ULSEB/ULEAC

Special Paper

Pure Mathematics Questions

June 1992 to June 1995

1. (*a*) By considering the recurring decimal 0.444 444 ... as the sum of an infinite geometric series express 0.444 444 ... in the form $\dfrac{p}{q}$, where p and q are integers.

(*b*) Find the sum of the first n terms of the series

$$4 + 44 + 444 + 4444 + 44444 + ...$$

(13 marks)

[June 1995 Module P1]

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2. The quadratic equation

$$4x^2 - 4(k + 3)x + 5k + 8 = 0,$$

where k is a real number, has roots α and β.

(*a*) Show that α and β are real and unequal for all values of k.

(*b*) Given that k varies, find the least value of $|\alpha - \beta|$.

Given that $\alpha = 2\beta$,

(c) determine the values of k and β.

(13 marks)

[June 1995 Module P1]

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3. The curve C has equation

$$y = \lambda x(x - 1)(x - 2),$$

where λ is a non-zero constant.

(*a*) Find the values of λ for which the tangent to C at the point (1, 0) is perpendicular to the tangent to C at the point (2, 0).

(*b*) Prove that $\displaystyle\int_{-m}^{(m+2)} y \; \mathrm{d}x = 0$, for all values of m.

(13 marks)

[June 1995 Module P1]

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4. (*a*) By using the substitution $x - \dfrac{1}{2} = \dfrac{\sqrt{3}}{2}\tan\theta$, or otherwise, show that

$$\int \frac{1}{x^2 - x + 1}\,\mathrm{d}x = \lambda \arctan\left(\frac{2x-1}{\sqrt{3}}\right) + k, \text{ where } \lambda \text{ and } k \text{ are constants.}$$

State the value of λ.

(*b*) Express $\dfrac{1}{x^3 + 1}$ as the sum of partial fractions and hence, or otherwise, find $\displaystyle\int \frac{1}{x^3+1}\,\mathrm{d}x$.

(*c*) Show that $\displaystyle\int_1^2 \frac{1}{x^3+1}\,\mathrm{d}x = \frac{1}{6}\ln\frac{3}{4} + \frac{\pi}{18}\sqrt{3}$.

(13 marks)

[June 1995 Module P2]

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5. Given that $y = e^{x\sqrt{3}} \cos x$,

(*a*) show that $\dfrac{dy}{dx} = 2e^{x\sqrt{3}} \cos (x + \dfrac{\pi}{6})$ and that $\dfrac{d^2y}{dx^2} = 4e^{x\sqrt{3}} \cos (x + \dfrac{\pi}{3})$.

(*b*) Sketch the curve with equation $y = e^{x\sqrt{3}} \cos x$ for $0 \le x \le \dfrac{\pi}{2}$ indicating clearly the turning point, and giving its coordinates.

(*c*) Show that the area of the finite region bounded by the *x*-axis, the *y*-axis, and the curve between (0, 1) and $(\dfrac{\pi}{2}, 0)$ is exactly $\frac{1}{4}\left(e^{\frac{\pi\sqrt{3}}{2}} - \sqrt{3}\right)$.

(13 marks)

[June 1995 Module P2]

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6. With respect to an origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively.

(*a*) Show that the point Q with position vector $(1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$ is collinear with A and B for all values of λ.

The point P is the foot of the perpendicular from O to the line AB.

(*b*) Show that P has position vector $\mathbf{p}$ where

$$\mathbf{p} = (1 - \mu)\mathbf{a} + \mu\mathbf{b} \text{ and } \mu = \frac{-\mathbf{a}.(\mathbf{b} - \mathbf{a})}{|\mathbf{b} - \mathbf{a}|^2}.$$

Given that A has coordinates (1, 2, 2) and B has coordinates (2, 4, 3),

(*c*) find the position vector of the point P.

The points L and M lie on this line AB and are such that OL and OM make angles of 45° with AB.

(*d*) Show that the values of λ at L and M are $\dfrac{-7 + \sqrt{5}}{6}$ and $\dfrac{-7 - \sqrt{5}}{6}$.

