use linear interpolation once on the interval [13, 14] to find an approximate value for α. Give your answer to 3 decimal places.

(4)

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3. (*a*) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that

$$\sum_{r=1}^{n} (r+1)(r+4) = \frac{n}{3}(n+4)(n+5)$$

for all positive integers n.

(5)

(*b*) Hence show that

$$\sum_{r=n+1}^{2n} (r+1)(r+4) = \frac{n}{3}(n+1)(an+b)$$

where a and b are integers to be found.

(3)

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4. $$z_1 = 3\text{i} \text{ and } z_2 = \frac{6}{1+\text{i}\sqrt{3}}.$$

(*a*) Express z_2 in the form $a + \text{i}b$, where a and b are real numbers.

(2)

(*b*) Find the modulus and the argument of z_2, giving the argument in radians in terms of π.

(4)

(*c*) Show the three points representing z_1, z_2 and $(z_1 + z_2)$ respectively, on a single Argand diagram.

(2)

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5. The rectangular hyperbola H has equation $xy = 9$.

The point A on H has coordinates $\left(6, \frac{3}{2}\right)$.

(*a*) Show that the normal to H at the point A has equation

$$2y - 8x + 45 = 0.$$

(5)

The normal at A meets H again at the point B.

(*b*) Find the coordinates of B.

(4)

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6. (i) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^n - 1) & 5^n \end{pmatrix}.$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(4n^2 - 1).$$

(6)

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7. (i) $$\mathbf{A} = \begin{pmatrix} 5k & 3k-1 \\ -3 & k+1 \end{pmatrix}, \quad \text{where } k \text{ is a real constant.}$$

Given that **A** is a singular matrix, find the possible values of *k*.

(4)

(ii) $$\mathbf{B} = \begin{pmatrix} 10 & 5 \\ -3 & 3 \end{pmatrix}$$

A triangle *T* is transformed onto a triangle *T′* by the transformation represented by the matrix **B**.

The vertices of triangle *T′* have coordinates (0, 0), (−20, 6) and $(10c, 6c)$, where *c* is a positive constant.

The area of triangle *T′* is 135 square units.

(*a*) Find the matrix $\mathbf{B}^{-1}$.

(2)

(*b*) Find the coordinates of the vertices of the triangle *T*, in terms of *c* where necessary.

(3)

(*c*) Find the value of *c*.

(3)

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8. The point $P(3p^2, 6p)$ lies on the parabola with equation $y^2 = 12x$ and the point *S* is the focus of this parabola.

(*a*) Prove that $SP = 3(1 + p^2)$.

(3)

The point $Q(3q^2, 6q)$, $p \neq q$, also lies on this parabola.

The tangent to the parabola at the point *P* and the tangent to the parabola at the point *Q* meet at the point *R*.

(*b*) Find the equations of these two tangents and hence find the coordinates of the point *R*, giving the coordinates in their simplest form.

(8)

(*c*) Prove that $SR^2 = SP.\ SQ$

(3)

TOTAL FOR PAPER: 75 MARKS

END

June 2015
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme	Marks
1.	$(x-5)$ is a factor of f(x) so f(x) = $(x-5)(9x^2 ... \quad)$	M1
	f(x) = $(x-5)(9x^2+12x+5)$	A1
	Solve $(9x^2+12x+5)=0$ to give $x=$	M1
	$(x=)-\frac{2}{3}-\frac{1}{3}\mathrm{i},\ -\frac{2}{3}+\frac{1}{3}\mathrm{i}$ or $-\frac{2}{3}\pm\frac{1}{3}\mathrm{i}$ or $\frac{-2\pm\mathrm{i}}{3}$ oe (and 5)	A1cao A1ft (5) **(5 marks)**
	Notes M1: Uses (x-5) as factor and begins division or process to obtain quadratic with $9x^2$. Award if no working but quadratic factor completely correct. A1: $9x^2+12x+5$ M1: Solves their quadratic by usual rules leading to $x=$ Award if one complex root correct with no working. Award for $(9x^2+...$ incorrectly factorised to $(3x+p)(3x+q)$, where $\lvert pq\rvert=5$ A1: One correct complex root. Accept any exact equivalent form. Accept single fraction and ± A1ft: Conjugate of their first complex root.	

