Single Pure - Binomial Expansion

1. Expand and simplify the following:

   (a) $(x+2)^3$. $\boxed{x^3+6x^2+12x+8}$

   (b) $(3x+1)^4$. $\boxed{81x^4+108x^3+54x^2+12x+1}$

   (c) $(x-3)^4$. $\boxed{x^4-12x^3+54x^2-108x+81}$

   (d) $(x-1)^5$. $\boxed{x^5-5x^4+10x^3-10x^2+5x-1}$

   (e) $\left(\frac{2}{x}+1\right)^4$. $\boxed{\frac{16}{x^4}+\frac{32}{x^3}+\frac{24}{x^2}+\frac{8}{x}+1}$

   (f) $(3-x^2)^5$. $\boxed{243-405x^2+270x^4-90x^6+15x^8-x^{10}}$

   (g) $(2x^2+3)^4$. $\boxed{16x^8+96x^6+216x^4+216x^2+81}$

   (h) $(2x-5y)^3$. $\boxed{8x^3-60x^2y+150xy^2-125y^3}$

   (i) $(p+3q^2)^5$. $\boxed{p^5+15p^4q^2+90p^3q^4+270p^2q^6+405pq^8+243q^{10}}$

   (j) $(ab-c)^4$. $\boxed{a^4b^4-4a^3b^3c+6a^2b^2c^2-4abc^3+c^4}$

   (k) $\left(x^2+\frac{2}{x}\right)^4$. $\boxed{x^8+8x^5+24x^2+\frac{32}{x}+\frac{16}{x^4}}$

   (l) $(2+\frac{1}{y^3})^3$. $\boxed{8+\frac{12}{y^3}+\frac{6}{y^6}+\frac{1}{y^9}}$

   (m) $(\sqrt{x}-y^2)^4$. $\boxed{x^2-4x^{\frac{3}{2}}y^2+6xy^4-4\sqrt{x}y^6+y^8}$

   (n) $\left(\frac{x^2}{2}+\frac{2y^3}{3}\right)^3$. $\boxed{\frac{x^6}{8}+\frac{x^4y^3}{2}+\frac{2x^2y^6}{3}+\frac{8y^9}{27}}$

   (o) $(x^2+x+1)^3$. $\boxed{x^6+3x^5+6x^4+7x^3+6x^2+3x+1}$

   (p) $(z^2-4z-3)^4$. $\boxed{z^8-16z^7+84z^6-112z^5-266z^4+336z^3+756z^2+432z+81}$

2. In each of the following, find the required coefficients. *Clearly* this should be done without doing the whole multiplication.

   (a) Coefficient of x^2 in $(2x+3)^8$. $\boxed{81648}$

   (b) Coefficient of m^4 in $(m-3)^7$. $\boxed{-945}$

   (c) Coefficient of r^4 in $(3r-r^2)^3$. $\boxed{-27}$

   (d) Coefficient of x^8 in $(1-\frac{3x^2}{2})^9$. $\boxed{\frac{5103}{8}}$

   (e) Coefficient of $x^{\frac{5}{2}}$ in $(2-5\sqrt{x})^7$. $\boxed{-262500}$

   (f) Coefficient of a^3b^5 in $(-2a-\frac{5}{2}b)^8$. $\boxed{43750}$

3. Find the value of each of the following:

   (a) $\binom{8}{5}$. $\boxed{56}$

   (b) ${}^7C_2$. $\boxed{21}$

   (c) $\binom{n}{n}$. $\boxed{1}$

   (d) $\binom{p}{p-1}$. $\boxed{p}$

   (e) $\binom{n}{n-2}$. $\boxed{\frac{n(n-1)}{2}}$

4. Explain why $\binom{42}{38}$ is the same as $\binom{42}{4}$.

5. Express the following in the form $a + b\sqrt{2}$:

   (a) $(3+\sqrt{2})^3$. $\boxed{45+29\sqrt{2}}$

   (b) $(1-3\sqrt{2})^4$. $\boxed{433-228\sqrt{2}}$

6. Simplify the following (but look for shortcuts!):

   (a) $(1+\sqrt{3})^4+(1-\sqrt{3})^4$. $\boxed{56}$

   (b) $(\sqrt{6}+\sqrt{3})^4-(\sqrt{6}-\sqrt{3})^4$. $\boxed{216\sqrt{2}}$

7. Find the term independent of x in the following expansions:

   (a) $\left(x+\frac{2}{x}\right)^4$. $\boxed{24}$

   (b) $\left(2x^2-\frac{3}{x}\right)^6$. $\boxed{4860}$

   (c) $\left(ax^n+\frac{b}{x^m}\right)^{5m+5n}$. $\boxed{\binom{5m+5n}{5m}a^{5m}b^{5n}}$

8. (a) Find the first three terms in the expansion of $(1+4x)^{12}$ in ascending powers of x. $\boxed{1+48x+1056x^2+\cdots}$

   (b) By substituting a suitable value for x, find an approximation to 1.004^{12} to 3 decimal places. $\boxed{1.049}$

9. Using the first 3 terms of $(2+x)^5$ (in ascending powers of x) find 2.001^5 to five decimal places without using a calculator. $\boxed{32.08008}$

10. Given that $(2+kx)^n = 64-576x+cx^2+\cdots$, find n, k and c. $\boxed{n=6, k=-3, c=2160}$

11. Given that $(1+cx)^n = 1+15x+90x^2+\cdots$, find c and n. $\boxed{c=3, n=5}$

12. Find the first four terms (in ascending powers of x) in the expansion of $(2+3x)(1+x)^{10}$. $\boxed{2+23x+120x^2+375x^3+\cdots}$

13. Find the first four terms (in ascending powers of x) in the expansion of $(1-2x)^2(1+x)^{11}$. $\boxed{1+7x+15x^2-11x^3+\cdots}$

14. Find the x^3 coefficient in the expansion of $(x-3)(x+2)^5$. $\boxed{-40}$

15. Find the x^4 coefficient in the expansion of $(x+1)(x-3)^{10}$. $\boxed{-109350}$

16. Find the x^5 coefficient in the expansion of $(x-2)^2(x+1)^8$. $\boxed{0}$

17. Find the x^6 coefficient in the expansion of $(2x-1)^2(x-2)^{10}$. $\boxed{89376}$

18. Find an expression for the x^3 coefficient in the expansion of $(1+ax)(x+b)^5$. $\boxed{10ab^3+10b^2}$

19. Find an expression for the x^4 coefficient in the expansion of $(1+ax)(ax-b)^7$. $\boxed{35a^4b^3(b-1)}$

20. Find an expression for the x^5 coefficient in the expansion of $(a+x)^2(ax-1)^{12}$. $\boxed{990a^5-792a^7-220a^3}$

21. ⋆ Prove that $\binom{n}{r-1}+2\binom{n}{r}+\binom{n}{r+1}=\binom{n+2}{r+1}$.

22. ⋆ Find the coefficient of x^6 in the expansion of

$$(1-2x+3x^2-4x^3+5x^4)^3.$$

$\boxed{369}$