(13 marks)

[June 1995 Module P2]

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7. (i) Determine the values of α, $0 \leq \alpha \leq 2\pi$, and β, $0 \leq \beta \leq 2\pi$, which satisfy simultaneously

$$2 \cos \alpha - \tan \beta = -2,$$
$$4 \sin^2 \beta + 3 \sec^2 \beta = 9.$$

(ii) Find the set of values of x, $0 \leq x \leq 360$, for which

$$2 \tan x° + \sec x° + 1 > 0,$$

giving your answers in degrees to one decimal place.

(13 marks)

[June 1994 Module P1]

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8. (*a*) Find the distance between the parallel lines with equations $x - 2y - 3 = 0$ and $x - 2y + 3 = 0$.

The circle C touches these lines. The centre of C is Q, and the tangents to C from the origin O meet C at the points A and B. The chord AB passes through the point (3, 2) and meets the line OQ at the point P.

(*b*) Find the distance OP.

Given that $\angle POA = \alpha$,

(*c*) show that $\sin \alpha = \frac{1}{3}$.

(*d*) Hence, or otherwise, find the coordinates of Q.

(13 marks)

[June 1994 Module P1]

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9. The binomial coefficients $\binom{n}{r}$, $0 \le r \le n$, are denoted by a_r and the binomial coefficients $\binom{n-1}{r}$, $0 \le r \le n$, by b_r.

(*a*) Show that $b_{r-1} = \frac{r}{n} a_r$, $(1 \le r \le n)$

(*b*) Show that $b_r + b_{r-1} = a_r$, $(1 \le r \le n - 1)$, and hence show that

$$a_0 - a_1 + a_2 - \ldots. + (-1)^r a_r = (-1)^r b_r, \ (0 \le r < n).$$

(*c*) By considering the coefficient of x^n on each side of the identity $(1 + x)^n (1 + x)^n \equiv (1 + x)^{2n}$, or otherwise, show that

$$a_0^2 + a_1^2 + a_2^2 + \ldots. + a_n^2 = \frac{(2n)!}{(n!)^2}.$$

(13 marks)

[June 1994 Module P1]

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10. Express $\cos 3\theta$ in terms of $\cos \theta$.

(*a*) Express $\cos 3\theta - 2 \cos 2\theta + 4 \cos \theta - 3$ as a cubic polynomial in $\cos \theta$.

By factorising this polynomial into the product of a linear and quadratic factor, show that, for all real values of θ,

$$\cos 3\theta - 2 \cos 2\theta + 4 \cos \theta \le 3.$$

(*b*) Show that the least value of $\cos 3\theta + \cos 2\theta - 2 \cos \theta$, for all real values of θ, is $-2\frac{1}{2}$.

(13 marks)

[June 1994 Module P2]

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11. The points P and Q represent the complex numbers 3i and $\frac{1}{2}(\sqrt{3} - \text{i})$ respectively. The internal bisector of the angle POQ meets PQ at the point R.

(*a*) Find the complex number r represented by R.

The mid-point S of PQ is represented by the complex number s.

(*b*) Express $\dfrac{s}{r}$ in the form $a + \text{i}b$, where a and b are real.

(*c*) Hence, or otherwise, find the area of the triangle ORS.

(13 marks)

[June 1994 Module P2]

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12. A long-distance runner sets out to run a 40 km race. He starts at a constant speed of U km h^{-1} and maintains this speed for a distance of $(40 - \frac{1}{20}U^2)$ km. After running this distance, his speed v km h^{-1} decreases at a rate of v km h^{-2}.

(*a*) Show that, at a time t hours after he has started to slow down his speed, the distance he has covered is

$$[40 - \tfrac{1}{20}U^2 + \text{U}(1 - \text{e}^{-t})] \text{ km}.$$

(*b*) Given that he finishes the race, show that $U < 20$.

(*c*) Show that, for differing values of U, his total time taken to complete the race is least if U satisfies the equation

$$U^3 + 800U - 16\,000 = 0.$$

Hence show that his fastest time for finishing the race is achieved if U lies between 15 and 16 km h^{-1}.

(13 marks)

[June 1994 Module P2]

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13. $$\mathrm{f}(x) \equiv ax^3 + x^2 + bx + c,$$

where a, b and c are positive constants.

