Mark Scheme (Results)

Summer 2017

Pearson Edexcel Advanced Extension Award
In Mathematics (9801/01)

Summer 2017
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - [or d… The second mark is dependent on gaining the first mark

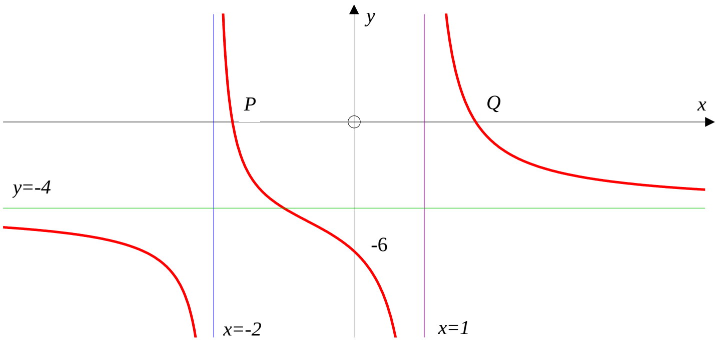
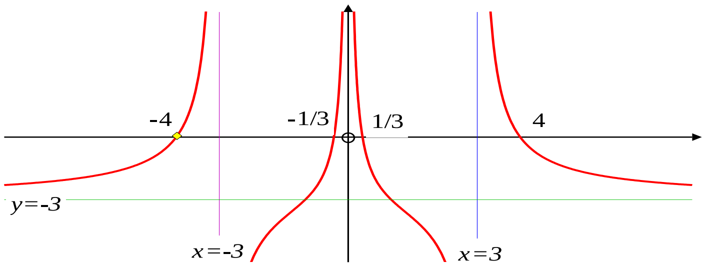
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question	Scheme	Marks	Notes
1. (a)	$\mathrm{f}^{-1}(x) = x^2 - 2$	B1	
	Domain is $x \in \mathbb{R},\ x \geqslant \sqrt{2}$	B1 (2)	
(b)	$\mathrm{g}(x) = (x-2)^2 + 1$ (or differentiation or equivalent)	M1	Suitable method to find min.
	So range is $\mathrm{g}(x) \geqslant 1$	A1 (2)	
(c)	fg(x) = x: $\sqrt{x^2 - 4x + 7} = x$ <u>or</u> $\mathrm{g}(x) = \mathrm{f}^{-1}(x)$: $x^2 - 4x + 5 = $(a)	M1	Attempt suitable eq
	$x^2 - 4x + 7 = x^2$ <u>or</u> $x^2 - 4x + 5 = x^2 - 2$	A1	Simplify x^2 +..= x^2
	$4x = 7$ so $x = \underline{\frac{7}{4}}$	A1 (3)	
		[7]	
2. (a)	$\dfrac{\sin x}{\cos x} = \dfrac{\sqrt{3}}{1 + 4\cos x}$	M1	Use of $\tan x = \frac{\sin x}{\cos x}$
	$4\sin x\cos x + \sin x = \sqrt{3}\cos x \quad \Rightarrow \quad 2\sin 2x + \sin x = \sqrt{3}\cos x$	M1	sin2x = 2sin$x$cos$x$
	$\sin 2x = \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \sin(60 - x)$	M1	Attempt sin(A $\underline{+}$ B)
	sin2x = sin(60 – x) (*)	A1 (4)	(cso)
(b)	2x = 60 – x , 2x = 180 – (60 – x) , 2x = 360 + (60 – x)	M1,M1	2nd and 3rd soln
	***x* = <u>20</u>**	B1	x = 20
	***x* = <u>120</u>**	A1	Ignore extras outside range. If x = 120 and 140 and extras in range then – 1 ee
	***x* = <u>140</u>**	A1 (5) **[9]**	
3. (a)	e.g. 1 – 2s = – 13 + 6t and 10 – 5s = – 1 + 3t	M1	Form 2 eqns in t, s
	[So 14 – 2s = 6t and 22 – 10s = 6t] gives 8 – 8s = 0	M1	Solving
	s = <u>**1**</u>, t = <u>**2**</u>	A1	
	s = 1 gives p = <u>**– 8**</u> and t = 2 gives **b** = $\underline{\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}}$	A1 A1	
		(5)	
(b)	$\overrightarrow{OC} = \begin{pmatrix} -13+6t \\ 7-2t \\ -1+3t \end{pmatrix}$ so $\overrightarrow{AC} = \begin{pmatrix} 6t-14 \\ -2t+15 \\ 3t-11 \end{pmatrix}$ and $\overrightarrow{AC} \bullet \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} = 0$	M1 dM1	Attempt $\overrightarrow{AC}$ in terms of t Intend correct ● = 0
	36t – 84 + 4t – 30 + 9t – 33 = 0 gives 49t = 147	dM1	Solve eqn in t
	t = 3	A1	(ignore t = 2)
	So $\overrightarrow{OC} = \underline{\begin{pmatrix} 5 \\ 1 \\ 8 \end{pmatrix}}$	A1	
		(5)	
(c)	$\lvert\overrightarrow{BC}\rvert = \sqrt{6^2 + (-2)^2 + 3^2} = \sqrt{49} = 7,\ \lvert\overrightarrow{AC}\rvert = \sqrt{4^2 + 9^2 + (-2)^2} = \sqrt{101}$	M1	Attempt at least one
	Area of ΔABD = 2(area ΔABC) = $2 \times \frac{1}{2} \times 7 \times \sqrt{101}$	M1	Correct method
	= $\underline{7\sqrt{101}}$	A1 (3) **[13]**	Correct ans 3/3