Question Number	Scheme	Marks
2. (a)	Let f(x) = $3+x\sin\left(\frac{x}{4}\right)$ then f (13)= 1.593 [and f(14) = -1.911 need not be seen in (a)]	
	f(13.5) = -0.122, so root in [13, 13.5]	M1 A1
	f(13.25) = 0.746 so root in [13.25, 13.5]	A1 (3)
(b)	$\frac{\alpha-13}{14-\alpha}=\frac{1.593}{1.911}$ or $\frac{\alpha-13}{1}=\frac{1.593}{1.593+1.911}$	M1 A1
	So $\alpha(1.911+1.593)=1.593\times14+13\times1.911$ and $\alpha=\frac{47.145}{3.504}=13.455$	dM1 A1 (4) **(7 marks)**

Notes

(a)
M1: Evaluate f(13) and f(13.5) giving at least positive, negative **OR** evaluate f(13.5) and f(13.25) to give at least negative, positive. Do not award if using degrees.
A1: f(13.5) = awrt -0.1, f(13.25) = awrt 0.7(5).
A1: Correct interval [13.25, 13.5] or equivalent form with or without boundaries.

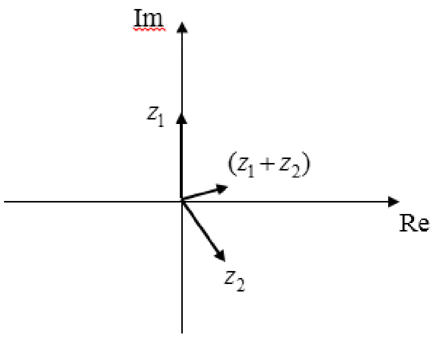
(b)
M1: Attempt at linear interpolation on either side of equation with correct signs.
A1: Correct equivalent statement
dM1: Makes alpha subject of formula
A1: cao. Award A0 for 13.456 and 13.454

ALT (b)
Using equation of line
M1: Attempt to find gradient $\frac{y_1-y_0}{x_1-x_0}=\frac{-1.911-1.593}{14-13}(=-3.504...)$, attempt to use $y-y_0=m(x-x_0)$ with either 13 or 14 (gives y = -3.504x + 47.145) and substitute $y=0$
A1: Correct statement after substituting $y=0$ in their equation i.e. 0 = -3.504x + 47.145
dM1: Makes x the subject of the formula
A1: cao. Award A0 for 13.456 and 13.454

Question Number	Scheme	Marks
3. (a)	$\sum_{r=1}^{n}(r+1)(r+4)$	
	$=\sum_{r=1}^{n} r^2+5r+4$	B1
	$=\frac{n}{6}(n+1)(2n+1)+5\frac{n}{2}(n+1)+4n$	M1 A1
	$=\frac{n}{6}\{(n+1)(2n+1)+15(n+1)+24\}$	dM1
	$=\frac{n}{6}\{(2n^2+3n+1)+15n+15+24\}$	
	$=\frac{n}{6}\left(2n^2+18n+40\right)$ or $=\frac{n}{3}\left(n^2+9n+20\right)$	
	$=\frac{n}{3}(n+4)(n+5)$ **** given answer****	A1*
		(5)
(b)	$\sum_{r=n+1}^{2n}(r+1)(r+4) \quad =\frac{2n}{3}(2n+4)(2n+5)-\frac{n}{3}(n+4)(n+5)$	M1
	$=\frac{n}{3}\{8n^2+36n+40-n^2-9n-20\}$	dM1
	$=\frac{n}{3}\{7n^2+27n+20\}=\frac{n}{3}(n+1)(7n+20)$ or a = 7, b = 20	A1
		(3)
		(8 marks)

Notes

(a)
B1: Expands bracket correctly to r^2+5r+4
M1: Uses $\frac{n}{6}(n+1)(2n+1)$ or $\frac{n}{2}(n+1)$ correctly.
A1: Completely correct expression.
dM1: Attempts to remove factor $\frac{n}{6}$ or $\frac{n}{3}$ to obtain a quadratic factor. Need not be 3 term.
A1: Completely correct work including a step with a collected **3 term** quadratic prior in the bracket with correct printed answer.
Accept approach which starts with LHS and then RHS which meet at $\frac{n^3}{3}+3n^2+\frac{20n}{3}$. Award marks as above.
NB If induction attempted then typically this may only score the first B1.
However, consider the solution carefully and award as above if seen in the body of the induction attempt.
(b) M1: Uses f(2n) – f(n) or f(2n) – f(n+1) correctly. Require all 3 terms in 2n (and n+1 if used).
dM1: Attempts to remove factor $\frac{n}{6}$ or $\frac{n}{3}$ to obtain a quadratic factor. Need not be 3 term.
A1: Either in expression or as above.