The graph of $y = \mathrm{f}(x)$ intersects the y-axis at the point P. The tangent at P to the curve $y = \mathrm{f}(x)$ meets the curve again at the point Q.

(*a*) Find, in terms of a, b and c, the coordinates of Q.

Given that the normal at P to the curve $y = \mathrm{f}(x)$ is also a tangent, at the point R, to the curve,

(*b*) determine an expression for a in terms of b,

(*c*) obtain, in terms of b and c, the coordinates of R.

Given also that $PQ = 4PR$,

(*d*) find the values of a and b.

(13 marks)

[June 1993 Module P1]

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14. A vertical tower of height h stands on a horizontal plane. An observer is standing at a point O on the plane, due south of the tower. From O, the angle of elevation of the top T of the tower is α. The observer walks from O on a bearing θ (east of north), $0° < \theta < 90°$, for a distance $2a$, reaching a point P. From P, the angle of elevation of T is again α.

(*a*) Express h in terms of a, α. and θ.

The observer continues to walk from P, on the same bearing, for a further distance a, reaching a point Q. From Q, the angle of elevation of T is β.

(*b*) Show that

$$\tan^2 \theta = \frac{4\tan^2 \beta - \tan^2 \alpha}{\tan^2 \alpha - \tan^2 \beta}.$$

Hence,

(*c*) show that $\tan \alpha < 2 \tan \beta$,

(*d*) determine the value of the ratio $(\tan \alpha) : (\tan \beta)$ when $\theta = 45°$.

(13 marks)

[June 1993 Module P1]

15.

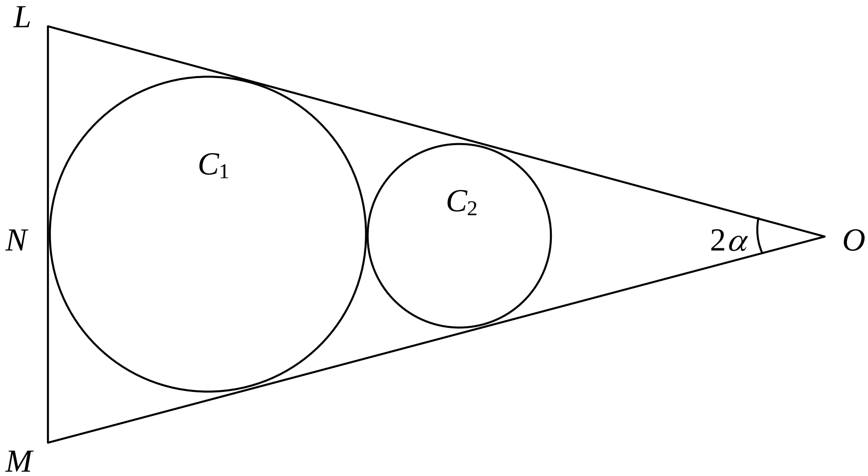

Fig. 1

Two straight lines, OL and OM, are such that $OL = OM$ and angle $LOM = 2\alpha$. Circles C_1 and C_2, with radii R_1 and R_2 respectively, where $R_2 < R_1$, touch OL, OM and each other, and C_1 touches LM at the point N, as shown in Fig. 1.

(*a*) Show that

$$R_2 = \left(\frac{1 - \sin\alpha}{1 + \sin\alpha}\right) R_1.$$

The infinite sequence of circles $C_1, C_2, C_3, \ldots,$ with radii $R_1, R_2, R_3, \ldots,$ and areas $A_1, A_2, A_3, \ldots,$ respectively, is such that, for $i \geq 1$, $R_{i+1} < R_i$ and C_{i+1} touches OL, OM and C_i.

Given that $\sin\alpha = \frac{7}{25}$ and $R_1 = a$,

(*b*) find, in terms of a, the length of ON,

(*c*) determine, in terms of π, the value of

$$\frac{\left(\sum_{i=1}^{\infty} A_i\right)}{(\text{Area of } \Delta OLM)}.$$

(13 marks)

[June 1993 Module P1]

16. The points $P\left(cp, \dfrac{c}{p}\right)$ and $Q\left(cq, \dfrac{c}{q}\right)$, $p \neq 0$, $q \neq 0$, lie on the rectangular hyperbola $xy = c^2$. The tangents to the hyperbola at P and Q intersect at the point R.

