

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 31 January 2011 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 10 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

1. $z = 5 - 3i, \quad w = 2 + 2i$

Express in the form $a + bi$, where a and b are real constants,

(a) z^2 , (2)

(b) $\frac{z}{w}$. (3)

2. $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$

(a) Find \mathbf{AB} . (3)

Given that

$$\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) describe fully the geometrical transformation represented by \mathbf{C} , (2)

(c) write down \mathbf{C}^{100} . (1)

3. $f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geq 0.$

The root α of the equation $f(x) = 0$ lies in the interval $[1.6, 1.8]$.

(a) Use linear interpolation once on the interval $[1.6, 1.8]$ to find an approximation to α .
Give your answer to 3 decimal places. (4)

(b) Differentiate $f(x)$ to find $f'(x)$. (2)

(c) Taking 1.7 as a first approximation to α , apply the Newton-Raphson process once to $f(x)$
to obtain a second approximation to α . Give your answer to 3 decimal places. (4)

4. Given that $2 - 4i$ is a root of the equation

$$z^2 + pz + q = 0,$$

where p and q are real constants,

- (a) write down the other root of the equation,

(1)

- (b) find the value of p and the value of q .

(3)

5. (a) Use the results for $\sum_{r=1}^n r$, $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r^3$, to prove that

$$\sum_{r=1}^n r(r+1)(r+5) = \frac{1}{4}n(n+1)(n+2)(n+7)$$

for all positive integers n .

(5)

- (b) Hence, or otherwise, find the value of

$$\sum_{r=20}^{50} r(r+1)(r+5).$$

(2)

6.

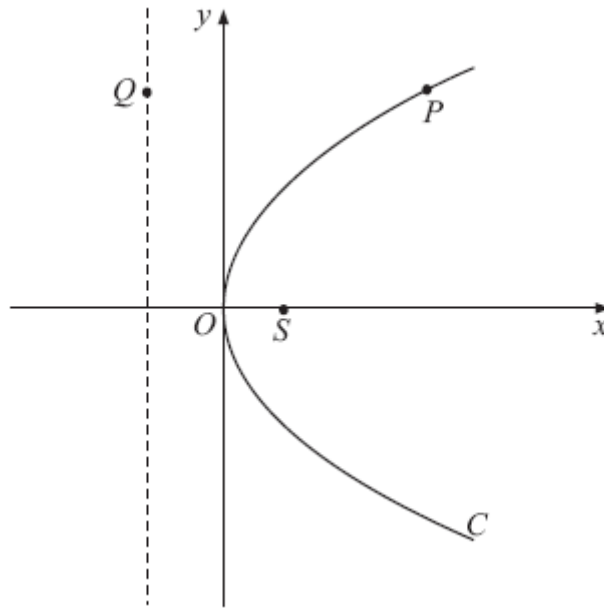


Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 36x$.
The point S is the focus of C .

- (a) Find the coordinates of S . (1)
- (b) Write down the equation of the directrix of C . (1)

Figure 1 shows the point P which lies on C , where $y > 0$, and the point Q which lies on the directrix of C . The line segment QP is parallel to the x -axis.

Given that the distance PS is 25,

- (c) write down the distance QP , (1)
- (d) find the coordinates of P , (3)
- (e) find the area of the trapezium $OSPQ$. (2)
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7. $z = -24 - 7i$

(a) Show z on an Argand diagram. (1)

(b) Calculate $\arg z$, giving your answer in radians to 2 decimal places. (2)

It is given that

$$w = a + bi, \quad a \in \mathbb{R}, \quad b \in \mathbb{R}.$$

Given also that $|w| = 4$ and $\arg w = \frac{5\pi}{6}$,

(c) find the values of a and b , (3)

(d) find the value of $|zw|$. (3)

8. $\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$

(a) Find $\det \mathbf{A}$. (1)

(b) Find \mathbf{A}^{-1} . (2)

The triangle R is transformed to the triangle S by the matrix \mathbf{A} .
Given that the area of triangle S is 72 square units,

(c) find the area of triangle R . (2)

The triangle S has vertices at the points $(0, 4)$, $(8, 16)$ and $(12, 4)$.

(d) Find the coordinates of the vertices of R . (4)

9. A sequence of numbers $u_1, u_2, u_3, u_4, \dots$, is defined by

$$u_{n+1} = 4u_n + 2, \quad u_1 = 2.$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{2}{3}(4^n - 1).$$

(5)

10. The point $P\left(6t, \frac{6}{t}\right)$, $t \neq 0$, lies on the rectangular hyperbola H with equation $xy = 36$.

(a) Show that an equation for the tangent to H at P is

$$y = -\frac{1}{t^2}x + \frac{12}{t}.$$

(5)

The tangent to H at the point A and the tangent to H at the point B meet at the point $(-9, 12)$.

