

Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Monday 2 June 2008 – Morning

Time: 1 hour 30 minutes



Materials required for examination

Mathematical Formulae (Green)

Items included with question papers

Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 11 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. Find $\int (2+5x^2) dx$. (3)

2. Factorise completely $x^3 - 9x$. (3)

3.

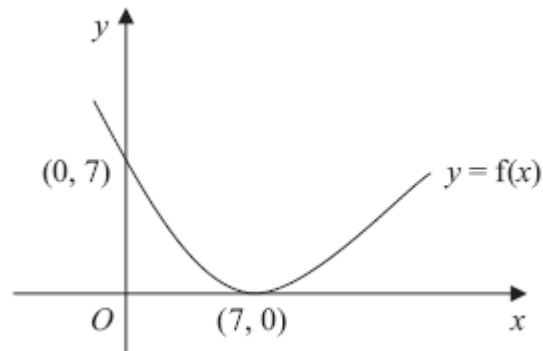


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve passes through the point $(0, 7)$ and has a minimum point at $(7, 0)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(x) + 3$, (3)

(b) $y = f(2x)$. (2)

On each diagram, show clearly the coordinates of the minimum point and the coordinates of the point at which the curve crosses the y-axis.

4. $f(x) = 3x + x^3, \quad x > 0$.

(a) Differentiate to find $f'(x)$. (2)

Given that $f'(x) = 15$,

(b) find the value of x . (3)

5. A sequence x_1, x_2, x_3, \dots is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, \quad n \geq 1,$$

where a is a constant.

- (a) Find an expression for x_2 in terms of a .

(1)

- (b) Show that $x_3 = a^2 - 3a - 3$.

(2)

Given that $x_3 = 7$,

- (c) find the possible values of a .

(3)

6. The curve C has equation $y = \frac{3}{x}$ and the line l has equation $y = 2x + 5$.

- (a) Sketch the graphs of C and l , indicating clearly the coordinates of any intersections with the axes.

(3)

- (b) Find the coordinates of the points of intersection of C and l .

(6)

7. Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

- (a) Show that on the 4th Saturday of training she runs 11 km.

(1)

- (b) Find an expression, in terms of n , for the length of her training run on the n th Saturday.

(2)

- (c) Show that the total distance she runs on Saturdays in n weeks of training is $n(n + 4)$ km.

(3)

On the n th Saturday Sue runs 43 km.

- (d) Find the value of n .

(2)

- (e) Find the total distance, in km, Sue runs on Saturdays in n weeks of training.

(2)

8. Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q .

(3)

9. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(a) Find $\frac{dy}{dx}$.

(2)

The point A with x -coordinate $-\frac{1}{2}$ lies on C . The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k ,

(4)

(c) the value of the y -coordinate of A .

(2)

10.

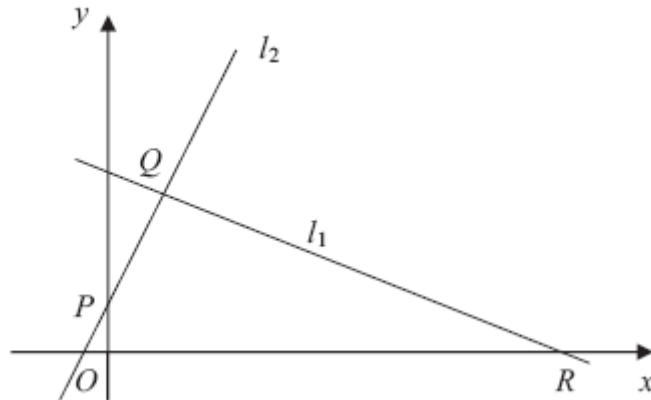


Figure 2

The points $Q(1, 3)$ and $R(7, 0)$ lie on the line l_1 , as shown in Figure 2.

The length of QR is $a\sqrt{5}$.

(a) Find the value of a .

(3)

The line l_2 is perpendicular to l_1 , passes through Q and crosses the y -axis at the point P , as shown in Figure 2. Find

(b) an equation for l_2 ,

(5)

(c) the coordinates of P ,

(1)

(d) the area of $\triangle PQR$.

(4)

11. The gradient of a curve C is given by $\frac{dy}{dx} = \frac{(x^2 + 3)^2}{x^2}$, $x \neq 0$.

(a) Show that $\frac{dy}{dx} = x^2 + 6 + 9x^{-2}$.