(*a*) Find an equation of the chord PQ.

(*b*) Determine the coordinates of R.

The points P and Q move on the hyperbola so that PQ is always normal at P to the hyperbola.

(*c*) Show that $p^3q = -1$.

(*d*) Hence show that the equation of the locus of R may be expressed in the form

$$(x^2 - y^2)^2 + kc^2xy = 0,$$

where k is a constant whose value is to be found.

(13 marks)

[June 1993 Module P2]

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17. (i) Find $\displaystyle\int x \log_{10} x \, \mathrm{d}x$.

(ii) (*a*) Differentiate with respect to x

$$x\sqrt{(c^2 - x^2)}, \ |x| < c,$$

where c is a positive constant.

(*b*) Sketch the curve with equation

$$y = \frac{c^2 + 2x^2}{\sqrt{(c^2 - x^2)}}, \ |x| < c.$$

(*c*) Using the result of (*a*), or otherwise, evaluate

$$\int_0^{\frac{c\sqrt{3}}{2}} \frac{c^2 + 2x^2}{\sqrt{(c^2 - x^2)}} \, \mathrm{d}x.$$

(13 marks)

[June 1993 Module P2]

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18. With respect to an origin O, the position vectors of the points A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, where

$$\mathbf{a} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k},$$
$$\mathbf{b} = 7\mathbf{i} - 2\mathbf{j} + \mathbf{k},$$
$$\mathbf{c} = 2\mathbf{i} + m\mathbf{j} + m\mathbf{k}.$$

Given that AC is perpendicular to AB,

(*a*) determine the value of m.

The point D on AB lies between A and B, such that $BD = 2DA$.

(*b*) Find a vector equation of the line AC, and also a vector equation of the line l which passes through B and is parallel to CD.

(*c*) Hence, or otherwise, show that the line l and the line through A and C intersect, and find the position vector of their point of intersection.

The point E is such that $CDBE$ is a parallelogram.

(*d*) Write down the position vector of E, and verify that $CD = DE$.

(13 marks)

[June 1993 Module P2]

19. Given that $x > 0$,

(*a*) using differentiation, or otherwise, show that

$$x - \sin x > 0,$$

(b) show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{x}\cot\frac{\pi}{x}\right] = \frac{\frac{2\pi}{x} - \sin\frac{2\pi}{x}}{2x^2\sin^2\frac{\pi}{x}}.$$

A regular n-sided polygon has a perimeter of length L.

(*c*) Show that the area of the polygon is

$$\frac{L^2}{4n}\cot\frac{\pi}{n}.$$

(*d*) Deduce that a regular $(n + 1)$-sided polygon with perimeter of length L has a greater area than a regular n-sided polygon with perimeter of length L.

(e) Hence, or otherwise, show that a circle of radius $\frac{L}{2\pi}$ has an area which is greater than the area of any regular n-sided polygon with perimeter of length L.

(17 marks)

[June 1992 – 9371 Section A: Pure Mathematics]

20. Sketch the curve C with parametric equations

$$x = 5\cos\theta,\ \ y = 3\sin\theta,\ 0 \le \theta < 2\pi.$$

Find an equation of the tangent l to C at the point where $\theta = \alpha$, $(\sin\alpha \neq 0)$.

The point A has coordinates $(-4, 0)$ and the point B has coordinates $(4, 0)$. The points P and Q lie on l and are such that the lines AP and BQ are perpendicular to l. Show that the coordinates of the mid-point of PQ are

$$\left(\frac{45\cos\alpha}{9\cos^2\alpha + 25\sin^2\alpha},\ \frac{75\sin\alpha}{9\cos^2\alpha + 25\sin^2\alpha}\right).$$

A circle is drawn with PQ as a diameter. Given that this circle touches the line AB, find the values of $\sin\alpha$.

(17 marks)

[June 1992 – 9371 Section A: Pure Mathematics]

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21. Given that k is a positive integer, state the set of values of x for which

$$\frac{k+1}{(1-x)(1+kx)}$$

may be expanded in the form

$$a_0 + a_1x + a_2x^2 + \ldots. + a_nx^n + \ldots,$$

where $a_0, a_1, a_2, \ldots, a_n, \ldots$ are constants.