Question	Scheme	Marks	Notes
4. (a)	Area of $\Delta LMN = \frac{1}{2}\times 2\times 2\times \sin 60\left[=\sqrt{3}\right]$	M1	Use of area
	Area of $\Delta LPQ = \left[\frac{\sqrt{3}}{2}\right] = \sin 60 = \frac{1}{2}xy\sin 60$ so $\underline{\boldsymbol{xy = 2}}$	A1cso	M1 scored and no incorrect working.
		(2)	
(b)	Let $PQ = d$ and cosine rule: $d^2 = x^2 + y^2 - 2xy\cos 60$	M1	Use of cos rule o.e.
	$d^2 = x^2 + y^2 - xy$ {or 2} and use symmetry or $x^2 + \frac{4}{x^2} - 2$ and diff'n	M1	Use of symmetry or differentiation
	Min when $x = y$ or $x = \sqrt{2}$	A1	
	Shortest length of PQ is $\underline{d = \sqrt{2}}$	M1A1	Method for *d* e.g. $x = y$ in $xy = 2$
		(5)	[S+ reason for min]
(c)	Area of sector is $\frac{1}{2}r^2\frac{\pi}{3}$	M1	
	So equation for *r* is $\frac{1}{2}r^2\frac{\pi}{3} = \frac{\sqrt{3}}{2}$	A1	A correct eqn for *r*
	So $r = \sqrt{\frac{3\sqrt{3}}{\pi}}$	M1	Attempt *r* (allow one slip)
	Arc = $r\theta$ So arc length is $\sqrt{\frac{\pi}{\sqrt{3}}}$	A1	Any correct simplified form
	Rearrange the 6 copies to form a hexagon centre *L*	B1g	Idea of hexagon
	Since circle is best curve for half the area of hexagon by symmetry the circular arc must be best for the triangle *LMN*	B1h	Complete argument
		(6)	
		[13]	
5. (a)	$a = \underline{\mathbf{1}}$; $b = \underline{\mathbf{3}}$	B1;B1	
		(2)	
(b)(i)		M1	A horizontal translation (left)
		M1	A vertical translation (down)
		A1ft	x = -2 and x = 1 ft their b – 2 Stated or on graph
		A1	y = -4&(0, -6) or -6 correctly marked
	$y = 0 \Rightarrow 4x + 4 = 4x^2 + 4x - 8$	M1	Attempt to find *P*,*Q* (correct eq'n)
	So $x = \pm\sqrt{3}$	A1 (6)	*P*,*Q* correctly marked
(b)(ii)		B1	Shape for $x > 0$ and crossing x-axis (x2)
		B1ft	Shape for $x < 0$ (Symmetry)
		B1	Asymptotes $x = \pm 3$ (both) $x = 0$ can be implied $y = -3$
	$f(x) - 3 = 0 \Rightarrow\ 4x - 4 - 3x^2 + 9x = 0$ [or $3x^2 - 13x + 4 = 0$]	M1	A correct equation
	$[(3x-1)(x-4) = 0]$ so $\underline{x = 4 \text{ or } \frac{1}{3}}$	A1	Points must be identifiable on sketch
	and $\underline{x = -4 \text{ or } -\frac{1}{3}}$	A1ft (6)	
		[14]	

