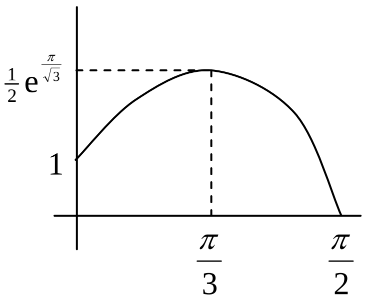
Question Number	Scheme	Marks
1. (*a*)	$0.\dot{4} = 0.4 + 0.4(0.1) + 0.4(0.1)^2 + \ldots$	M1
	Geometric series $a = 0.4$, $r = 0.1$	A1
	$S_\infty = \dfrac{0.4}{1-0.1} = \dfrac{0.4}{0.9} = \dfrac{4}{9}$	M1 A1
	$(\therefore p = 4, q = 9)$	(4)
(*b*)	$T_n = \left(\dfrac{4}{9} - \dfrac{4}{9\times 10^n}\right)10^n$	M1 A1 A1
	$S_n = \dfrac{4}{9}(10 + 10^2 + \ldots 10^n) - \dfrac{4}{9}n$	M1 A1 A1
	$= \dfrac{4}{9}\left(\dfrac{10(1-10^n)}{1-10}\right) - \dfrac{4}{9}n$	M1 A1
	$= \dfrac{4}{9}\left(\dfrac{10}{9}(10^n - 1) - n\right)$ $\quad\left[= \dfrac{4}{81}(10^{n+1} - 10 - 9n)\right]$	A1 (9)
		(13 marks)
Alt (*b*)	$T_n = 4(1 + 10 + 100 + \ldots + 10^n) = \dfrac{4}{9}(10^{n+1} - 1)$	M1 A1 A1
	$S_n = \dfrac{4}{9}\sum_{m=0}^{n} 10^{n+1} - \dfrac{4}{9}\sum_{m=0}^{n} 1$	M1 A1 A1
	$= \dfrac{4}{81}(10^{n+1} - 1) - \dfrac{4}{9}(n+1)$ $\quad\left[= \dfrac{4}{81}(10^{n+1} - 10 - 9n)\right]$	M1 A1 A1 (9)

Question Number	Scheme	Marks
2. (*a*)	$b^2 - 4ac = (4(k+3))^2 - 4(4)(5k+8)$	M1
	$= 16(k^2 + k + 1)$	A1
	$= 16((k + \frac{1}{2})^2 + \frac{3}{4})$	M1
	which is positive, $\therefore$ α, β real and different	A1 (4)
(*b*)	$(\alpha - \beta) = (\alpha + \beta)^2 - 4\alpha\beta$	M1 A1
	$= (k+3)^2 - 4\left(\frac{5k+8}{4}\right)$	M1
	$= k^2 + k + 1$	
	$= (k + \frac{1}{2})^2 + \frac{3}{4}$	
	$\therefore$ $\lvert \alpha - \beta \rvert$ least when $k = -\frac{1}{2}$, $\therefore$ $\alpha - \beta = \frac{\sqrt{3}}{2}$	A1 (4)
(*c*)	$(\alpha + \beta) = k + 3$ $\quad \alpha\beta = \frac{5k+8}{4}$	
	If $\alpha = 2\beta$ then equations become	M1
	$\left.\begin{aligned} 3\beta &= k+3 \\ 2\beta^2 &= \frac{5k+8}{4} \end{aligned}\right\}$	A1
	Solving gives $k = 0$ and $\beta = 1$	M1 A1
	or $k = -\frac{3}{8}$ and $\beta = \frac{7}{8}$	A1 (5)
		(13 marks)

Question Number	Scheme	Marks
3. (*a*)	$y = \lambda x(x-1)(x-2) = \lambda x^3 - 3\lambda x^2 + 2\lambda x$	
	$\frac{dy}{dx} = 3\lambda x^2 - 6\lambda x + 2\lambda$	M1 A1
	when $x = 1$, $\frac{dy}{dx} = -\lambda$; when $x = 2$, $\frac{dy}{dx} = 2\lambda$	M1 A1 ft
	For these to be perpendicular we need	
	$(-\lambda)(2\lambda) = -1$	M1
	$\therefore\ \lambda = \pm\frac{1}{\sqrt{2}}$	A1 (6)
(*b*)	$\int_{-m}^{(m+2)} y\ dx = \lambda\left[\frac{x^4}{4} - x^3 + x^2\right]_{-m}^{m+2}$	M1 A1
	$= \lambda\left[\left(\frac{(m+2)^4}{4} - (m+2)^3 + (m+2)^2\right) - \left(\frac{(-m)^4}{4} - (-m)^3 + (-m)^2\right)\right]$	M1
	$= \lambda[\frac{1}{4}(m^4 + 8m^3 + 24m^2 + 32m + 16) - (m^3 + 6m^2 + 12m + 8) + (m^2 + 4m + 4)$ $- (\frac{1}{4}m^4 + m^3 + m^2)]$	M1 A1 A1
	$= 0$ (*)	A1 (7)
		(13 marks)