(b) Find the coordinates of A and B .

(7)

TOTAL FOR PAPER: 75 MARKS

END

January 2011
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme	Marks
<p>1.</p> <p>(a)</p>	$z = 5 - 3i, w = 2 + 2i$ $z^2 = (5 - 3i)(5 - 3i)$ $= 25 - 15i - 15i + 9i^2$ $= 25 - 15i - 15i - 9$ $= 16 - 30i$	<p>An attempt to multiply out the brackets to give four terms (or four terms implied). zw is M0</p> <p>M1</p> <p>16 - 30i A1</p> <p>Answer only 2/2 (2)</p>
<p>(b)</p>	$\frac{z}{w} = \frac{(5 - 3i)}{(2 + 2i)}$ $= \frac{(5 - 3i)}{(2 + 2i)} \times \frac{(2 - 2i)}{(2 - 2i)}$ $= \frac{10 - 10i - 6i - 6}{4 + 4}$ $= \frac{4 - 16i}{8}$ $= \frac{1}{2} - 2i$	<p>Multiplies $\frac{z}{w}$ by $\frac{(2 - 2i)}{(2 - 2i)}$</p> <p>M1</p> <p>Simplifies realising that a real number is needed on the denominator and applies $i^2 = -1$ on their numerator expression and denominator expression.</p> <p>M1</p> <p>$\frac{1}{2} - 2i$ or $a = \frac{1}{2}$ and $b = -2$ or equivalent A1</p> <p>Answer as a single fraction A0</p> <p>(3) [5]</p>

Question Number	Scheme	Marks
2. (a)	$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$ $\mathbf{AB} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$ $= \begin{pmatrix} 2(-3) + 0(5) & 2(-1) + 0(2) \\ 5(-3) + 3(5) & 5(-1) + 3(2) \end{pmatrix}$ $= \begin{pmatrix} -6 & -2 \\ 0 & 1 \end{pmatrix}$	<p>A correct method to multiply out two matrices. Can be implied by two out of four correct elements. M1</p> <p>Any three elements correct A1</p> <p>Correct answer A1</p> <p>Correct answer only 3/3 (3)</p>
(b)	Reflection; about the y -axis.	<p><u>Reflection</u> <u>y-axis</u> (or $x = 0$.) M1</p> <p>A1</p> <p>(2)</p>
(c)	$\mathbf{C}^{100} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	<p>$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or \mathbf{I} B1</p> <p>(1)</p> <p>[6]</p>

Question Number	Scheme	Marks
<p>3.</p> <p>(a)</p>	$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geq 0$ $f(1.6) = -1.29543081\dots$ $f(1.8) = 0.5401863372\dots$ $\frac{\alpha - 1.6}{\text{"1.29543081..."}} = \frac{1.8 - \alpha}{\text{"0.5401863372..."}}$ $\alpha = 1.6 + \left(\frac{\text{"1.29543081..."}}{\text{"0.5401863372..." + "1.29543081..."}} \right) 0.2$ $= 1.741143899\dots$	<p>awrt -1.30 B1</p> <p>awrt 0.54 B1</p> <p>Correct linear interpolation method with signs correct. Can be implied by working below. M1</p> <p>awrt 1.741 A1</p> <p>Correct answer seen 4/4 (4)</p>
(b)	$f'(x) = 10x - 6x^{\frac{1}{2}}$	<p>At least one of $\pm ax$ or $\pm bx^{\frac{1}{2}}$ correct. M1</p> <p>Correct differentiation. A1</p> <p>(2)</p>
(c)	$f(1.7) = -0.4161152711\dots$ $f'(1.7) = 9.176957114\dots$ $\alpha_2 = 1.7 - \left(\frac{\text{"-0.4161152711..."}}{\text{"9.176957114..."}} \right)$ $= 1.745343491\dots$ $= 1.745 \text{ (3dp)}$	<p>$f(1.7) =$ awrt -0.42 B1</p> <p>$f'(1.7) =$ awrt 9.18 B1</p> <p>Correct application of Newton-Raphson formula using their values. M1</p> <p>1.745 A1 cao</p> <p>Correct answer seen 4/4 (4)</p> <p>[10]</p>