Question Number	Scheme	Marks
4. (a)	$z_2 = \dfrac{6(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})} = \dfrac{6(1-i\sqrt{3})}{4}$	M1
	$z_2 = \dfrac{6(1-i\sqrt{3})}{4}\left(=\dfrac{3}{2}-i\dfrac{3}{2}\sqrt{3}\right)$	A1 (2)
(b)	$\lvert z_2\rvert = \sqrt{\left(\dfrac{3}{2}\right)^2+\left(\dfrac{3\sqrt{3}}{2}\right)^2} = \sqrt{\dfrac{9}{4}+\dfrac{27}{4}}$	M1
	The modulus of z_2 is 3	A1
	$\tan\theta = (\pm)\sqrt{3}$ and attempts to find θ	M1
	and the argument is $-\dfrac{\pi}{3}$	A1 (4)
(c)		M1 A1 (2)
		(8 marks)

Notes

(a) M1: Multiplies numerator and denominator by $1-i\sqrt{3}$
A1: any correct equivalent with real denominator.
(b) M1: Uses correct method for modulus for their z_2 in part (a)
A1: for 3 only
M1: Uses tan or inverse tan
A1: $-\dfrac{\pi}{3}$ accept $\dfrac{5\pi}{3}$
NB Answers only then award 4/4 but arg must be in terms of π
(c) M1:
Either z_1 on imaginary axis and labelled with z_1 or 3i or (0,3) or axis labelled 3;
or their z_2 in the correct quadrant labelled z_2 or $\dfrac{3}{2}-i\dfrac{3}{2}\sqrt{3}$ or $\left(\dfrac{3}{2},-\dfrac{3}{2}\sqrt{3}\right)$ or axes labelled or their $a+bi$ or their (a,b) or axes labelled.
Axes need not be labelled Re and Im.
A1: All 3 correct ie z_1 on positive imaginary axis, z_2 in 4th quadrant and z_1+z_2 in the first quadrant.
Accept points or lines. Arrows not required.

Question Number	Scheme	Marks
5. (a)	$\frac{dy}{dx} = -\frac{9}{x^2}$ or $\frac{dy}{dx} = -\frac{y}{x}$ or $\frac{dy}{dx} = -\frac{1}{t^2}$	M1
	so gradient at $x = 6$ or $t = 2$ is $-\frac{9}{36}$ or $-\frac{\frac{3}{2}}{6}$ or $-\frac{1}{4}$ o.e.	A1
	Gradient of normal is $-\frac{1}{m}$ $(= 4)$	M1
	Equation of normal is $y - \frac{3}{2} = 4(x-6)$	dM1
	So $2y - 8x + 45 = 0$ ****given answer****	A1 * (5)
(b)	$\frac{18}{x} - 8x + 45 = 0$ or $2y - \frac{72}{y} + 45 = 0$ or $x(4x - 22.5) = 9$ or $y\left(\frac{y}{4} + \frac{45}{8}\right) = 9$ o.e.	M1
	$8x^2 - 45x - 18 = 0$ or $2y^2 + 45y - 72 = 0$	
	So $x = -\frac{3}{8}$ or $y = -24$	A1
	Finds other ordinate: $\left(-\frac{3}{8}, -24\right)$	M1 A1
ALT	Sub $\left(3t, \frac{3}{t}\right)$ in $2y - 8x + 45 = 0 \Rightarrow t = -\frac{1}{8}$	M1A1
	Sub $t = -\frac{1}{8}$ in $\left(3t, \frac{3}{t}\right) \Rightarrow \left(-\frac{3}{8}, -24\right)$	M1A1
		(4) **(9 marks)**

Notes

(a) M1: Differentiates to obtain $\frac{k}{x^2}$ and substitutes $x = 6$

or uses implicit differentiation $\frac{dy}{dx} = -\frac{y}{x}$ and substitutes x and y

or uses parametric differentiation $\frac{dy}{dx} = -\frac{1}{t^2}$ and substitutes $t = 2$

A1: For grad of tangent – accept any equivalent i.e. - 0.25 etc

M1: Uses negative reciprocal of their gradient.

dM1: $y - y_1 = m(x - x_1)$ with $\left(6, \frac{3}{2}\right)$ or $y = mx + c$ and sub $\left(6, \frac{3}{2}\right)$ to find c=.

A1: cso: Correct answer with no errors seen in the solution.

