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British Mathematical Olympiad

Round 2 : Tuesday, 31 January 2006

Time allowed *Three and a half hours.*

Each question is worth 10 marks.

Instructions

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt.*

  Rough work should *be handed in, but should be clearly marked.*
- *One or two* complete *solutions will gain far more credit than partial attempts at all four problems.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Staple all the pages neatly together in the top* left *hand corner, with questions 1,2,3,4 in order, and the cover sheet at the front.*

In early March, twenty students will be invited to attend the training session to be held at Trinity College, Cambridge (6-10 April). At the training session, students sit a pair of IMO-style papers and 8 students will be selected for further training. Those selected will be expected to participate in correspondence work and to attend further training. The UK Team of 6 for this summer's International Mathematical Olympiad (to be held in Ljubljana, Slovenia 10-18 July) will then be chosen.

Do not turn over until **told to do so.**

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2005/6 British Mathematical Olympiad
Round 2

1. Find the minimum possible value of $x^2 + y^2$ given that x and y are real numbers satisfying
$$xy(x^2 - y^2) = x^2 + y^2 \text{ and } x \neq 0.$$

2. Let x and y be positive integers with no prime factors larger than 5. Find all such x and y which satisfy
$$x^2 - y^2 = 2^k$$
for some non-negative integer k.

3. Let ABC be a triangle with $AC > AB$. The point X lies on the side BA extended through A, and the point Y lies on the side CA in such a way that $BX = CA$ and $CY = BA$. The line XY meets the perpendicular bisector of side BC at P. Show that
$$\angle BPC + \angle BAC = 180^o.$$

4. An exam consisting of six questions is sat by 2006 children. Each question is marked either right or wrong. Any three children have right answers to at least five of the six questions between them. Let N be the total number of right answers achieved by all the children (i.e. the total number of questions solved by child 1 + the total solved by child 2 + $\cdots$ + the total solved by child 2006). Find the least possible value of N.