Question Number	Scheme	Marks
4. (a)	$z^2 + pz + q = 0, z_1 = 2 - 4i$ $z_2 = 2 + 4i$	$2 + 4i$ B1 (1)
(b)	$(z - 2 + 4i)(z - 2 - 4i) = 0$ $\Rightarrow z^2 - 2z - 4iz - 2z + 4 - 8i + 4iz - 8i + 16 = 0$ $\Rightarrow z^2 - 4z + 20 = 0$	An attempt to multiply out brackets of two complex factors and no i^2 . Any one of $p = -4, q = 20$. Both $p = -4, q = 20$. $\Rightarrow z^2 - 4z + 20 = 0$ only 3/3 M1 A1 A1 (3) [4]

Question Number	Scheme	Marks
5	<p>(a) $\sum_{r=1}^n r(r+1)(r+5)$</p> <p>$= \sum_{r=1}^n r^3 + 6r^2 + 5r$</p> <p>$= \frac{1}{4}n^2(n+1)^2 + 6 \cdot \frac{1}{6}n(n+1)(2n+1) + 5 \cdot \frac{1}{2}n(n+1)$</p> <hr/> <p>$= \frac{1}{4}n^2(n+1)^2 + n(n+1)(2n+1) + \frac{5}{2}n(n+1)$</p> <p>$= \frac{1}{4}n(n+1)(n(n+1) + 4(2n+1) + 10)$</p> <p>$= \frac{1}{4}n(n+1)(n^2 + n + 8n + 4 + 10)$</p> <p>$= \frac{1}{4}n(n+1)(n^2 + 9n + 14)$</p>	<p>Multiplying out brackets and an attempt to use at least one of the standard formulae correctly. M1</p> <p><u>Correct expression.</u> A1</p> <p>Factorising out at least $n(n+1)$ dM1</p> <p>Correct 3 term quadratic factor A1</p>
	<p>$= \frac{1}{4}n(n+1)(n+2)(n+7) *$</p>	<p>Correct proof. No errors seen. A1</p> <p>(5)</p>
	<p>(b) $S_n = \sum_{r=20}^{50} r(r+1)(r+5)$</p> <p>$= S_{50} - S_{19}$</p> <p>$= \frac{1}{4}(50)(51)(52)(57) - \frac{1}{4}(19)(20)(21)(26)$</p> <p>$= 1889550 - 51870$</p> <p>$= 1837680$</p>	<p>Use of $S_{50} - S_{19}$ M1</p> <p>1837680 A1</p> <p>Correct answer only 2/2</p> <p>(2) [7]</p>

Question Number	Scheme	Marks
6. (a)	$C: y^2 = 36x \Rightarrow a = \frac{36}{4} = 9$ $S(9, 0)$	$(9, 0)$ B1 (1)
(b)	$x + 9 = 0$ or $x = -9$	$x + 9 = 0$ or $x = -9$ or ft using their a from part (a). B1 $\sqrt{\quad}$ (1)
(c)	$PS = 25 \Rightarrow \underline{QP = 25}$	Either 25 by itself or $PQ = 25$. Do not award if just $PS = 25$ is seen. B1 (1)
(d)	x -coordinate of $P \Rightarrow x = 25 - 9 = 16$ $y^2 = 36(16)$ $\underline{y} = \sqrt{576} = \underline{24}$ Therefore $P(16, 24)$	$x = 16$ Substitutes their x -coordinate into equation of C . $\underline{y} = 24$ A1 (3)
(e)	$\text{Area } OSPQ = \frac{1}{2}(9 + 25)24$ $= \underline{408} \text{ (units)}^2$	$\frac{1}{2}(\text{their } a + 25)(\text{their } y)$ or rectangle and 2 distinct triangles, correct for their values. 408 A1 (2) [8]

Question Number	Scheme	Marks
8. (a)	$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$ $\det \mathbf{A} = 2(3) - (-1)(-2) = 6 - 2 = \underline{4}$	$\underline{4}$ B1 (1)
(b)	$\mathbf{A}^{-1} = \frac{1}{4} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	$\frac{1}{\det \mathbf{A}} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$ $\frac{1}{4} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$ M1 A1 (2)
(c)	$\text{Area}(R) = \frac{72}{4} = \underline{18} \text{ (units)}^2$	$\frac{72}{\text{their det } \mathbf{A}} \text{ or } 72 \text{ (their det } \mathbf{A})$ $\underline{18} \text{ or ft answer.}$ M1 A1 $\sqrt{\quad}$ (2)
(d)	$\mathbf{AR} = \mathbf{S} \Rightarrow \mathbf{A}^{-1} \mathbf{AR} = \mathbf{A}^{-1} \mathbf{S} \Rightarrow \mathbf{R} = \mathbf{A}^{-1} \mathbf{S}$ $\mathbf{R} = \frac{1}{4} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 8 & 12 \\ 4 & 16 & 4 \end{pmatrix}$ $= \frac{1}{4} \begin{pmatrix} 8 & 56 & 44 \\ 8 & 40 & 20 \end{pmatrix}$ $= \begin{pmatrix} 2 & 14 & 11 \\ 2 & 10 & 5 \end{pmatrix}$ <p>Vertices are (2, 2), (14, 10) and (11, 5).</p>	<p>At least one attempt to apply \mathbf{A}^{-1} by any of the three vertices in \mathbf{S}.</p> <p>At least one correct column o.e.</p> <p>At least two correct columns o.e.</p> <p>All three coordinates correct.</p> M1 A1 $\sqrt{\quad}$ A1 A1 (4) [9]