(b) M1: Obtains equation in one variable, x or y

A1: Correct value of x or correct value of y

M1: Finds second coordinate using $xy = 9$ or solving second quadratic or equation of the normal

A1: Correct coordinates that can be written as $x = ..., y = ...$

Question Number	Scheme	Marks
6. (i)	If $n = 1$, $\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^1 = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^1-1) & 5^1 \end{pmatrix}$ so **true** for ***n* = 1**	B1
	Assume result true for $n = k$ $\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k-1) & 5^k \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k-1) & 5^k \end{pmatrix}$	M1
	$\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k-1)-5^k & 5\times 5^k \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ -1-5.\frac{1}{4}(5^k-1) & 5\times 5^k \end{pmatrix}$	M1 A1
	$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}5^k+\frac{1}{4}-5^k & 5^{k+1} \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ -1-\frac{1}{4}5^{k+1}+\frac{5}{4} & 5^{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^{k+1}-1) & 5^{k+1} \end{pmatrix}$	A1
	True for ***n* = *k* + 1** if **true** for ***n* = *k***, (and **true** for ***n* = 1**) so **true** by induction for all $\boldsymbol{n} \in \boldsymbol{Z}^+$ **.**	A1cso (6)
(ii)	If $n = 1$, $\sum_{r=1}^{n}(2r-1)^2 = 1$ and $\frac{1}{3}n(4n^2-1) = 1$, so **true** for ***n* = 1**.	B1
	Assume result true for $n = k$ so $\sum_{r=1}^{k+1}(2r-1)^2 = \frac{1}{3}k(4k^2-1)+(2(k+1)-1)^2$	M1
	$= \sum_{r=1}^{k+1}(2r-1)^2 = \frac{1}{3}(2k+1)\{(2k^2-k)+(3(2k+1))\}$	M1 A1
	$= \frac{1}{3}(2k+1)\{(2k^2+5k+3)\}$ or $\frac{1}{3}(k+1)(4k^2+8k+3)$ or $\frac{1}{3}((2k+3)(2k^2+3k+1)\}$	
	$= \frac{1}{3}(k+1)(2k+1)(2k+3) \quad = \frac{1}{3}(k+1)(4(k+1)^2-1)$	dA1
	True for ***n* = *k* + 1** if **true** for ***n* = *k***, (and **true** for ***n* = 1**) so **true** by induction for all $\boldsymbol{n} \in \boldsymbol{Z}^+$	A1cso (6) **12 marks**

Notes

(i) B1: Checks $n = 1$ on both sides and states true for $n = 1$ seen anywhere.
M1: Assumes true for $n = k$ and indicates intention to multiply power k by power 1 either way around.
M1: Multiplies matrices. Condone one slip. A1: Correct unsimplified matrix
A1: Intermediate step required cao
A1: cso Makes correct induction statement including at least statements in bold.
Statement **true** for ***n* = 1** here could contribute to B1 mark earlier.
(ii) B1: Checks $n = 1$ on both sides and states true for $n = 1$ seen anywhere.
M1: Assumes true for $n = k$ and adds $(k+1)^{th}$ term to sum of k terms. Accept $4(k+1)^2-4(k+1)+1$ or $(2k+1)^2$ for $(k+1)^{th}$ term. M1: Factorises out a linear factor of the three possible - usually $2k+1$
A1: Correct expression with one linear and one quadratic factor.
dA1: Need to see $\frac{1}{3}(k+1)(4(k+1)^2-1)$ somewhere dependent upon previous A1.

Accept assumption plus $(k+1)^{th}$ term and $\frac{1}{3}(k+1)(4(k+1)^2-1)$ both leading to $\frac{1}{3}(4k^3+12k^2+11k+3)$ then award for expressions seen as above.
A1: cso Makes correct complete induction statement including at least statements in bold. Statement true for $n = 1$ here could contribute to B1 mark earlier.

Question Number	Scheme	Marks
7. (i)	$5k(k+1)$ - $-3(3k-1)=0$	M1
	$5k^2+5k+9k-3=0$	A1
	$(5k-1)(k+3)=0$ so $k=$	M1
	$k=\frac{1}{5}$ or -3	A1
		(4)
(ii)(a)	$\mathbf{B}^{-1}=\frac{1}{45}\begin{pmatrix}3 & -5\\ 3 & 10\end{pmatrix}$	M1 A1 (2)
(b)	$\frac{1}{45}\begin{pmatrix}3 & -5\\ 3 & 10\end{pmatrix}\begin{pmatrix}0 & -20 & 10c\\ 0 & 6 & 6c\end{pmatrix}=$	M1
	$\frac{1}{45}\begin{pmatrix}0 & -90 & 0\\ 0 & 0 & 90c\end{pmatrix}$	
	Vertices at (0, 0) (-2, 0) (0, 2c)	A1,A1
		(3)
ALT	$\begin{pmatrix}10 & 5\\ -3 & 3\end{pmatrix}\begin{pmatrix}a & d & f\\ b & e & g\end{pmatrix}=\begin{pmatrix}0 & -20 & 10c\\ 0 & 6 & 6c\end{pmatrix}$ and attempt to form simultaneous equations	M1
	$10a+5b=0, -3a+3b=0$	
	$10d+5e=-20, -3d+3e=6$ all correct oe	A1
	$10f+5g=10c, -3f+3g=6c$	
	Vertices at (0, 0) (-2, 0) (0, 2c)	A1
(c)	Area of T is $\frac{1}{2}\times 2\times 2c=2c$ — OR Area of $T'=\frac{1}{2}\begin{vmatrix}0 & -20 & 10c & 0\\ 0 & 6 & 6c & 0\end{vmatrix}=90c$	B1
	Area of $T\times$determinant $=135$ — OR Their area = 135	M1
	So $c=\frac{3}{2}$ — OR So $c=\frac{3}{2}$	A1 (3)
		(12 marks)