(2)

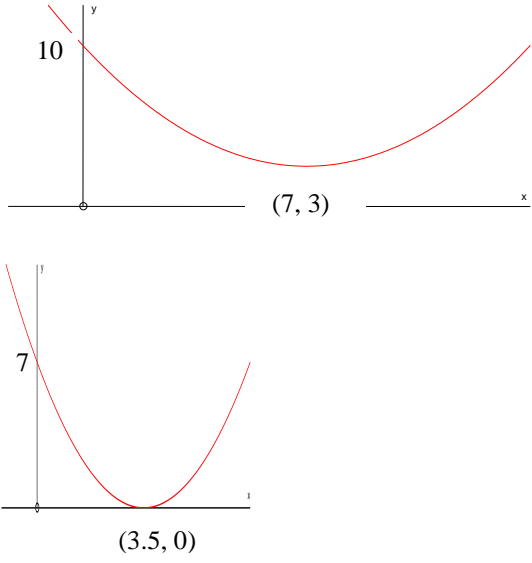
The point $(3, 20)$ lies on C .

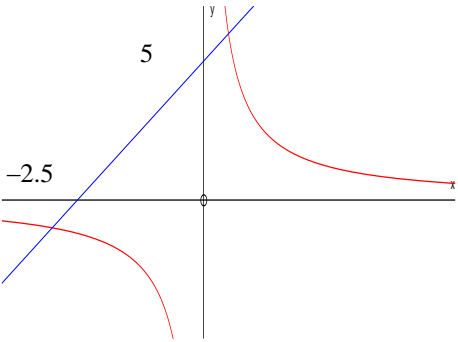
(b) Find an equation for the curve C in the form $y = f(x)$.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$2x + \frac{5}{3}x^3 + c$	M1 A1 A1 (3) (3 marks)
2.	$x(x^2 - 9)$ or $(x \pm 0)(x^2 - 9)$ or $(x - 3)(x^2 + 3x)$ or $(x + 3)(x^2 - 3x)$ $x(x - 3)(x + 3)$ (7, 3)	B1 M1 A1 (3) (3 marks)
3. (a)		B1 B1 B1 (3) B1 B1 (2) (5 marks)
4. (a) (b)	$f'(x) = 3 + 3x^2$ $3 + 3x^2 = 15$ and start to try and simplify $x^2 = k \rightarrow x = \sqrt{k}$ (ignore \pm) $x = 2$ (ignore $x = -2$)	M1 A1 (2) M1 M1 A1 (3) (8 marks)

Question Number	Scheme	Marks
<p>5. (a)</p> <p>(b)</p> <p>(c)</p>	<p>$[x_2 =] a - 3$</p> <p>$[x_3 =] ax_2 - 3$ or $a(a - 3) - 3$</p> <p>$= a(a - 3) - 3 = a^2 - 3a - 3$ (*)</p> <p>$a^2 - 3a - 3 = 7$</p> <p>$a^2 - 3a - 10 = 0$ or $a^2 - 3a = 10$</p> <p>$(a - 5)(a + 2) = 0$</p> <p>$a = 5$ or -2</p>	<p>B1 (1)</p> <p>B1</p> <p>A1 cso (2)</p> <p>M1</p> <p>M1</p> <p>A1 (3)</p> <p>(6 marks)</p>
<p>6. (a)</p> <p>(b)</p>	 <p>$2x + 5 = \frac{3}{x}$</p> <p>$2x^2 + 5x - 3 [= 0]$ or $2x^2 + 5x = 3$</p> <p>$(2x - 1)(x + 3) [= 0]$</p> <p>$x = -3$ or $\frac{1}{2}$</p> <p>$y = \frac{3}{-3}$ or $2 \times (-3) + 5$ or $y = \frac{3}{\frac{1}{2}}$ or $2 \times (\frac{1}{2}) + 5$</p> <p>Points are <u>$(-3, -1)$ and $(\frac{1}{2}, 6)$</u> (correct pairings)</p>	<p>B1</p> <p>M1</p> <p>A1 (3)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 ft (6)</p> <p>(10 marks)</p>