Find a_0, a_1 and a_2 and show that

$$a_n = 1 + k(-k)^n.$$

Find the sum to infinity of the series

$$a_0^2 + a_1^2x + a_2^2x^2 + \ldots. + a_n^2x^n + \ldots,$$

giving your answer as a single algebraic fraction.

State the set of values of x for which this series is convergent.

(17 marks)

[June 1992 – 9371 Section A: Pure Mathematics]

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22. Relative to an origin O, the lines L_1 and L_2 have equations

$$L_1\text{: } \mathbf{r} = (7\mathbf{i} + 4\mathbf{j} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} + 4\mathbf{k})$$

$$L_2\text{: } \mathbf{r} = (\mathbf{i} + \mathbf{j} - 5\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{k}),$$

where λ and μ are scalar parameters.

(*a*) Find the acute angle between L_1 and L_2, giving your answer to the nearest tenth of a degree.

Given that P is the point on L_1 with parameter λ and that Q is the point on L_2 with parameter μ,

(*b*) find, in terms of λ and μ, the vector $\overrightarrow{PQ}$.

(*c*) Find the value of λ and the value of μ for which PQ is perpendicular to L_1 and L_2.

(*d*) Hence find the position vectors of P and Q.

(17 marks)

[June 1992 – 9371 Section A: Pure Mathematics]

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23. Given that k is a positive integer,

(*a*) show that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin(2k+1)x}{\sin x}\,dx - \int_0^{\frac{\pi}{2}} \frac{\sin(2k-1)x}{\sin x}\,dx = 0.$

(*b*) deduce that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin(2k+1)x}{\sin x}\,dx = \frac{\pi}{2}.$

(*c*) Show also that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin^2(k+1)x}{\sin^2 x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin^2 kx}{\sin^2 x}\,dx + \frac{\pi}{2}.$

(*d*) Deduce that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin^2 kx}{\sin^2 x}\,dx = k\frac{\pi}{2}.$

(17 marks)

[June 1992 – 9371 Section A: Pure Mathematics]

Pure Mathematics Module P1 (6380): Syllabus from June 1993 – June 1995

	SYLLABUS	*NOTES*
1.	Algebraic operations on polynomials and rational functions.	Addition, subtraction, multiplication and division and the confident use of brackets.
	The concept of a one-one or many-one mapping from $\mathbb{R}$ (or a subset of $\mathbb{R}$) to $\mathbb{R}$. Domain and range of a function. The inverse of a one-one function. Composite functions.	To include simple odd, even and composite functions and their graphical representations. The notation f:$x \to$ will be used. fg will mean "do g first, then f". The range of f is the domain of f^{-1}. Graphical illustration of the relationship between a one-one function and its inverse.
2.	Identities. Factor and remainder theorems applied to polynomials with real coefficients and their real factorization.	The solution of a polynomial equation of degree n, $n \leq 3$, containing at least one rational root may be set. The emphasis will be on simple questions designed to test fundamental principles.
3.	The theory of quadratic functions. Solution of quadratic equations by factorization, completing the square and use of the quadratic formula. Simple cases of symmetric functions of the roots of $ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$, $a \neq 0$, without solving the equation, and applications.	To include completing the square to determine maxima and minima, sketching graphs of curves with equations such as $y = ax^2 + bx + c$ and $y = (ax + b)^2$. The solution of a linear and a quadratic equation in two variables.
4.	Positive and negative rational indices. Surds.	An understanding of the equivalence of $a^{m/n}$ and $^n\sqrt{a^m}$ is required.
5.	The manipulation and solution of simple linear and quadratic inequalities in one variable. The geometrical representation of linear inequalities in two dimensions.	Use of the number line is required. $ax + b \geq cx + d$ $ax^2 + bx + c \geq 0$ $ax + by + c > 0$ Questions will not be set on linear programming but the solution set of simultaneous linear inequalities is required.
6.	Use of the binomial series for a positive integral index.	
7.	Arithmetic and geometric series. Use of the Σ notation. The sum to infinity of a convergent geometric series.	The general terms and the sum to n terms are required. The terms "arithmetic mean" and "geometric mean" should be known.
8.	Rectangular Cartesian coordinates in the xy-plane. Distance between two points, gradient of a line through two points, coordinates of the point dividing a line segment in a given ratio.	
	The straight line and its equation. Conditions for a pair of lines to be parallel or to be perpendicular.	$y - y_1 = m(x - x_1)$ *and* $y = mx + c$ forms of the equation are required. The determination of the acute angle between a general line and a coordinate axis is required.