Notes

(i) M1: Puts determinant equal to zero
A1: cao as three or four term quadratic
M1: Solve their quadratic to find k
A1: cao – need both correct answers

(ii) (a) M1 Uses correct method for inverse with fraction $\frac{1}{45}$ or $\frac{1}{\text{their det}}$

A1: All correct oe
(b) M1: Post multiplies their inverse by **2 by 3 matrix or 2 by 2 matrix excluding the origin** or does not use inverse and attempts to form simultaneous equations. Can exclude origin.
A1: (-2,0) and (0,2c).Can be written as column vectors. Accept seen in final two columns of single matrix
A1: (0,0). Can be written as column vectors. Award if seen as first column of single matrix.
(c) B1: Area of T given as 2c or area of $T'=90c$ Accept $\pm$
M1: Either method using their area of T and their det or their area of T'
A1: $c=\frac{3}{2}$ cao

Question Number	Scheme	Marks
8(a)	$SP=\sqrt{(3p^2-a)^2+36p^2}$, with $a = 3$	M1, B1
	$SP=\sqrt{9p^4+18p^2+9}$ $= 3(1+p^2)$ ****given answer****	A1 *
		(3)
ALT	For parabola, perpendicular distance from *P* to directrix = *SP*	M1
	Directrix $x=-3$	B1
	So $SP=3+3p^2=3(1+p^2)$	A1
(b)		
	$y^2=12x\Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}=12$ or $y=\sqrt{12x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{3}x^{-\frac{1}{2}}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}p}}{\frac{\mathrm{d}x}{\mathrm{d}p}}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}q}}{\frac{\mathrm{d}x}{\mathrm{d}q}}$	M1
	The tangent at *P* has gradient $=\frac{1}{p}$ or the tangent at *Q* has gradient $\frac{1}{q}$	A1
	and equation is $y-6p=\frac{1}{p}(x-3p^2)$ or $py=x+3p^2$ o.e.	A1
	Tangent at *Q* is $y-6q=\frac{1}{q}(x-3q^2)$ or $qy=x+3q^2$ o.e.	B1
	Eliminate *x* or *y:* So $x=3pq$ or $y=3(p+q)=3p+3q$	M1 A1
	Substitute for second variable so $x=3pq$ and $y=3(p+q)=3p+3q$	M1 A1
		(8)
(c)	$SR^2=(3-3pq)^2+(3p+3q)^2(=9+9p^2q^2+9p^2+9q^2)$	M1
	$SP.SQ=3(1+p^2)\ 3(1+q^2)$ $(=9+9p^2q^2+9p^2+9q^2)$	M1
	So $SR^2=SP.SQ$ as required	A1
		(3)
		(14 marks)

Notes

(a) M1: Uses distance between two points or states perpendicular distance from *P* to directrix required.
B1: States or uses focus at (3,0) or focus at $a=3$ or directrix as $x = -3$
A1: cso
(b) M1: Calculus method for finding gradient and substitutes *x* value at either point
A1: Either correct. Accept unsimplified.
A1: One equation of tangent correct. B1: Both correct
M1: Eliminate *x* or *y*. A1: Obtain first variable
M1: Substitute or eliminate again. A1: Both variables correct in simplest form as above.
(c) M1:Find their $SR^2=(3-3pq)^2+(3p+3q)^2\ (=9+9p^2q^2+9p^2+9q^2)$
M1: Find their $SP.SQ=3(1+p^2)\ 3(1+q^2)$ $(=9+9p^2q^2+9p^2+9q^2)$
A1: Deduce equal after no errors seen. Concluding statement required cso.