Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Monday 15 June 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

$$f(x) = \frac{1}{\sqrt{(4+x)}}, \quad |x| < 4.$$

Find the binomial expansion of f (x) in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(6)

2.

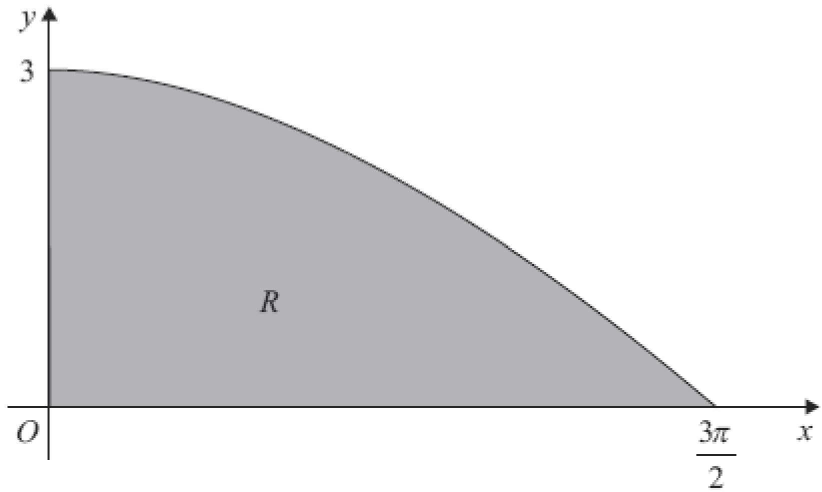

Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis and the curve with equation $y = 3\cos\left(\frac{x}{3}\right)$, $0 \leq x \leq \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3\cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

(*a*) Copy and complete the table above giving the missing value of y to 5 decimal places.

(1)

(*b*) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R, giving your answer to 3 decimal places.

(4)

(*c*) Use integration to find the exact area of R.

(3)

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3. $$\mathrm{f}(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+3)}.$$

(*a*) Find the values of the constants *A*, *B* and *C*.

(4)

(*b*) (i) Hence find $\int \mathrm{f}(x)\ \mathrm{d}x$.

(3)

(ii) Find $\int_0^2 \mathrm{f}(x)\ \mathrm{d}x$ in the form ln *k*, where *k* is a constant.

(3)

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4. The curve *C* has the equation $ye^{-2x} = 2x + y^2$.

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of *x* and *y*.

(5)

The point *P* on *C* has coordinates (0, 1).

(*b*) Find the equation of the normal to *C* at *P*, giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(4)

5.

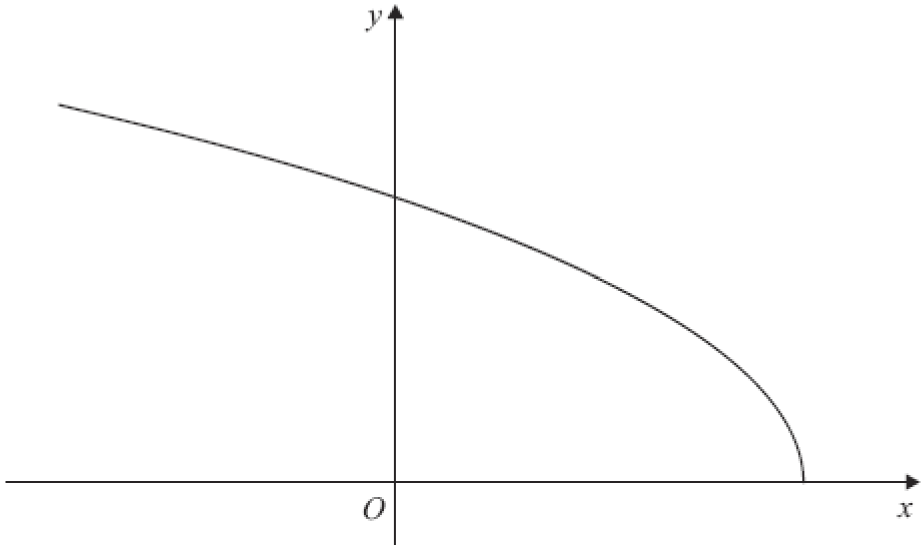


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \quad y = 6\sin t, \quad 0 \leq t \leq \frac{\pi}{2}.$$

(a) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

(4)

(b) Find a cartesian equation of the curve in the form

$$y = \mathrm{f}(x), \quad -k \leq x \leq k,$$

stating the value of the constant k.