Question Number	Scheme	Marks
4. (*a*)	$x^2 - x + 1 = (x - \tfrac{1}{2})^2 + \tfrac{3}{4}$	M1
	$x - \tfrac{1}{2} = \dfrac{\sqrt{3}}{2}\tan\theta, \quad \therefore \dfrac{dx}{d\theta} = \dfrac{\sqrt{3}}{2}\sec^2\theta$	M1
	$\displaystyle\int \frac{4}{3(\tan^2\theta + 1)}\frac{\sqrt{3}}{2}\sec^2\theta \, d\theta = \frac{2}{\sqrt{3}}\theta + k = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + k$	A1
	$\therefore \lambda = \dfrac{2\sqrt{3}}{3}$	A1 (4)
(*b*)	$\dfrac{1}{x^3+1} \equiv \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2-x+1}$	M1 A1
	$\equiv \dfrac{\frac{1}{3}}{x+1} + \dfrac{-\frac{1}{3}x + \frac{2}{3}}{x^2-x+1}$	A1 A1
	$\displaystyle\int \frac{1}{x^3+1}dx = \tfrac{1}{3}\ln\lvert x+1\rvert + \int \frac{2-x}{3(x^2-x+1)}dx$	M1
	$\displaystyle = \tfrac{1}{3}\ln\lvert x+1\rvert + \int \frac{4-2x}{6(x^2-x+1)}dx$	
	$\displaystyle = \tfrac{1}{3}\ln\lvert x+1\rvert + \frac{1}{6}\int \frac{1-2x}{x^2-x+1}dx + \frac{1}{6}\int \frac{3}{x^2-x+1}dx$	M1
	$\displaystyle = \tfrac{1}{3}\ln\lvert x+1\rvert + \tfrac{1}{6}\ln\lvert x^2-x+1\rvert + \frac{1}{2}\int \frac{1}{x^2-x+1}dx$	
	$\displaystyle = \tfrac{1}{3}\ln\lvert x+1\rvert + \tfrac{1}{6}\ln\lvert x^2-x+1\rvert + \frac{1}{2}\left(\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right) + c$	
	$\displaystyle = \tfrac{1}{3}\ln\lvert x+1\rvert + \tfrac{1}{6}\ln\lvert x^2-x+1\rvert + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c$	A1 (7)
(*c*)	$\displaystyle\int_1^2 \frac{1}{x^3+1}dx = \left(\frac{1}{3}\ln 3 - \frac{1}{6}\ln 3 + \frac{1}{\sqrt{3}}\tan^{-1}\sqrt{3}\right) - \left(\frac{1}{3}\ln 2 - \frac{1}{6}\ln 1 + \frac{1}{\sqrt{3}}\tan^{-1}\frac{1}{\sqrt{3}}\right)$	M1
	$\displaystyle = \frac{1}{6}\left(2\ln 3 - \ln 3 - 2\ln 2 + \ln 1\right) + \frac{\sqrt{3}}{3}\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$	
	$\displaystyle = \frac{1}{6}\ln\frac{3}{4} + \frac{\pi\sqrt{3}}{18}$ (*)	A1 (2)
		(13 marks)