SYLLABUS		*NOTES*
9.	The six basic trigonometric functions for any angle. Knowledge of the graphs, periodic properties and symmetries of the curves with equations $y = \sin x$, $y = \cos x$ and $y = \tan x$. The use for a circle of $S = r\theta$ and $A = \frac{1}{2}r^2\theta$.	Angles measured in both degrees and radians. Knowledge of the effect of simple transformations like $y = 3\sin x$, $y = \sin(x + \pi/6)$, $y = 3\sin 2x$ are expected.
	Knowledge and use of the identities $\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \text{cosec}^2\theta$. The use of the sine and cosine rules. Applications to simple problems in 2 and 3 dimensions.	Equivalent forms will be expected. The determination of angles between a line and a plane and between 2 planes is expected but in simple cases only.
	Solution of trigonometric equations in a given interval.	e.g. $\sin(x - \pi/2) = \frac{3}{4}$ for $0 < \boldsymbol{x} < 2\pi$, $\cos(x + 30°) = \frac{1}{2}$ for $-180° < x < 180°$, $\tan 2x = 1$ for $90° < \boldsymbol{x} < 270°$.
10.	Differentiation and integration of x^n (where n is rational and excluding x^{-1} for integration). Application of differentiation to gradients. Applications of calculus to simple linear kinematics and to the determination of areas of regions enclosed between a curve, given lines parallel to the coordinate axes and a coordinate axis. Maxima and minima. The equations of tangents and normals to curves with equations in the form $y = \mathrm{f}(x)$.	Notation such as $y = \mathrm{f}(x)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}'(x)$ and $\frac{\mathrm{d}(\mathrm{f}(x))}{\mathrm{d}x} = \mathrm{f}'(x)$ will be used. Questions on maxima and minima may be set in the form of a practical problem where $\mathrm{f}(x)$ has to be determined first. Integrals like $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$ will be required to be formed and evaluated and, as with tangents and normals, $\mathrm{f}(x)$ will take a form such as $ax^m \pm bx^n$ or $(ax + b)^k$ where m, n are rational and k is a small positive integer.

Pure Mathematics Module P2 (6381): Syllabus from June 1993 – June 1995

	SYLLABUS	*NOTES*
1.	The exponential and logarithmic functions. The definition $a^x = e^{x \ln a}$.	Simple properties and graphs. (Series expansions are excluded.)
	The use, manipulation and graphs of simple algebraic, trigonometrical, logarithmic and exponential functions, and combinations of these functions. Curves and equations in cartesian form.	In particular, ability to sketch curves with equations such as $y = x^n$, for integral and simple rational n, $ax + by = c$, $\frac{x^2}{y^2} + \frac{y^2}{b^2} = 1$. Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x - a)$ and $y = f(ax)$. The relation of the equation of a graph to its symmetries.
2.	The function $\lvert x \rvert$. The manipulation of simple algebraic inequalities and inequations including the use of the modulus sign.	The solution of inequalities such as $\frac{1}{x-a} > \frac{x}{x-b}$, $\lvert x - a \rvert > k\lvert x - b \rvert$.
	Concept of a linear relation. Reduction of a given relation to linear form and determination of the constants from graphs.	For example, relations such as: (i) $y = ax^n$, (ii) $y = ab^x$, (iii) $\frac{1}{x} + \frac{1}{y} = \frac{1}{a}$, (iv) $y = ax^2 + b$, (v) $y = ax^2 + bx$ may be considered.
3.	Partial fractions.	The denominators in the partial fractions will be at most of degree 3 and may contain repeated factors. Candidates may be required to find partial fractions for use in summation of series or for simplifying integration or differentiation.
4.	The use of the binomial series $(1 + x)^n$ where n is rational and $\lvert x \rvert < 1$.	
5.	Summation of simple finite series.	For example, $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$, $\sum_{r=1}^{n} r(r+1)$.
6.	Treatment of curves with simple parametric equations.	Questions may be set involving tangents, normals or areas under curves where x and y may be expressed parametrically as either algebraic or trigonometric functions.
	Loci using cartesian or parametric forms.	Geometrical properties of the circle will be expected but not of the parabola, ellipse or hyperbola. The parametric forms considered will include $(ct, c/t)$, $(at^2, 2at)$, $(a \cos t, b \sin t)$ and (at^2, at^3).

