Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary

Friday 1 June 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. $$f(x) = 2x^3 - 6x^2 - 7x - 4.$$

(*a*) Show that f(4) = 0.

(1)

(*b*) Use algebra to solve f(*x*) = 0 completely.

(4)

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2. (*a*) Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix},$$

find **AB**.

(2)

(*b*) Given that

$$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

and

$$\mathbf{E} = \mathbf{C} + \mathbf{D},$$

find the value of *k* for which **E** has no inverse.

(4)

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3. $$f(x) = x^2 + \frac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0.$$

A root α of the equation f(*x*) = 0 lies in the interval [3, 5].

Taking 4 as a first approximation to α, apply the Newton-Raphson process once to f(*x*) to obtain a second approximation to α. Give your answer to 2 decimal places.

(6)

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4. (*a*) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n}(r^3 + 6r - 3) = \frac{1}{4}n^2(n^2 + 2n + 13)$$

for all positive integers n.

(5)

(*b*) Hence find the exact value of

$$\sum_{r=16}^{30}(r^3 + 6r - 3).$$

(2)

5.

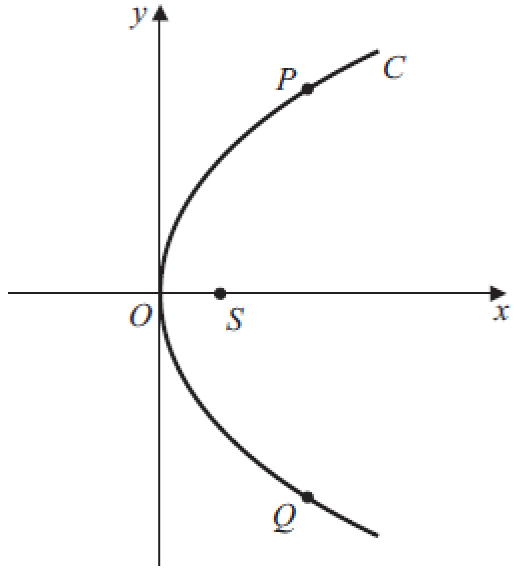

Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 8x$.
The point P lies on C, where $y > 0$, and the point Q lies on C, where $y < 0$.
The line segment PQ is parallel to the y-axis.

Given that the distance PQ is 12,

(*a*) write down the y-coordinate of P,

(1)

(*b*) find the x-coordinate of P.

(2)

Figure 1 shows the point S which is the focus of C.

The line l passes through the point P and the point S.

(*c*) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

6. $$\mathrm{f}(x) = \tan\left(\frac{x}{2}\right) + 3x - 6, \quad -\pi < x < \pi.$$

(*a*) Show that the equation $\mathrm{f}(x) = 0$ has a root α in the interval [1, 2].

(2)

(*b*) Use linear interpolation once on the interval [1, 2] to find an approximation to α.
Give your answer to 2 decimal places.

(3)

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7. $$z = 2 - \mathrm{i}\sqrt{3}.$$

(*a*) Calculate arg z, giving your answer in radians to 2 decimal places.

(2)

Use algebra to express

(*b*) $z + z^2$ in the form $a + b\mathrm{i}\sqrt{3}$, where a and b are integers,

(3)

(*c*) $\dfrac{z+7}{z-1}$ in the form $c + d\mathrm{i}\sqrt{3}$, where c and d are integers.

(4)

Given that

$$w = \lambda - 3\mathrm{i},$$

where λ is a real constant, and $\arg(4 - 5\mathrm{i} + 3w) = -\dfrac{\pi}{2}$,

(*d*) find the value of λ.

(2)

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8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on H.

(*a*) Show that an equation for the tangent to H at P is

$$x + t^2 y = 2ct.$$

(4)

The tangent to H at the point P meets the x-axis at the point A and the y-axis at the point B.

Given that the area of the triangle OAB, where O is the origin, is 36,

(*b*) find the exact value of c, expressing your answer in the form $k\sqrt{2}$, where k is an integer.

(4)

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9. $$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}.$$

(*a*) Find det **M**.

(1)

The transformation represented by **M** maps the point $S(2a - 7, a - 1)$, where a is a constant, onto the point $S'(25, -14)$.

(*b*) Find the value of a.

(3)

The point R has coordinates (6, 0).

Given that O is the origin,

(*c*) find the area of triangle ORS.

(2)

Triangle ORS is mapped onto triangle $OR'S'$ by the transformation represented by **M**.

(*d*) Find the area of triangle $OR'S'$.

(2)

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(*e*) describe fully the single geometrical transformation represented by **A**.

(2)

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by **M**.

(*f*) Find **B**.

(4)

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10. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1}$$

is divisible by 5.

(6)

TOTAL FOR PAPER: 75 MARKS

END