Question Number	Scheme	Marks
9.	<p>$u_{n+1} = 4u_n + 2$, $u_1 = 2$ and $u_n = \frac{2}{3}(4^n - 1)$</p> <p>$n = 1$; $u_1 = \frac{2}{3}(4^1 - 1) = \frac{2}{3}(3) = 2$</p> <p>So u_n is true when $n = 1$.</p> <p>Assume that for $n = k$ that, $u_k = \frac{2}{3}(4^k - 1)$ is true for $k \in \mathbb{Z}^+$.</p> <p>Then $u_{k+1} = 4u_k + 2$</p> $= 4\left(\frac{2}{3}(4^k - 1)\right) + 2$ $= \frac{8}{3}(4)^k - \frac{8}{3} + 2$ $= \frac{2}{3}(4)(4)^k - \frac{2}{3}$ $= \frac{2}{3}4^{k+1} - \frac{2}{3}$ $= \frac{2}{3}(4^{k+1} - 1)$ <p>Therefore, the general statement, $u_n = \frac{2}{3}(4^n - 1)$ is true when $n = k + 1$. (As u_n is true for $n = 1$,) then u_n is true for all positive integers by mathematical induction</p>	<p>Check that $u_n = \frac{2}{3}(4^n - 1)$ yields 2 when $n = 1$.</p> <p>B1</p> <p>Substituting $u_k = \frac{2}{3}(4^k - 1)$ into $u_{n+1} = 4u_n + 2$.</p> <p>M1</p> <p>An attempt to multiply out the brackets by 4 or $\frac{8}{3}$</p> <p>M1</p> <p>$\frac{2}{3}(4^{k+1} - 1)$</p> <p>A1</p> <p>Require 'True when $n=1$', 'Assume true when $n=k$' and 'True when $n = k + 1$' then true for all n o.e.</p> <p>A1</p> <p>(5) [5]</p>

Question Number	Scheme	Marks
<p>10.</p> <p>(a)</p>	<p>$xy = 36$ at $(6t, \frac{6}{t})$.</p> <p>$y = \frac{36}{x} = 36x^{-1} \Rightarrow \frac{dy}{dx} = -36x^{-2} = -\frac{36}{x^2}$</p> <p>At $(6t, \frac{6}{t})$, $\frac{dy}{dx} = -\frac{36}{(6t)^2}$</p> <p>So, $m_T = \frac{dy}{dx} = -\frac{1}{t^2}$</p> <p>T: $y - \frac{6}{t} = -\frac{1}{t^2}(x - 6t)$</p> <p>T: $y - \frac{6}{t} = -\frac{1}{t^2}x + \frac{6}{t}$</p> <p>T: $y = -\frac{1}{t^2}x + \frac{6}{t} + \frac{6}{t}$</p> <p>T: $y = -\frac{1}{t^2}x + \frac{12}{t}$*</p>	<p>An attempt at $\frac{dy}{dx}$.</p> <p>or $\frac{dy}{dt}$ and $\frac{dx}{dt}$</p> <p>An attempt at $\frac{dy}{dx}$ in terms of t</p> <p>$\frac{dy}{dx} = -\frac{1}{t^2}$ *</p> <p>Must see working to award here</p> <p>Applies $y - \frac{6}{t} = \text{their } m_T(x - 6t)$</p> <p>Correct solution .</p> <p>A1 cso (5)</p>
<p>(b)</p>	<p>Both T meet at $(-9, 12)$ gives</p> <p>$12 = -\frac{1}{t^2}(-9) + \frac{12}{t}$</p> <p>$12 = \frac{9}{t^2} + \frac{12}{t} \quad (\times t^2)$</p> <p>$12t^2 = 9 + 12t$</p> <p>$12t^2 - 12t - 9 = 0$</p> <p>$4t^2 - 4t - 3 = 0$</p> <p>$(2t - 3)(2t + 1) = 0$</p> <p>$t = \frac{3}{2}, -\frac{1}{2}$</p> <p>$t = \frac{3}{2} \Rightarrow x = 6(\frac{3}{2}) = 9, y = \frac{6}{(\frac{3}{2})} = 4 \Rightarrow (9, 4)$</p> <p>$t = -\frac{1}{2} \Rightarrow x = 6(-\frac{1}{2}) = -3,$ $y = \frac{6}{(-\frac{1}{2})} = -12 \Rightarrow (-3, -12)$</p>	<p>Substituting $(-9, 12)$ into T.</p> <p>An attempt to form a "3 term quadratic"</p> <p>An attempt to factorise.</p> <p>$t = \frac{3}{2}, -\frac{1}{2}$</p> <p>An attempt to substitute either their $t = \frac{3}{2}$ or their $t = -\frac{1}{2}$ into x and y.</p> <p>At least one of $(9, 4)$ or $(-3, -12)$.</p> <p>Both $(9, 4)$ and $(-3, -12)$.</p> <p>(7) [12]</p>

