C2 January 2007

1. $$f(x) = x^3 + 3x^2 + 5.$$

Find

(a) $f''(x)$,

(3)

(b) $\int_1^2 f(x)\,dx$.

(4)

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2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1-2x)^5$. Give each term in its simplest form.

(4)

(b) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1+x)(1-2x)^5 \approx 1-9x.$$

(2)

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3. The line joining the points $(-1, 4)$ and $(3, 6)$ is a diameter of the circle C.

Find an equation for C.

(6)

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4. Solve the equation

$$5^x = 17,$$

giving your answer to 3 significant figures.

(3)

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5. $$f(x) = x^3 + 4x^2 + x - 6.$$

(a) Use the factor theorem to show that $(x+2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

(c) Write down all the solutions to the equation

$$x^3 + 4x^2 + x - 6 = 0.$$

(1)

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6. Find all the solutions, in the interval $0 \leqslant x < 2\pi$, of the equation

$$2\cos^2 x + 1 = 5\sin x,$$

giving each solution in terms of π.

(6)

7.

Figure 1

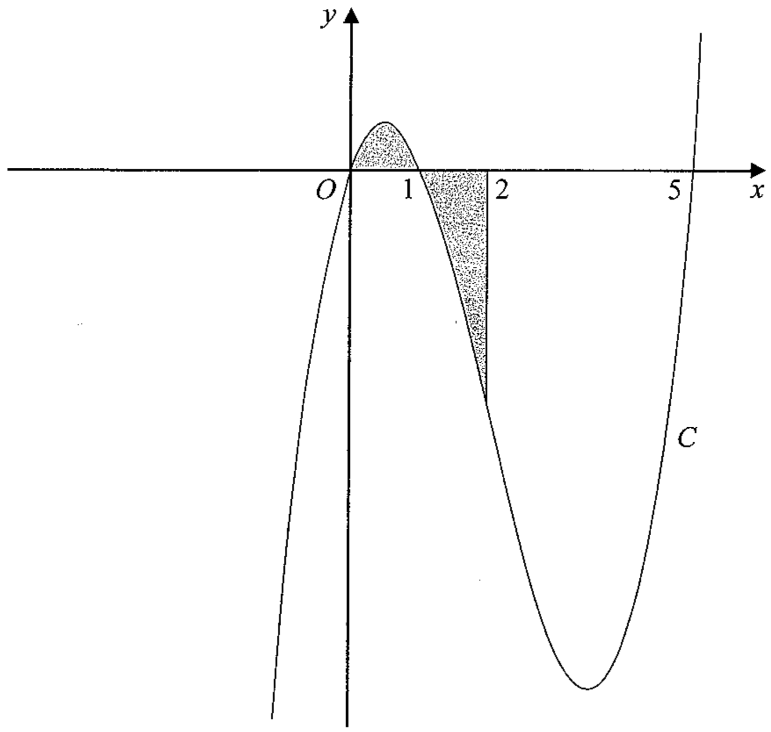

Figure 1 shows a sketch of part of the curve C with equation

$$y = x(x-1)(x-5).$$

Use calculus to find the total area of the finite region, shown shaded in Figure 1, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(9)

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8. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum.

(5)

(b) Find $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ and hence verify that C is a minimum for this value of v.

(2)

(c) Calculate the minimum total cost of the journey.

(2)

9.

Figure 2

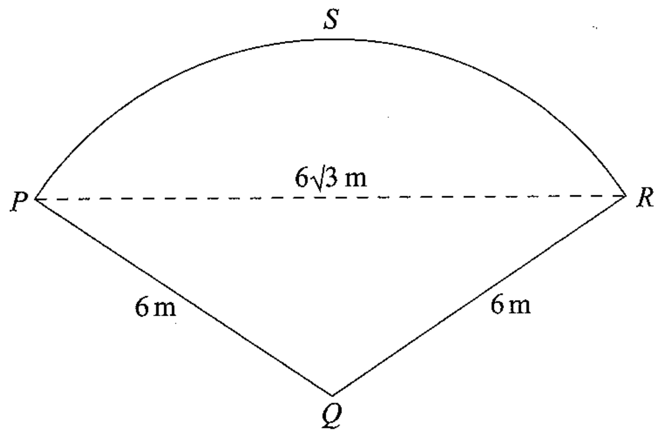

Figure 2 shows a plan of a patio. The patio *PQRS* is in the shape of a sector of a circle with centre *Q* and radius 6 m.

Given that the length of the straight line *PR* is $6\sqrt{3}$ m,

(a) find the exact size of angle *PQR* in radians.

(3)

(b) Show that the area of the patio *PQRS* is $12\pi\ \text{m}^2$.

(2)

(c) Find the exact area of the triangle *PQR*.

(2)

(d) Find, in m^2 to 1 decimal place, the area of the segment *PRS*.

(2)

(e) Find, in m to 1 decimal place, the perimeter of the patio *PQRS*.

(2)

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10. A geometric series is $a + ar + ar^2 + ...$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

(4)

(b) Find

$$\sum_{k=1}^{10} 100(2^k).$$

(3)

(c) Find the sum to infinity of the geometric series

$$\frac{5}{6} + \frac{5}{18} + \frac{5}{54} + ...$$

(3)

(d) State the condition for an infinite geometric series with common ratio r to be convergent.

(1)

TOTAL FOR PAPER: 75 MARKS