Question Number	Scheme	Marks
7.	<p>(a) 5, 7, 9, 11 or $5 + 2 + 2 + 2 = 11$ or $5 + 6 = 11$ use $a = 5, d = 2, n = 4$ and $t_4 = 5 + 3 \times 2 = 11$</p> <p>(b) $t_n = a + (n-1)d$ with one of $a = 5$ or $d = 2$ correct $= 5 + 2(n - 1)$ or $2n + 3$ or $1 + 2(n + 1)$</p> <p>(c) $S_n = \frac{n}{2}[2 \times 5 + 2(n-1)]$ or use of $\frac{n}{2}(5 + \text{"their } 2n + 3\text{"})$ $= \{n(5 + n - 1)\} = n(n + 4)$ (*)</p> <p>(d) $43 = 2n + 3$ $[n] = 20$</p> <p>(e) $S_{20} = 20 \times 24, = \underline{480}$ (km)</p>	<p>B1 (1)</p> <p>M1</p> <p>A1 (2)</p> <p>M1 A1</p> <p>A1 cso (3)</p> <p>M1</p> <p>A1 (2)</p> <p>M1 A1 (2)</p> <p>(12 marks)</p>
8.	<p>(a) [No real roots implies $b^2 - 4ac < 0$.] $b^2 - 4ac = q^2 - 4 \times 2q \times (-1)$ So $q^2 - 4 \times 2q \times (-1) < 0$ i.e. $q^2 + 8q < 0$ (*)</p> <p>(b) $q(q + 8) = 0$ or $(q \pm 4)^2 \pm 16 = 0$ $(q) = 0$ or -8 (2 cvs) $-8 < q < 0$ or $q \in (-8, 0)$ or $q < 0$ and $q > -8$</p> <p>(c)</p>	<p>M1</p> <p>A1 cso (2)</p> <p>M1</p> <p>A1</p> <p>A1 ft (3)</p> <p>(5 marks)</p>
9.	<p>(a) $\left[\frac{dy}{dx} = \right] 3kx^2 - 2x + 1$</p> <p>(b) Gradient of line is $\frac{7}{2}$ When $x = -\frac{1}{2}$: $3k \times \left(\frac{1}{4}\right) - 2 \times \left(-\frac{1}{2}\right) + 1, = \frac{7}{2}$ $\frac{3k}{4} = \frac{3}{2} \Rightarrow k = 2$</p> <p>(c) $x = -\frac{1}{2} \Rightarrow y = k \times \left(-\frac{1}{8}\right) - \left(\frac{1}{4}\right) - \frac{1}{2} - 5, = -6$</p>	<p>M1 A1 (2)</p> <p>B1</p> <p>M1</p> <p>A1 A1 (4)</p> <p>M1 A1 (2)</p> <p>(8 marks)</p>

Question Number	Scheme	Marks
<p>10. (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	$QR = \sqrt{(7-1)^2 + (0-3)^2}$ $= \sqrt{36+9} \text{ or } \sqrt{45}$ $= 3\sqrt{5} \text{ or } a=3$ <p>Gradient of QR (or l_1) = $\frac{3-0}{1-7}$ or $\frac{3}{-6}, = -\frac{1}{2}$</p> <p>Gradient of l_2 is $-\frac{1}{-\frac{1}{2}}$ or 2</p> <p>Equation for l_2 is: $y-3=2(x-1)$ or $\frac{y-3}{x-1}=2$ [or $y=2x+1$]</p> <p>P is (0, 1) (allow “$x=0, y=1$” but it must be clearly identifiable as P)</p> $PQ = \sqrt{(1-x_p)^2 + (3-y_p)^2}$ $PQ = \sqrt{1^2 + 2^2} = \sqrt{5}$ <p>Area of triangle is $\frac{1}{2}QR \times PQ = \frac{1}{2}3\sqrt{5} \times \sqrt{5}, = \frac{15}{2}$ or 7.5</p>	<p>M1</p> <p>A1</p> <p>A1 (3)</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1 ft (5)</p> <p>B1 (1)</p> <p>M1</p> <p>A1</p> <p>M1 A1 (4)</p> <p>(13 marks)</p>
<p>11. (a)</p> <p>(b)</p>	$(x^2 + 3)^2 = x^4 + 3x^2 + 3x^2 + 3^2$ $\frac{(x^2 + 3)^2}{x^2} = \frac{x^4 + 6x^2 + 9}{x^2} = x^2 + 6 + 9x^{-2} \quad (*)$ $y = \frac{x^3}{3} + 6x + \frac{9}{-1}x^{-1} (+c)$ $20 = \frac{27}{3} + 6 \times 3 - \frac{9}{3} + c$ $c = -4$ $[y =] \frac{x^3}{3} + 6x - 9x^{-1} - 4$	<p>M1</p> <p>A1 cso (2)</p> <p>M1 A1 A1</p> <p>M1</p> <p>A1</p> <p>A1 ft (6)</p> <p>(8 marks)</p>