Diophantine Equations

Note that the set of *integers* is

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

Note also that the set of *positive integers* is

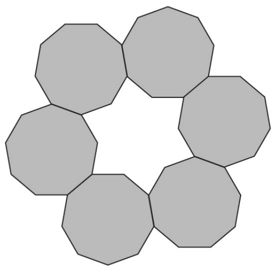
$$\{1, 2, 3, 4, 5, \ldots\}.$$

1. Find all integer solutions to $mn = 2$. $(m, n) = (2, 1), (1, 2), (-1, -2), (-2, -1)$
2. Find all integer solutions to $mn = -3$. $(m, n) = (-3, 1), (3, -1)$
3. Find all integer solutions to $(m - 5)n = 2$. $(m, n) = (7, 1), (6, 2), (4, -2), (3, -1)$
4. Find all integer solutions to $mn + m = -4$. $(m, n) = (1, -5), (-1, 3), (-2, 1), (2, -3), (-4, 0), (4, -2)$
5. Find all integer solutions to $(m - 3)(n + 10) = 1$. $(m, n) = (4, -9), (2, -11)$
6. Find all integer solutions to $mn + m + 2n = 1$. $(m, n) = (1, 0), (-1, 2), (-5, -2), (-3, -4)$
7. Find all integer solutions to $mn + 4m = 3n + 16$. $(m, n) = (7, -3), (4, 0), (5, -2), (-1, -5), (1, -6), (2, -8)$
8. Find all integer solutions to $\frac{6}{mn} + \frac{7}{n} + \frac{1}{m} + 1 = 0$. $(m, n) = (-2, -8)$ (only)
9. Find all integer solutions to $1 = \frac{1}{m} + \frac{2}{n}$. $(m, n) = (3, 3), (2, 4), (-1, 1)$ (only)
10. Find all integer solutions to $1 = \frac{3}{m} + \frac{2}{n}$. $(m, n) = (4, 8), (2, -4), (9, 3), (-3, 1), (5, 5), (1, -1), (6, 4)$
11. Find all integer solutions to $0 = 1 + \frac{3}{m} + \frac{4}{n} + \frac{11}{mn}$. □
12. Maclaurin 2015 problem.

    A symmetrical ring of m identical regular n-sided polygons is formed according to the rules:

    (a) each polygon in the ring meets exactly two others;
    (b) two adjacent polygons have only an edge in common; and
    (c) the perimeter of the inner region enclosed by the ring consists of exactly two edges of each polygon.

    The example in the figure shows a ring with $m = 6$ and $n = 9$. For how many different values of n is such a ring possible?



13. Maclaurin 2011 problem.

    How many solutions are there to the equation $x^2 + y^2 = x^3$, where x and y are positive integers and x is less than 2011?

14. The final digit of a door entry code is 9. If this digit (9) is moved from the end of the code to the very start of the code, and all other digits are left unchanged, the resulting number is 4 times the door entry code. Find the code.

15. BMO problem.

    N is a four-digit positive integer, not ending in zero, and $R(N)$ is the four-digit integer obtained by reversing the digits of N. For example, $R(3475) = 5743$. Determine all such integers N for which $R(N) = 4N + 3$.

16. STEP problem.

    (a) Find all sets of positive integers a, b and c that satisfy the equation

    $$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1.$$

    (b) Determine the sets of positive integers a, b and c that satisfy the inequality

    $$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geqslant 1.$$