(4)

(c) Write down the range of f(x).

(2)

6. (*a*) Find $\int \sqrt{(5-x)}\ \mathrm{d}x$.

(2)

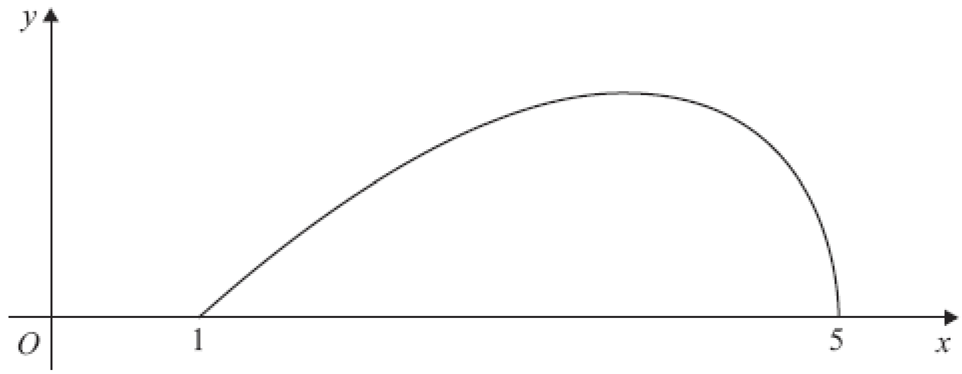


Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = (x-1)\sqrt{(5-x)}, \quad 1 \leq x \leq 5$$

(*b*) (i) Using integration by parts, or otherwise, find $\int (x-1)\sqrt{(5-x)}\ \mathrm{d}x$.

(4)

(ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)}\ \mathrm{d}x$..

(2)

7. Relative to a fixed origin O, the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B.

(*a*) Find a vector equation for the line l.
(3)

(*b*) Find $\left|\overrightarrow{CB}\right|$.
(2)

(*c*) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place.
(3)

(*d*) Find the shortest distance from the point C to the line l.
(3)

The point X lies on l. Given that the vector $\overrightarrow{CX}$ is perpendicular to l,

(*e*) find the area of the triangle CXB, giving your answer to 3 significant figures.
(3)

8. (*a*) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta \, d\theta$.

(2)

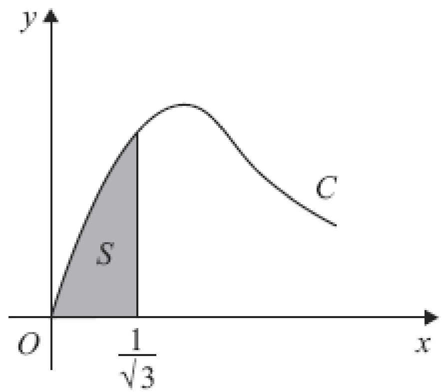


Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan\theta, \quad y = 2\sin 2\theta, \quad 0 \le \theta < \frac{\pi}{2}.$$

The finite shaded region S shown in Figure 4 is bounded by C, the line $x = \frac{1}{\sqrt{3}}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(*b*) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta,$$

where k is a constant.

(5)

(*c*) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)

