Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Monday 24 January 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H35404A

1. (*a*) Express $7 \cos x - 24 \sin x$ in the form $R \cos (x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

(*b*) Hence write down the minimum value of $7 \cos x - 24 \sin x$.

(1)

(*c*) Solve, for $0 \leq x < 2\pi$, the equation

$$7 \cos x - 24 \sin x = 10,$$

giving your answers to 2 decimal places.

(5)

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2. (*a*) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$\mathrm{f}(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \qquad x > 1,$$

(*b*) show that

$$\mathrm{f}(x) = \frac{3}{2x-1}.$$

(2)

(*c*) Hence differentiate $\mathrm{f}(x)$ and find $\mathrm{f}'(2)$.

(3)

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3. Find all the solutions of

$$2 \cos 2\theta = 1 - 2 \sin \theta$$

in the interval $0 \leq \theta < 360°$.

(6)

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4. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ °C, of the tea is modelled by the equation

$$\theta = 20 + A\mathrm{e}^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90 °C,

(*a*) find the value of A.

(2)

The tea takes 5 minutes to decrease in temperature from 90 °C to 55 °C.

(*b*) Show that $k = \frac{1}{5}\ln 2$.

(3)

(*c*) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places.

(3)

5.

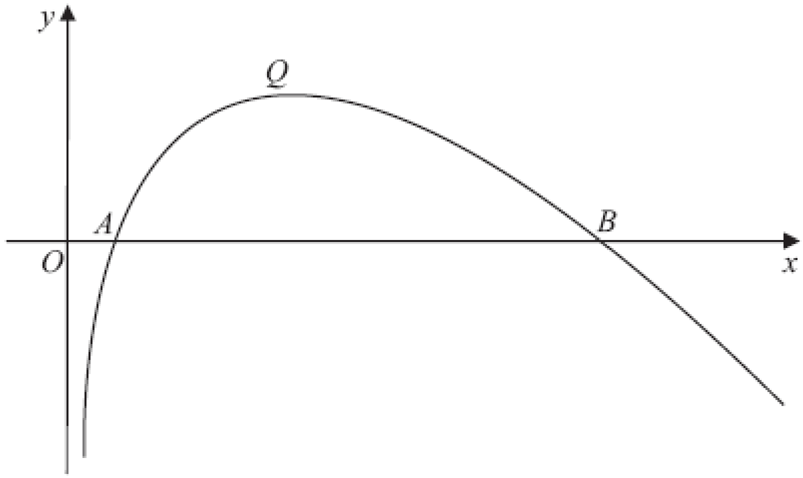

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (8 - x)\ln x, \qquad x > 0.$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 1.

(*a*) Write down the coordinates of A and the coordinates of B.

(2)

(*b*) Find $\mathrm{f}'(x)$.

(3)

(*c*) Show that the x-coordinate of Q lies between 3.5 and 3.6

(2)

(*d*) Show that the x-coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x}.$$

(3)

To find an approximation for the x-coordinate of Q, the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(*e*) Taking $x_0 = 3.55$, find the values of x_1, x_2 and x_3.
Give your answers to 3 decimal places.

(3)

6. The function f is defined by

$$\text{f}: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \quad x \neq 5.$$

(*a*) Find $\text{f}^{-1}(x)$.

(3)

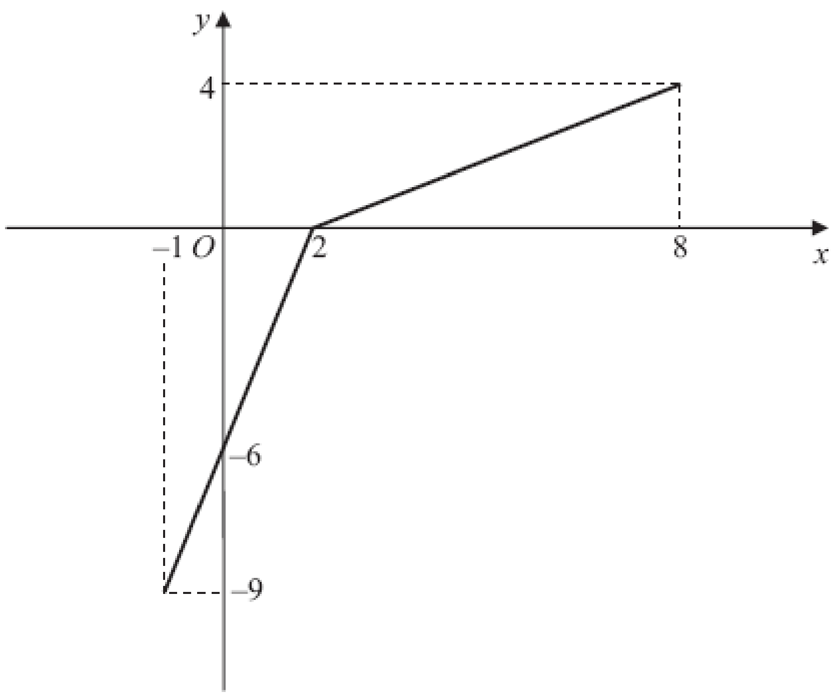

Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from (–1, –9) to (2, 0) and from (2, 0) to (8, 4). Figure 2 shows a sketch of the graph of $y = \text{g}(x)$.