	SYLLABUS	*NOTES*
	Sketching of curves given in either Cartesian or parametric form. Asymptotes in simple cases.	Questions will not be set involving oblique asymptotes.
7.	The trigonometric addition and product formulae. Their uses and applications. Expression of $a \cos \theta + b \sin \theta$ in the form $r \cos(\theta + a)$.	Deductions of double and half-angle formulae from the addition formulae will be included.
	General solution of simple trigonometric equations to include those of the form $a \cos \theta + b \sin \theta = c$.	To include graphical interpretation. Involved trigonometric identity questions will *not* be set, but general solutions, and particular solutions within given intervals, to simple equations requiring the use of the addition formulae will be expected.
	The approximations sin x x, tan x x, cos x =1— **~x2.**	Knowledge that these approximations are correct to order x^2 is expected.
	The inverse functions of sine, cosine and tangent defined over suitable intervals and the graphs of these functions.	For example, f^{-1} where $f: x \to \sin x, (x \in \mathbb{R}, \pi/2 \le \mathbf{x} \le \pi/2)$.
8.	Vectors in two and three dimensions. Use of vectors to establish simple properties of geometrical figures. Algebraic operations of the addition of two vectors and the multiplication of a vector by a scalar; their geometrical significance. The orthogonal unit vectors **i**, **j**, **k**, and the cartesian components of a vector. Position vectors and displacement vectors. The equation of a line in the form r = **a** + *t***b**.	
	Scalar product of two vectors; its geometrical significance and algebraic properties.	To include its use for calculating the angle between two lines, including skew lines.
9.	Complex numbers. The Argand diagram. Modulus and argument of complex numbers. Sum, product and quotient of complex numbers. Geometrical representation of sums, products and quotients of complex numbers.	Use of conjugate numbers will be expected.
10.	The idea of a limit and the derivative defined as a limit. The gradient of a tangent as the limit of the gradient of a chord.	
	Differentiation of standard functions.	Differentiation may be used in any part of the syllabus where appropriate. To include the derivatives of x^n, $\sin x$, $\cos x$, $\tan x$, $\arcsin x$, $\arccos x$, $\arctan x$, e^x, a^x, $\ln x$.

SYLLABUS		*NOTES*
	Differentiation of sunis, products and quotients. Differentiation of composite and inverse functions. Differentiation of functions defined implicitly or parametrically. Applications of differentiation to rates of change, tangents and normals, maxima, minima and points of inflexion, curve-sketching, connected rates of change, small increments and approximations.	Skill will be expected in the differentiation of functions generated from standard forms by these operations. Differentiation of inverse functions will be limited to inverse trigonometric functions. $\frac{d^2y}{dx^2}$ will not be required when the curve is given in parametric form.
11.	The idea of the area under a curve as the limit of a sum of areas of rectangles. Integration as the inverse of differentiation. Indefinite and definite integrals.	
	Integration of standard functions.	To include the integrals of l/x, e^x, sin x, cos x, l/(l + x^2), l/√(l – x^2).
	Simple techniques of integration to include decomposition, linear and non-linear substitutions and simple integration by parts. Simple applications of integration using the concept of a limit of a sum including plane areas, volumes of revolution. Approximations to definite integrals using the trapezium rule.	Questions may be set which require more than application of integration by parts. Substitutic~ will be given except in the easiest cases.
	Solution of simple first order differential equations with variables which can be separated.	Determination of arbitrary constants, and sketching of members of families of solution curves, may be required.