TOTAL FOR PAPER: 75 MARKS

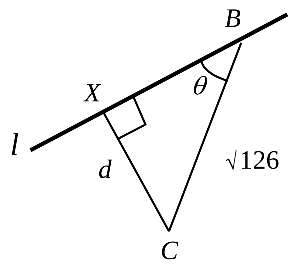
END

Question Number	Scheme	Marks
1.	$\mathrm{f}(x)=\frac{1}{\sqrt{(4+x)}}=(4+x)^{-\frac{1}{2}}$	M1
	$=(4)^{-\frac{1}{2}}(1+\ \dots\ )^{\dots}$ $\quad\frac{1}{2}(1+\ \dots\ )^{\dots}$ or $\frac{1}{2\sqrt{(1+\dots)}}$	B1
	$=\ \dots\ \left(1+(-\tfrac{1}{2})\left(\frac{x}{4}\right)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2}\left(\frac{x}{4}\right)^2+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}\left(\frac{x}{4}\right)^3+\ \dots\right)$	M1 A1ft
	ft their $\left(\frac{x}{4}\right)$	
	$=\frac{1}{2}-\frac{1}{16}x,+\frac{3}{256}x^2-\frac{5}{2048}x^3+\ \dots$	A1, A1 (6)
		(6 marks)
2. (a)	1.14805 $\quad$ awrt 1.14805	B1 (1)
(b)	$A\approx\frac{1}{2}\times\frac{3\pi}{8}(\ \dots\ )$	B1
	$=\ \dots\ (3+2(2.77164+2.12132+1.14805)+0)$ $\quad$ 0 can be implied	M1
	$=\frac{3\pi}{16}(3+2(2.77164+2.12132+1.14805))$ $\quad$ ft their (a)	A1ft
	$=\frac{3\pi}{16}\times15.08202\ \dots\ =8.884$ $\quad$ cao	A1 (4)
(c)	$\int 3\cos\left(\frac{x}{3}\right)\mathrm{d}x=\frac{3\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$	M1 A1
	$=9\sin\left(\frac{x}{3}\right)$	
	$A=\left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}}=9-0=9$ $\quad$ cao	A1 (3)
		(8 marks)

Question Number	Scheme	Marks
3. (a)	$\mathrm{f}(x) = \dfrac{4-2x}{(2x+1)(x+1)(x+3)} = \dfrac{A}{2x+1} + \dfrac{B}{x+1} + \dfrac{C}{x+3}$	
	$4-2x = A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)$	M1
	A method for evaluating one constant	M1
	$x \to -\frac{1}{2},\quad 5 = A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right) \Rightarrow A = 4$ any one correct constant	A1
	$x \to -1,\quad 6 = B(-1)(2) \Rightarrow B = -3$	
	$x \to -3,\quad 10 = C(-5)(-2) \Rightarrow C = 1$ all three constants correct	A1 (4)
(b) (i)	$\displaystyle\int \left(\frac{4}{2x+1} - \frac{3}{x+1} + \frac{1}{x+3} \right) \mathrm{d}x$	
	$= \frac{4}{2}\ln(2x+1) - 3\ln(x+1) + \ln(x+3) + C$ A1 two ln terms correct	M1 A1ft
	All three ln terms correct and "+C" ; ft constants	A1ft (3)
(ii)	$\left[2\ln(2x+1) - 3\ln(x+1) + \ln(x+3)\right]_0^2$	
	$= (2\ln 5 - 3\ln 3 + \ln 5) - (2\ln 1 - 3\ln 1 + \ln 3)$	M1
	$= 3\ln 5 - 4\ln 3$	
	$= \ln\left(\dfrac{5^3}{3^4}\right)$	M1
	$= \ln\left(\dfrac{125}{81}\right)$	A1 (3)
		(10 marks)

Question Number	Scheme	Marks
4. (a)	$e^{-2x}\frac{dy}{dx} - 2ye^{-2x} = 2 + 2y\frac{dy}{dx}$ A1 correct RHS	M1 A1
	$\frac{d}{dx}\left(ye^{-2x}\right) = e^{-2x}\frac{dy}{dx} - 2ye^{-2x}$	B1
	$\left(e^{-2x} - 2y\right)\frac{dy}{dx} = 2 + 2ye^{-2x}$	M1
	$\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$	A1 (5)
(b)	At P, $\frac{dy}{dx} = \frac{2 + 2e^0}{e^0 - 2} = -4$	M1
	Using $mm' = -1$ $m' = \frac{1}{4}$	M1
	$y - 1 = \frac{1}{4}(x - 0)$	M1
	$x - 4y + 4 = 0$ or any integer multiple	A1 (4)
		(9 marks)
5. (a)	$\frac{dx}{dt} = -4\sin 2t$, $\frac{dy}{dt} = 6\cos t$	B1, B1
	$\frac{dy}{dx} = -\frac{6\cos t}{4\sin 2t}$ $\left(= -\frac{3}{4\sin t}\right)$	M1
	At $t = \frac{\pi}{3}$, $m = -\frac{3}{4 \times \frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1 (4)
(b)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1
	$\cos 2t = \frac{x}{2}$, $\sin t = \frac{y}{6}$	
	$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$	M1
	Leading to $y = \sqrt{(18 - 9x)}$ $\left(= 3\sqrt{(2 - x)}\right)$ cao	A1
	$-2 \le x \le 2$ $k = 2$	B1 (4)
(c)	$0 \le f(x) \le 6$ either $0 \le f(x)$ or $f(x) \le 6$	B1
	Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1 (2)
		(10 marks)