(*b*) Write down the range of g.

(1)

(*c*) Find gg(2).

(2)

(*d*) Find fg(8).

(2)

(*e*) On separate diagrams, sketch the graph with equation

(i) $y = |\text{g}(x)|$,

(ii) $y = \text{g}^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(*f*) State the domain of the inverse function g^{-1}.

(1)

7. The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}.$$

(*a*) Show that

$$\frac{dy}{dx} = \frac{6\sin 2x + 4\cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(*b*) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

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8. Given that

$$\frac{d}{dx}(\cos x) = -\sin x,$$

(*a*) show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that

$$x = \sec 2y,$$

(*b*) find $\frac{dx}{dy}$ in terms of y.

(2)

(*c*) Hence find $\frac{dy}{dx}$ in terms of x.

(4)

TOTAL FOR PAPER: 75 MARKS

END

January 2011
Core Mathematics C3 6665
Mark Scheme

Question Number	Scheme		Marks
1.			
(a)	$7\cos x - 24\sin x = R\cos(x+\alpha)$		
	$7\cos x - 24\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
	Equate $\cos x$: $7 = R\cos\alpha$ Equate $\sin x$: $24 = R\sin\alpha$		
	$R = \sqrt{7^2+24^2}\,;= 25$	$R = 25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 1.287002218...^c$	$\tan\alpha = \frac{24}{7}$ or $\tan\alpha = \frac{7}{24}$ awrt 1.287	M1 A1
	Hence, $7\cos x - 24\sin x = 25\cos(x+1.287)$		(3)
(b)	Minimum value = $\underline{-25}$	-25 or $-R$	B1ft (1)
(c)	$7\cos x - 24\sin x = 10$		
	$25\cos(x+1.287) = 10$		
	$\cos(x+1.287) = \frac{10}{25}$	$\cos(x \pm \text{their } \alpha) = \frac{10}{(\text{their } R)}$	M1
	$\text{PV} = 1.159279481...^c$ or $66.42182152...^\circ$	For applying $\cos^{-1}\left(\frac{10}{\text{their } R}\right)$	M1
	So, $x + 1.287 = \{1.159279...^c, 5.123906...^c, 7.442465...^c\}$	either 2π + or − their PV^c or 360° + or − their PV°	M1
	gives, $x = \{3.836906..., 6.155465...\}$	awrt 3.84 OR 6.16 awrt 3.84 AND 6.16	A1 A1
			(5) **[9]**

Question Number	Scheme		Marks
2. (a)	$\dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)}$		
	$= \dfrac{(4x-1)(2x-1)-3}{2(x-1)(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{8x^2-6x-2}{\{2(x-1)(2x-1)\}}$	Simplifies to give a correct quadratic numerator over a correct quadratic denominator	A1 aef
	$= \dfrac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$	An attempt to factorise a 3 term quadratic numerator	M1
	$= \dfrac{4x+1}{2x-1}$		A1 (4)
(b)	$f(x) = \dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)} - 2, \quad x > 1$		
	$f(x) = \dfrac{(4x+1)}{(2x-1)} - 2$		
	$= \dfrac{(4x+1)-2(2x-1)}{(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{4x+1-4x+2}{(2x-1)}$		
	$= \dfrac{3}{(2x-1)}$	Correct result	A1 * (2)
(c)	$f(x) = \dfrac{3}{(2x-1)} = 3(2x-1)^{-1}$		
	$f'(x) = 3(-1)(2x-1)^{-2}(2)$	$\pm k(2x-1)^{-2}$	M1
			A1 aef
	$f'(2) = \dfrac{-6}{9} = -\dfrac{2}{3}$	Either $\dfrac{-6}{9}$ or $-\dfrac{2}{3}$	A1
			(3) **[9]**

