Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/ Advanced Subsidiary

Wednesday 22 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P38168A

1. $$f(x) = 3^x + 3x - 7$$

(*a*) Show that the equation f $(x) = 0$ has a root α between $x = 1$ and $x = 2$.

(2)

(*b*) Starting with the interval [1, 2], use interval bisection twice to find an interval of width 0.25 which contains α.

(3)

---

2. $$z_1 = -2 + i$$

(*a*) Find the modulus of z_1.

(1)

(*b*) Find, in radians, the argument of z_1, giving your answer to 2 decimal places.

(2)

The solutions to the quadratic equation

$$z^2 - 10z + 28 = 0$$

are z_2 and z_3.

(*c*) Find z_2 and z_3, giving your answers in the form $p \pm i\sqrt{q}$, where p and q are integers.

(3)

(*d*) Show, on an Argand diagram, the points representing your complex numbers z_1, z_2 and z_3.

(2))

---

3. (*a*) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix},$$

(i) find $\mathbf{A}^2$,

(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$.

(4)

(*b*) Given that

$$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

describe fully the geometrical transformation represented by **B**.

(2)

(*c*) Given that

$$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix},$$

where k is a constant, find the value of k for which the matrix **C** is singular.

(3)

---

4. $$\mathrm{f}(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0.$$

(*a*) Use differentiation to find $\mathrm{f}'(x)$.

(2)

The root α of the equation $\mathrm{f}(x) = 0$ lies in the interval [0.7, 0.9].

(*b*) Taking 0.8 as a first approximation to α, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

---

5. $$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that the matrix **A** maps the point with coordinates (4, 6) onto the point with coordinates (2, −8),

(*a*) find the value of *a* and the value of *b*.

(4)

A quadrilateral *R* has area 30 square units.
It is transformed into another quadrilateral *S* by the matrix **A**.
Using your values of *a* and *b*,

(*b*) find the area of quadrilateral *S*.

(4)

---

6. Given that $z = x + \mathrm{i}y$, find the value of *x* and the value of *y* such that

$$z + 3\mathrm{i}z^* = -1 + 13\mathrm{i}$$

where z^* is the complex conjugate of *z*.

(7)

---

7. (*a*) Use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(2n+1)(2n-1)$$

for all positive integers *n*.

(6)

(*b*) Hence show that

$$\sum_{r=n+1}^{3n} (2r-1)^2 = \frac{2}{3}n(an^2+b)$$

where *a* and *b* are integers to be found.

(4)

---

8. The parabola C has equation $y^2 = 48x$.

The point $P(12t^2, 24t)$ is a general point on C.

(*a*) Find the equation of the directrix of C.

(2)

(*b*) Show that the equation of the tangent to C at $P(12t^2, 24t)$ is

$$x - ty + 12t^2 = 0.$$

(4)

The tangent to C at the point (3, 12) meets the directrix of C at the point X.

(*c*) Find the coordinates of X.

(4)

---

9. Prove by induction, that for $n \in \mathbb{Z}^+$,

(*a*) $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$

(6)

(*b*) $\mathrm{f}(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

TOTAL FOR PAPER: 75 MARKS

END