Paper Reference(s)

9801

Edexcel GCE

Mathematics

Advanced Extension Award

Friday 2 July 2004 – Afternoon

Time: 3 hours

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, other names and signature.

Answers should be given in as simple a form as possible, e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae including Statistical Formulae and Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

P18543A

1. Solve the equation $\cos x + \sqrt{(1 - \tfrac{1}{2}\sin 2x)} = 0$, in the interval $0° \leq x < 360°$.

(9)

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2. (*a*) For the binomial expansion of $\dfrac{1}{(1-x)^2}$, $|x| < 1$, in ascending powers of x,

(i) find the first four terms,

(ii) write down the coefficient of x^n.

(2)

(*b*) Hence, show that, for $|x| < 1$, $\displaystyle\sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2}$.

(2)

(*c*) Prove that, for $|x| < 1$, $\displaystyle\sum_{n=1}^{\infty} (an+1)x^n = \frac{(a+1)x - x^2}{(1-x)^2}$, where a is a constant.

(4)

(*d*) Hence evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{5n+1}{2^{3n}}$.

(2)

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3. $f(x) = x^3 - (k+4)x + 2k$, where k is a constant.

(*a*) Show that, for all values of k, the curve with equation $y = f(x)$ passes through the point (2, 0).

(1)

(*b*) Find the values of k for which the equation $f(x) = 0$ has exactly two distinct roots.

(5)

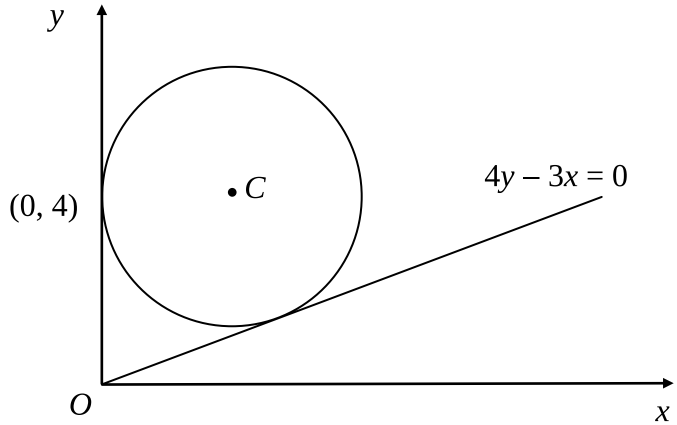
Given that $k > 0$, that the x-axis is a tangent to the curve with equation $y = f(x)$, and that the line $y = p$ intersects the curve in three distinct points,

(*c*) find the set of values that p can take.

(5)

4.

Figure 1



The circle, with centre C and radius r, touches the y-axis at (0, 4) and also touches the line with equation $4y - 3x = 0$, as shown in Fig. 1.

(*a*) (i) Find the value of r.

(ii) Show that $\arctan\left(\frac{3}{4}\right) + 2\ \text{acrtan}\left(\frac{1}{2}\right) = \frac{1}{2}\pi$.

(8)

The line with equation $4x + 3y = q$, $q > 12$, is a tangent to the circle.

(*b*) Find the value of q.

(4)

TURN OVER FOR QUESTION 5

5. (*a*) Given that $y = \ln [t + \sqrt{(1 + t^2)}]$, show that $\dfrac{dy}{dt} = \dfrac{1}{\sqrt{(1+t^2)}}$.

(3)

The curve C has parametric equations

$$x = \frac{1}{\sqrt{(1+t^2)}}, \quad y = \ln [t + \sqrt{(1 + t^2)}], \quad t \in \mathbb{R}.$$

A student was asked to prove that, for $t > 0$, the gradient of the tangent to C is negative.

The attempted proof was as follows:

$$\begin{aligned} y &= \ln\left(t + \frac{1}{x}\right) \\ &= \ln\left(\frac{tx+1}{x}\right) \\ &= \ln(tx + 1) - \ln x \\ \therefore \frac{dy}{dx} &= \frac{t}{tx+1} - \frac{1}{x} \\ &= \frac{\frac{t}{x}}{t + \frac{1}{x}} - \frac{1}{x} \\ &= \frac{t\sqrt{(1+t^2)}}{t + \sqrt{(1+t^2)}} - \sqrt{(1+t^2)} \\ &= -\frac{(1+t^2)}{t+\sqrt{(1+t^2)}} \end{aligned}$$

As $(1 + t^2) > 0$, and $t + \sqrt{(1 + t^2)} > 0$ for $t > 0$, $\dfrac{dy}{dx} < 0$ for $t > 0$.

(*b*) (i) Identify the error in this attempt.

(ii) Give a correct version of the proof.

(6)

(*c*) Prove that $\ln [-t + \sqrt{(1 + t^2)}] = -\ln [t + \sqrt{(1 + t^2)}]$.

(3)

(*d*) Deduce that C is symmetric about the x-axis and sketch the graph of C.

(3)

6. $$f(x) = x - [x], \quad x \geq 0$$

where $[x]$ is the largest integer $\leq x$.

For example, $f(3.7) = 3.7 - 3 = 0.7$; $f(3) = 3 - 3 = 0$.

(*a*) Sketch the graph of $y = f(x)$ for $0 \leq x < 4$.

(3)

(*b*) Find the value of p for which $\int_2^p f(x)\,dx = 0.18$.

(3)

Given that

$$g(x) = \frac{1}{1+kx}, \quad x \geq 0, \quad k > 0,$$

and that $x_0 = \frac{1}{2}$ is a root of the equation $f(x) = g(x)$,

(*c*) find the value of k.

(2)

(*d*) Add a sketch of the graph of $y = g(x)$ to your answer to part (*a*).

(1)

The root of $f(x) = g(x)$ in the interval $n < x < n + 1$ is x_n, where n is an integer.

(*e*) Prove that

$$2x_n^2 - (2n - 1)x_n - (n + 1) = 0.$$

(4)

(*f*) Find the smallest value of n for which $x_n - n < 0.05$.

(4)

TURN OVER FOR QUESTION 7

7. Triangle ABC, with $BC = a$, $AC = b$ and $AB = c$ is inscribed in a circle. Given that AB is a diameter of the circle and that a^2, b^2 and c^2 are three consecutive terms of an arithmetic progression (arithmetic series),

(*a*) express b and c in terms of a,

(4)

(*b*) verify that cot A, cot B and cot C are consecutive terms of an arithmetic progression.

(3)

In an acute-angled triangle PQR the sides QR, PR and PQ have lengths p, q and r respectively.

(*c*) Prove that

$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R}.$$

(3)

Given now that triangle PQR is such that p^2, q^2 and r^2 are three consecutive terms of an arithmetic progression,

(*d*) use the cosine rule to prove that $\dfrac{2\cos Q}{q} = \dfrac{\cos P}{p} + \dfrac{\cos R}{r}$.

(6)

(*e*) Using the results given in parts (*c*) and (*d*), prove that cot P, cot Q and cot R are consecutive terms in an arithmetic progression.

(3)

Marks for style, clarity and presentation: 7

TOTAL FOR PAPER: 100 MARKS

END