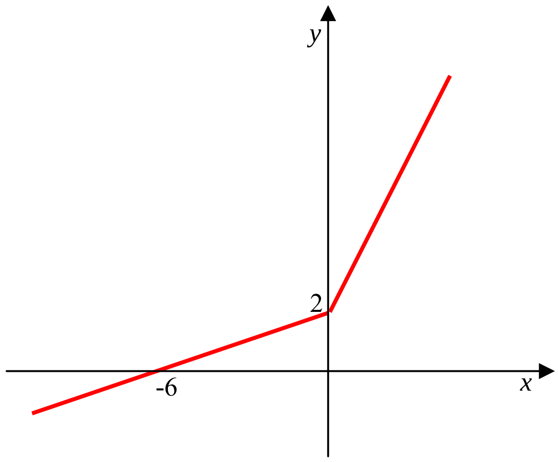
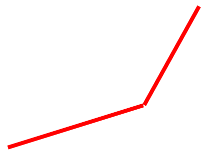
Question Number	Scheme		Marks
3.	$2\cos 2\theta = 1 - 2\sin\theta$		
	$2\left(1 - 2\sin^2\theta\right) = 1 - 2\sin\theta$	Substitutes either $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$.	M1
	$2 - 4\sin^2\theta = 1 - 2\sin\theta$		
	$4\sin^2\theta - 2\sin\theta - 1 = 0$	Forms a "quadratic in sine" = 0	M1(*)
	$\sin\theta = \dfrac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$	Applies the quadratic formula See notes for alternative methods.	M1
	PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$		
	$\theta = \{54, 126, 198, 342\}$	Any one correct answer 180-their pv All four solutions correct.	A1 dM1(*) A1 **[6]**

Question Number	Scheme		Marks
4.			
(a)	$\theta = 20 + Ae^{-kt}$ (eqn *)		
	$\{t = 0, \theta = 90 \Rightarrow\}$ $90 = 20 + Ae^{-k(0)}$	Substitutes $t = 0$ and $\theta = 90$ into eqn *	M1
	$90 = 20 + A \Rightarrow \underline{A = 70}$	$\underline{A = 70}$	A1
			(2)
(b)	$\theta = 20 + 70e^{-kt}$		
	$\{t = 5, \theta = 55 \Rightarrow\}$ $55 = 20 + 70e^{-k(5)}$ $\frac{35}{70} = e^{-5k}$	Substitutes $t = 5$ and $\theta = 55$ into eqn * and rearranges eqn * to make $e^{\pm 5k}$ the subject.	M1
	$\ln\left(\frac{35}{70}\right) = -5k$	Takes 'lns' and proceeds to make '$\pm 5k$' the subject.	dM1
	$-5k = \ln\left(\frac{1}{2}\right)$		
	$-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2}$	Convincing proof that $k = \frac{1}{5}\ln 2$	A1 *
			(3)
(c)	$\theta = 20 + 70e^{-\frac{1}{5}t\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{1}{5}\ln 2.(70)e^{-\frac{1}{5}t\ln 2}$	$\pm\alpha e^{-kt}$ where $k = \frac{1}{5}\ln 2$ $-14\ln 2 e^{-\frac{1}{5}t\ln 2}$	M1 A1 oe
	When $t = 10$, $\frac{d\theta}{dt} = -14\ln 2 e^{-2\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{7}{2}\ln 2 = -2.426015132...$		
	Rate of decrease of $\theta = 2.426\ ^{\circ}C/\text{min}$ (3 dp.)	awrt ± 2.426	A1
			(3) **[8]**

Question Number	Scheme		Marks
5. **(a)**	Crosses x-axis $\Rightarrow \mathrm{f}(x) = 0 \Rightarrow (8 - x)\ln x = 0$		
	Either $(8 - x) = 0$ or $\ln x = 0 \Rightarrow x = 8, 1$	Either one of {x}=1 OR x={8}	B1
	Coordinates are $A(1, 0)$ and $B(8, 0)$.	Both $A(1, \{0\})$ and $B(8, \{0\})$	B1
			(2)
(b)	Apply product rule: $\left\{\begin{array}{ll} u = (8 - x) & v = \ln x \\ \frac{\mathrm{d}u}{\mathrm{d}x} = -1 & \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{x} \end{array}\right\}$	$vu' + uv'$	M1
	$\mathrm{f}'(x) = -\ln x + \dfrac{8-x}{x}$	Any one term correct	A1
		Both terms correct	A1
			(3)
(c)	$\mathrm{f}'(3.5) = 0.032951317...$ $\mathrm{f}'(3.6) = -0.058711623...$ Sign change (and as $\mathrm{f}'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate **both** $\mathrm{f}'(3.5)$ and $\mathrm{f}'(3.6)$	M1
		both values correct to at least 1 sf, sign change and conclusion	A1
			(2)
(d)	At Q, $\mathrm{f}'(x) = 0 \Rightarrow -\ln x + \dfrac{8-x}{x} = 0$	Setting $\mathrm{f}'(x) = 0$.	M1
	$\Rightarrow -\ln x + \dfrac{8}{x} - 1 = 0$	Splitting up the numerator and proceeding to x=	M1
	$\Rightarrow \dfrac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$		
	$\Rightarrow x = \dfrac{8}{\ln x + 1}$ (as required)	For correct proof. No errors seen in working.	A1
			(3)

