

C1 January 2007

1. Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0,$$

find $\frac{dy}{dx}$.

(4)

2. (a) Express
- $\sqrt{108}$
- in the form
- $a\sqrt{3}$
- , where
- a
- is an integer.

(1)

- (b) Express
- $(2 - \sqrt{3})^2$
- in the form
- $b + c\sqrt{3}$
- , where
- b
- and
- c
- are integers to be found.

(3)

3. Given that

$$f(x) = \frac{1}{x}, \quad x \neq 0,$$

- (a) sketch the graph of
- $y = f(x) + 3$
- and state the equations of the asymptotes.

(4)

- (b) Find the coordinates of the point where
- $y = f(x) + 3$
- crosses a coordinate axis.

(2)

4. Solve the simultaneous equations

$$y = x - 2,$$

$$y^2 + x^2 = 10.$$

(7)

5. The equation
- $2x^2 - 3x - (k + 1) = 0$
- , where
- k
- is a constant, has no real roots.

Find the set of possible values of k .

(4)

6. (a) Show that
- $(4 + 3\sqrt{x})^2$
- can be written as
- $16 + k\sqrt{x} + 9x$
- , where
- k
- is a constant to be found.

(2)

- (b) Find
- $\int (4 + 3\sqrt{x})^2 dx$
- .

(3)

7. The curve C has equation $y = f(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C . Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

- (a) find $f(x)$. (5)
- (b) Find an equation for the tangent to C at the point P , giving your answer in the form $y = mx + c$, where m and c are integers. (4)
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8. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

- (a) Find an expression for $\frac{dy}{dx}$. (3)
- (b) Show that the point $P(4, 8)$ lies on C . (1)
- (c) Show that an equation of the normal to C at the point P is

$$3y = x + 20. \quad (4)$$

The normal to C at P cuts the x -axis at the point Q .

- (d) Find the length PQ , giving your answer in a simplified surd form. (3)
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9. Ann has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:

Row 1 □

Row 2 □□

Row 3 □□□

She notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares.

- (a) Find an expression, in terms of n , for the number of sticks required to make a similar arrangement of n squares in the n th row. (3)

Ann continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

- (b) Find the total number of sticks Ann uses in making these 10 rows. (3)

Ann started with 1750 sticks. Given that Ann continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k+1)$ th row,

- (c) show that k satisfies $(3k-100)(k+35) < 0$. (4)

- (d) Find the value of k . (2)
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10. (a) On the same axes sketch the graphs of the curves with equations

(i) $y = x^2(x-2)$, (3)

(ii) $y = x(6-x)$, (3)

and indicate on your sketches the coordinates of all the points where the curves cross the x -axis.

- (b) Use algebra to find the coordinates of the points where the graphs intersect. (7)
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TOTAL FOR PAPER: 75 MARKS