Question Number	Scheme	Marks
5. (*a*)	$y = e^{x\sqrt{3}} \cos x$	
	$\frac{dy}{dx} = -e^{x\sqrt{3}} \sin x + \sqrt{3}\, e^{x\sqrt{3}} \cos x$	M1
	$= 2\, e^{x\sqrt{3}} \cos + \left(x + \frac{\pi}{6}\right)$ (*)	M1 A1
	$\frac{d^2 y}{dx^2} = 4\, e^{x\sqrt{3}} \cos + \left(x + \frac{\pi}{3}\right)$ (*)	B1 (4)
(*b*)	TP when $2e^{x\sqrt{3}} \cos + \left(x + \frac{\pi}{6}\right) = 0 \Rightarrow x = \frac{\pi}{3}, y = \frac{1}{2} e^{\frac{\pi}{\sqrt{3}}}, y'' > 0$	M1 A1
		G1 (3)
(*c*)	Area $= \int_0^{\frac{\pi}{2}} e^{x\sqrt{3}} \cos x \; dx = \left[e^{x\sqrt{3}} \sin x \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sqrt{3} e^{x\sqrt{3}} \sin x \; dx$	M1 A1
	$= e^{\frac{\pi\sqrt{3}}{2}} - \sqrt{3}\left(\left[-e^{x\sqrt{3}} \cos x \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \sqrt{3} e^{x\sqrt{3}} \cos x \; dx \right)$	M1 A1
	$= e^{\frac{\pi\sqrt{3}}{2}} - \sqrt{3} - 3\int_0^{\frac{\pi}{2}} e^{x\sqrt{3}} \cos x \; dx$	
	$\therefore 4\int_0^{\frac{\pi}{2}} e^{x\sqrt{3}} \cos x \; dx = e^{\frac{\pi\sqrt{3}}{2}} - \sqrt{3}$	M1
	$\therefore \int_0^{\frac{\pi}{2}} e^{x\sqrt{3}} \cos x \; dx = \frac{1}{4}\left(e^{\frac{\pi\sqrt{3}}{2}} - \sqrt{3} \right)$ (*)	A1 (6)
		(13 marks)

Question Number	Scheme	Marks
6. (*a*)	$\overrightarrow{AQ} = -\mathbf{a} + (1-\lambda)\mathbf{a} + \lambda\mathbf{b} = \lambda(\mathbf{b}-\mathbf{a})$ which is parallel to $\overrightarrow{AB}$, $\therefore Q$ lies on $\overrightarrow{AB}$	B1 (1)
(*b*)	$\mathbf{p}.(\mathbf{b}-\mathbf{a}) = 0 \Rightarrow ((1-\mu)\mathbf{a} + \mu\mathbf{b}).(\mathbf{b}-\mathbf{a}) = 0$	M1
	$\therefore\ \mathbf{a}.(\mathbf{b}-\mathbf{a}) - \mu\mathbf{a}.(\mathbf{b}-\mathbf{a}) + \mu\mathbf{b}.(\mathbf{b}-\mathbf{a}) = 0$	
	$\mathbf{a}.(\mathbf{b}-\mathbf{a}) - \mu(\mathbf{b}-\mathbf{a}) + (\mathbf{b}-\mathbf{a}).(\mathbf{b}-\mathbf{a}) = 0$	
	$\mu = \dfrac{-\mathbf{a}.(\mathbf{b}-\mathbf{a})}{\lvert\mathbf{b}-\mathbf{a}\rvert^2}$ (*)	A1 (2)
(*c*)	$\mu = \dfrac{-\begin{pmatrix}1\\2\\2\end{pmatrix}.\left(\begin{pmatrix}2\\4\\3\end{pmatrix} - \begin{pmatrix}1\\2\\2\end{pmatrix}\right)}{1+4+1} = \dfrac{-\begin{pmatrix}1\\2\\2\end{pmatrix}.\begin{pmatrix}1\\2\\1\end{pmatrix}}{6}$	M1
	$= -\dfrac{7}{6}$	A1
	$\therefore \mathbf{p} = \dfrac{13}{6}\begin{pmatrix}1\\2\\2\end{pmatrix} - \dfrac{7}{6}\begin{pmatrix}2\\4\\3\end{pmatrix} = \tfrac{1}{6}(-\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$	M1 A1 (4)
(*d*)	$\overrightarrow{OL}.\overrightarrow{AB} = \begin{pmatrix}1+\lambda\\2+2\lambda\\2+\lambda\end{pmatrix}.\begin{pmatrix}1\\2\\1\end{pmatrix} = \dfrac{1}{\sqrt{2}}$	M1
	$\therefore\ \dfrac{7+6\lambda}{\sqrt{(1+\lambda)^2 + (2+2\lambda)^2 + (2+\lambda)^2} \times \sqrt{6}} = \dfrac{1}{\sqrt{2}}$	A1
	$\therefore\ 7 + 6\lambda = \sqrt{6\lambda^2 + 14\lambda + 9} \times \sqrt{3}$	M1
	$\therefore\ 18\lambda^2 + 42\lambda + 22 = 0$	A1
	$\lambda = \dfrac{-42 \pm \sqrt{1764 - 4\times 18\times 22}}{2\times 18} = \dfrac{-42\pm\sqrt{180}}{36}$	M1
	$= \dfrac{-7\pm\sqrt{5}}{6}$	A1 (6)
		(13 marks)

Question Number	Scheme	Marks
7. – 23.	In production – to be circulated when finalised…	