Question Number	Scheme	Marks
6. (a)	$\int \sqrt{(5-x)}\,dx = \int (5-x)^{\frac{1}{2}}\,dx = \frac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}} \quad (+C)$	M1 A1 (2)
	$\left(= -\frac{2}{3}(5-x)^{\frac{3}{2}} + C\right)$	
(b)(i)	$\int (x-1)\sqrt{(5-x)}\,dx = -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \frac{2}{3}\int (5-x)^{\frac{3}{2}}\,dx$	M1 A1ft
	$= \quad \ldots \quad + \frac{2}{3} \times \frac{(5-x)^{\frac{5}{2}}}{-\frac{5}{2}} \quad (+C)$	M1
	$= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}} \quad (+C)$	A1 (4)
(ii)	$\left[-\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}}\right]_1^5 = (0-0) - \left(0 - \frac{4}{15} \times 4^{\frac{5}{2}}\right)$	
	$= \frac{128}{15}\left(= 8\frac{8}{15} \approx 8.53\right)$ awrt 8.53	M1 A1 (2)
		(8 marks)

Question Number	Scheme	Marks
7. (a)	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}10\\14\\-4\end{pmatrix}-\begin{pmatrix}8\\13\\-2\end{pmatrix}=\begin{pmatrix}2\\1\\-2\end{pmatrix}$ or $\overrightarrow{BA}=\begin{pmatrix}-2\\-1\\2\end{pmatrix}$	M1
	$\mathbf{r}=\begin{pmatrix}8\\13\\-2\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}10\\14\\-4\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ accept equivalents	M1 A1ft (3)
(b)	$\overrightarrow{CB}=\overrightarrow{OB}-\overrightarrow{OC}=\begin{pmatrix}10\\14\\-4\end{pmatrix}-\begin{pmatrix}9\\9\\6\end{pmatrix}=\begin{pmatrix}1\\5\\-10\end{pmatrix}$ or $\overrightarrow{BC}=\begin{pmatrix}-1\\-5\\10\end{pmatrix}$	
	$CB=\sqrt{\left(1^2+5^2+(-10)^2\right)}=\sqrt{(126)}\quad\left(=3\sqrt{14}\approx 11.2\right)$ awrt 11.2	M1 A1 (2)
(c)	$\overrightarrow{CB}.\overrightarrow{AB}=\lvert\overrightarrow{CB}\rvert\lvert\overrightarrow{AB}\rvert\cos\theta$	
	$(\pm)(2+5+20)=\sqrt{126}\sqrt{9}\cos\theta$	M1 A1
	$\cos\theta=\dfrac{3}{\sqrt{14}}\Rightarrow\theta\approx 36.7°$ awrt 36.7°	A1 (3)
(d)	 $\dfrac{d}{\sqrt{126}}=\sin\theta$	M1 A1ft
	$d=3\sqrt{5}\,(\approx 6.7)$ awrt 6.7	A1 (3)
(e)	$BX^2=BC^2-d^2=126-45=81$	M1
	! $CBX=\dfrac{1}{2}\times BX\times d=\dfrac{1}{2}\times 9\times 3\sqrt{5}=\dfrac{27\sqrt{5}}{2}\,(\approx 30.2)$ awrt 30.1 or 30.2	M1 A1 (3)
		(14 marks)

Question Number	Scheme	Marks
8. (a)	$\int \sin^2\theta \, d\theta = \frac{1}{2}\int (1-\cos 2\theta)\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \quad (+C)$	M1 A1 (2)
	$x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta$	
(b)	$\pi\int y^2\, dx = \pi\int y^2 \frac{dx}{d\theta} d\theta = \pi\int (2\sin 2\theta)^2 \sec^2\theta \, d\theta$	M1 A1
	$= \pi\int \frac{(2\times 2\sin\theta\cos\theta)^2}{\cos^2\theta} d\theta$	M1
	$= 16\pi\int \sin^2\theta \, d\theta \qquad k = 16\pi$	A1
	$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0, \quad x = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	B1 (5)
(c)	$\left(V = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta\right)$	
	$V = 16\pi\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$	M1
	$= 16\pi\left[\left(\frac{\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3}\right) - (0-0)\right]$ Use of correct limits	M1
	$= 16\pi\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = \frac{4}{3}\pi^2 - 2\pi\sqrt{3} \qquad p = \frac{4}{3},\ q = -2$	A1 (3)
		(10 marks)