Question Number	Scheme		Marks
(e)	Iterative formula: $x_{n+1} = \dfrac{8}{\ln x_n + 1}$		
	$x_1 = \dfrac{8}{\ln(3.55)+1}$	An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$ $x_2 = 3.538246011...$ $x_3 = 3.534144722...$	Both $x_1 =$ awrt 3.529 and $x_2 =$ awrt 3.538	A1
	$x_1 = 3.529,\ x_2 = 3.538,\ x_3 = 3.534,$ to 3 dp.	x_1, x_2, x_3 all stated correctly to 3 dp	A1
			(3) **[13]**

Question Number	Scheme		Marks
6.			
(a)	$y = \dfrac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make x (or swapped y) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$	Collect x terms together and factorise.	M1
	$\Rightarrow x = \dfrac{3+5y}{y+2} \quad \therefore f^{-1}(x) = \underline{\dfrac{3+5x}{x+2}}$	$\underline{\dfrac{3+5x}{x+2}}$	A1 oe
			(3)
(b)	Range of g is $-9 \le g(x) \le 4$ or $-9 \le y \le 4$	<u>Correct Range</u>	B1 (1)
(c)		Deduces that $g(2)$ is 0. Seen or implied.	M1
	g g(2)= g (0) $= -6$, from sketch.	-6	A1 (2)
(d)	$fg(8) = f(4)$	Correct order g followed by f	M1
	$= \dfrac{3-4(2)}{4-5} = \dfrac{-5}{-1} = \underline{5}$	5	A1
			(2)

Question Number	Scheme	Marks
(e)(ii)	 Correct shape 	B1
	Graph goes through $(\{0\}, 2)$ and $(-6, \{0\})$ which are marked.	B1
		(4)
(f)	Domain of g^{-1} is $-9 \le x \le 4$ — Either correct answer or a follow through from part (b) answer	B1 $\sqrt{}$ (1) **[13]**

Question Number	Scheme		Marks
7 **(a)**	$y = \dfrac{3+\sin 2x}{2+\cos 2x}$ Apply quotient rule: $\left\{\begin{matrix} u = 3+\sin 2x & v = 2+\cos 2x \\ \dfrac{du}{dx} = 2\cos 2x & \dfrac{dv}{dx} = -2\sin 2x \end{matrix}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{2\cos 2x(2+\cos 2x) - -2\sin 2x(3+\sin 2x)}{(2+\cos 2x)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$ Any one term correct on the numerator Fully correct (unsimplified).	M1 A1 A1
	$= \dfrac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2+\cos 2x)^2}$ $= \dfrac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2+\cos 2x)^2}$ $= \dfrac{4\cos 2x + 6\sin 2x + 2}{(2+\cos 2x)^2}$ (as required)	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. No errors seen in working.	A1* (4)
(b)	When $x = \frac{\pi}{2}$, $y = \dfrac{3+\sin\pi}{2+\cos\pi} = \dfrac{3}{1} = 3$	$y = 3$	B1
	At $\left(\frac{\pi}{2}, 3\right)$, $m(\mathbf{T}) = \dfrac{6\sin\pi + 4\cos\pi + 2}{(2+\cos\pi)^2} = \dfrac{-4+2}{1^2} = -2$	$m(\mathbf{T}) = -2$	B1
	Either $\mathbf{T}$: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2\left(\frac{\pi}{2}\right) + c \Rightarrow c = 3 + \pi$;	$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';	M1
	$\mathbf{T}$: $y = -2x + (\pi + 3)$	$y = -2x + \pi + 3$	A1 (4) **[8]**

Question Number	Scheme		Marks
8.			
(a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both underlined steps.	A1 **AG**
			(3)
(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4}, n \in \mathbb{Z}.$		
	$\dfrac{dx}{dy} = 2\sec 2y \tan 2y$	$K \sec 2y \tan 2y$	M1
		$2\sec 2y \tan 2y$	A1
			(2)
(c)	$\dfrac{dy}{dx} = \dfrac{1}{2\sec 2y \tan 2y}$	Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{2x \tan 2y}$	Substitutes x for $\sec 2y$.	M1
	$1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$	Attempts to use the identity $1 + \tan^2 A = \sec^2 A$	M1
	So $\tan^2 2y = x^2 - 1$		
	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	A1
			(4)
			[9]