Question	Scheme	Marks	Notes
6 (a)	$\frac{\mathrm{d}}{\mathrm{d}u}\ln\left(u+\sqrt{u^2-1}\right)=\frac{1}{u+\sqrt{u^2-1}}\times\left[1+\frac{1}{\cancel{2}}\left(u^2-1\right)^{-\frac{1}{2}}\times\cancel{2}u\right]$	M1	For an attempt at chain rule. Allow one slip.
	$=\left\{\frac{1}{u+\sqrt{u^2-1}}\times\left[\sqrt{u^2-1}+u\right]\times\frac{1}{\sqrt{u^2-1}}\right\}=\underline{\frac{1}{\sqrt{u^2-1}}}$ **(*)**	A1cso (2)	No incorrect working seen
(b)	$\mathrm{d}x=-\frac{1}{t^2}\ \mathrm{d}t\quad\Rightarrow,\ \ I=-\int\frac{1}{t^2}\times\frac{t}{1}\times\frac{1}{\sqrt{\frac{2}{t}-6+7}}\ \mathrm{d}t$	M1, M1	d*x* = …d*t* Integrand in *t*
	$I=-\int\frac{1}{\sqrt{t^2+2t}}\ \mathrm{d}t\quad;\quad\text{so }\ I=-\int\frac{1}{\sqrt{(t+1)^2-1}}\ \mathrm{d}t$	A1; M1	Correct & simplified Attempt to complete the square to use (a)
	$I=-\ln\left[\left(t+1\right)+\sqrt{\left(t+1\right)^2-1}\right]\quad(+c)$	M1	Use of (a)
	$=-\ln\left[\frac{x+4}{x+3}+\sqrt{\frac{2x+7}{\left(x+3\right)^2}}\right]$ <u>or</u> $=\ln\left(x+3\right)-\ln\left(x+4+\sqrt{2x+7}\right)$	A1 (6)	Correct integral in terms of *x*
(c)	$\frac{1}{2x^2+13x+21}=\frac{A}{x+3}+\frac{B}{2x+7}\ ;\ =\frac{1}{x+3}-\frac{2}{2x+7}$	M1; A1 (2)	Correct split Correct *A* and *B*
(d)	$\int J=\int\frac{1}{(x+3)\sqrt{2x+7}}\ \mathrm{d}x-\int\frac{2}{(2x+7)\sqrt{2x+7}}\ \mathrm{d}x$	M1	Use the P/F to split the integral
	$=\left[I\ \text{or (b)}\right]-\int 2(2x+7)^{-\frac{3}{2}}\ \mathrm{d}x$	M1	Prep 2nd integral $(..)^{\pm\frac{3}{2}}$
	$=[\ I\ \text{or their (b)}]\ +\ 2(2x+7)^{-\frac{1}{2}}\quad(+c)$	A1	Correct form for 2nd integral., ignore +*c*
	$\int_1^9 J=\left(\ln 12-\ln\left(13+5\right)+\frac{2}{5}\right)-\left(\ln 4-\ln\left(5+3\right)+\frac{2}{3}\right)$	M1	Clear use of both limits. Ft their integr'
	$=\ln\left(\frac{12}{4}\right)+\ln\left(\frac{8}{18}\right)-\frac{4}{15}\quad;\quad=\underline{\ln\left(\frac{4}{3}\right)-\frac{4}{15}}$	M1; A1 (6) **[16]**	Some correct use of ln*a* – ln*b* rule. Correct *r* and *s*

