

Sequences

When given a sequence of numbers it is often useful to look at the difference between terms to find a pattern. For example given the triangular numbers we can see the difference has a readily identifiable pattern.

△ Numbers	1	3	6	10	15	21	28
Difference	+2	+3	+4	+5	+6	+7	

We can use this pattern to work out the next few terms.

Linear Sequences

When there is a constant difference between the terms then the sequence is *linear*. The formula for the n^{th} term is given by:

$$T = (\text{diff. between terms})n + (\text{zeroth term}).$$

So for example

$$\begin{aligned} 5, 8, 11, 14, 17 \dots &\Rightarrow T = 3n + 2 \\ 5, 1, -3, -7, -11 \dots &\Rightarrow T = -4n + 9 \\ -3, -1, 1, 3, 5 \dots &\Rightarrow T = 2n - 5 \end{aligned}$$

Quadratic Sequences

If the second difference is constant then you are dealing with a *quadratic sequence*. To work out the formula we do the following;

1. Halve the second difference to get the number of n^2 s (e.g. if you had a second difference of +6 you would have $+3n^2$).
2. Write out the original sequence above the terms of your number of n^2 s.
3. Subtract the n^2 s from the sequence to give the RESIDUE.
4. The residue will either be constant or a linear sequence. If it is a linear sequence then work out its formula.
5. Finally add the number of n^2 s to the formula for the residue and this will be the formula for the original sequence.

Two Examples

- Find the formula for the sequence 7, 16, 31, 52, 79...

Difference table:

7	16	31	52	79
+9	+15	+21	+27	
	+6	+6	+6	

So we have quadratic sequence with $+3n^2$ (half of +6).

So write out sequence and $+3n^2$ and subtract to find residue.

SEQUENCE	7	16	31	52	79
$3n^2$	3	12	27	48	75
RESIDUE	4	4	4	4	4

So the residue is just 4 so the overall formula for the sequence 7, 16, 31, 52, 79... is $T = 3n^2 + 4$.

- Find the formula for the sequence 3, 13, 27, 45, 67...

Difference table:

3	13	27	45	67
+10	+14	+18	+22	
	+4	+4	+4	

So we have quadratic sequence with $+2n^2$ (half of +4).

So write out sequence and $+2n^2$ and subtract to find residue.

SEQUENCE	3	13	27	45	67
$2n^2$	2	8	18	32	50
RESIDUE	1	5	9	13	17

So the formula for the residue is $4n - 3$ so the overall formula for the sequence 3, 13, 27, 45, 67... is $T = 2n^2 + 4n - 3$.