Other Possible Solutions

Question Number	Scheme	Marks
<p>4.</p> <p>(a) (i) <i>Aliter</i> (ii) Way 2</p>	<p>$z^2 + pz + q = 0, z_1 = 2 - 4i$</p> <p>$z_2 = 2 + 4i$</p> <p>Product of roots = $(2 - 4i)(2 + 4i)$</p> <p style="text-align: center;">$= 4 + 16 = 20$</p> <p>or $b^2 - 4ac = (8i)^2$</p> <p>Sum of roots = $(2 - 4i) + (2 + 4i) = 4$</p> <p>$= z^2 - 4z + 20 = 0$</p>	<p style="text-align: right;">$2 + 4i$</p> <p>B1</p> <p>M1</p> <p>No i^2. Attempt Sum and Product of roots or Sum and discriminant</p> <p>Any one of $p = -4, q = 20$. A1</p> <p>Both $p = -4, q = 20$. A1</p> <p style="text-align: right;">(4)</p>
<p>4.</p> <p>(a) (i) <i>Aliter</i> (ii) Way 3</p>	<p>$z^2 + pz + q = 0, z_1 = 2 - 4i$</p> <p>$z_2 = 2 + 4i$</p> <p>$(2 - 4i)^2 + p(2 - 4i) + q = 0$</p> <p>$-12 - 16i + p(2 - 4i) + q = 0$</p> <p>Imaginary part: $-16 - 4p = 0$</p> <p>Real part: $-12 + 2p + q = 0$</p> <p>$4p = -16 \Rightarrow p = -4$</p> <p>$q = 12 - 2p \Rightarrow q = 12 - 2(-4) = 20$</p>	<p style="text-align: right;">$2 + 4i$</p> <p>B1</p> <p>M1</p> <p>An attempt to substitute either z_1 or z_2 into $z^2 + pz + q = 0$ and no i^2.</p> <p>Any one of $p = -4, q = 20$. A1</p> <p>Both $p = -4, q = 20$. A1</p> <p style="text-align: right;">(4)</p>

Question Number	Scheme	Marks
<p><i>Aliter</i> 7. (c) Way 2</p>	<p>$w = 4, \arg w = \frac{5\pi}{6}$ and $w = a + ib$</p> <p>$w = 4 \Rightarrow a^2 + b^2 = 16$</p> <p>$\arg w = \frac{5\pi}{6} \Rightarrow \arctan\left(\frac{b}{a}\right) = \frac{5\pi}{6} \Rightarrow \frac{b}{a} = -\frac{1}{\sqrt{3}}$</p> <p>$a = -\sqrt{3}b \Rightarrow a^2 = 3b^2$</p> <p>So, $3b^2 + b^2 = 16 \Rightarrow b^2 = 4$</p> <p>$\Rightarrow b = \pm 2$ and $a = \mp 2\sqrt{3}$</p> <p>As w is in the second quadrant</p> <p>$w = -2\sqrt{3} + 2i$</p> <p>$a = -2\sqrt{3}, b = 2$</p>	<p>Attempts to write down an equation in terms of a and b for either the modulus or the argument of w. Either $a^2 + b^2 = 16$ or $\frac{b}{a} = -\frac{1}{\sqrt{3}}$</p> <p>M1</p> <p>A1</p> <p>either $-2\sqrt{3} + 2i$ or awrt $-3.5 + 2i$</p> <p>A1</p> <p>(3)</p>