Questions	Mark	Awarding of S and T marks
2, 3	S1	For a fully correct solution that is succinct or includes an S+ point
4-7	S2	For a fully correct solution that is succinct or includes an S+ point
4-7	S1	For a fully correct solution that is succinct but has an S – point
4-7	S1	For a fully correct solution that is slightly laboured but includes an S+ point
4-7	S1	For a score of *n* -1 but solution is otherwise succinct or contains an S+ point
		Maximum of 6 S marks
ALL	T1	For at least half marks on every question

Question	Scheme	Marks	Notes
7 (a)	Curve – Line (gives LHS)	B1g	Reason for LHS
	Has roots $x = p$ and $x = q$, since L is a tangent roots are "double" or since these are only roots need squares to balance powers	B1h	Mention roots & reason for squares not e.g. $(x-p)(x-q)^3$
		(2)	
(b)	$\int_p^q (C-L)\,\mathrm{d}x = \int_p^q (x-p)^2(x-q)^2\,\mathrm{d}x = \int_p^q (x-p)^2\,\mathrm{d}\left[\frac{(x-q)^3}{3}\right]$	M1	Attempt 1st step of integration by parts
	$= \cancel{\left[(x-p)^2\frac{(x-q)^3}{3}\right]_p^q} - \int_p^q 2(x-p)\frac{(x-q)^3}{3}\,\mathrm{d}x$	A1 A1	Correctly get 1st integral = 0 Correct 2nd integral
	$= -\int_p^q 2(x-p)\,\mathrm{d}\left[\frac{(x-q)^4}{12}\right]$	M1	Attempt 2nd step of integration by parts
	$= \cancel{\left[-2(x-p)\frac{(x-q)^4}{12}\right]_p^q} - -\int_p^q 2\frac{(x-q)^4}{12}\,\mathrm{d}x = \int_p^q \frac{(x-q)^4}{6}\,\mathrm{d}x$	A1	Correct work leading to this single integral including zeros seen
	$= \left[\frac{(x-q)^5}{30}\right]_p^q = 0 - \frac{(p-q)^5}{30} = \frac{(q-p)^5}{30}$	A1cso	No incorrect working seen leading to this.
	No correct use of the limits can score M1A0A1M1A0A0		
		(6)	
(c)	$(x-p)^2(x-q)^2 = (x^2-2px+p^2)(x^2-2qx+q^2)$ (o.e.)	M1	1st step(⇒ by S, T, U)
	$= x^4 - 2(p+q)x^3; + (\underline{p^2+q^2+4pq})x^2 + \ldots$ (o.e.)	A1cso; $\underline{\text{A1}}$	1st 2 terms cso Correct expr' for $\underline{S}$
	$\ldots -(\underline{2pq^2+2qp^2})x + \underline{\underline{p^2q^2}}$ (o.e.)	A1 A1	Correct $\underline{T}$ and $\underline{\underline{U}}$
		(5)	
(d)	$x^3 \Rightarrow p+q=5$	B1	
	$x^2 \Rightarrow p^2+q^2+4pq=33$ so $33=(p+q)^2+2pq$ [so $pq = 4$]	M1	2nd eqn and use of $p + q = k$
	$q(5-q)=4 \Rightarrow q^2-5q+4=0$ or $(q-4)(q-1)=0$	M1	Solving eqn in 1 variable
	So q = 1 or 4	A1	For 1 and 4
	Since $q > p$ (from diagram) **$\underline{p = 1}$** and **$\underline{q = 4}$**	A1	For p = 1 and q = 4
	Using p and q the value of $T = 2pq(p+q) = 40$	M1	Use of T
	Comparing x gives : $34 + m = 40$ so $m = 6$	A1	$m = 6$
	$c = -U = -p^2q^2 = -4^2 = -16$ so equation of L is **$\underline{y = 6x - 16}$**	A1	For y = "6"x – 16
		(8) **[21]**	

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