Single Pure – Partial Fractions

Patrons are reminded that in order to dive straight in to 'pure' partial fractions, the order of the numerator must be less that the order of the denominator. If this is *not* the case, then you must carry out some sort of polynomial division first. Also, if you see a polynomial on the denominator, your best first move is to see if you can factorise it; for cubics this may mean using factor theorem.

1. Split the following simple expressions into partial fractions.

   (a) $\dfrac{3x-2}{x^2-x}$. $\boxed{\frac{2}{x}+\frac{1}{x-1}}$

   (b) $\dfrac{7x+1}{x^2+x-6}$. $\boxed{\frac{3}{x-2}+\frac{4}{x+3}}$

   (c) $\dfrac{4x-19}{x^2-11x+28}$. $\boxed{\frac{1}{x-4}+\frac{3}{x-7}}$

   (d) $\dfrac{x-25}{x^2-25}$. $\boxed{\frac{3}{x+5}-\frac{2}{x-5}}$

   (e) $\dfrac{19x+21}{3x^2+13x+4}$. $\boxed{\frac{4}{3x+1}+\frac{5}{x+4}}$

   (f) $\dfrac{6-4x}{4x^2+4x-3}$. $\boxed{\frac{1}{2x-1}-\frac{3}{2x+3}}$

   (g) $\dfrac{15x+9}{5x^2+4x-1}$. $\boxed{\frac{10}{5x-1}+\frac{1}{x+1}}$

   (h) $\dfrac{41-10x}{8x^2+18x-5}$. $\boxed{\frac{7}{4x-1}-\frac{6}{2x+5}}$

   (i) $\dfrac{6x^2-2x-2}{x^3-x}$. $\boxed{\frac{3}{x+1}+\frac{2}{x}+\frac{1}{x-1}}$

   (j) $\dfrac{2x^2+x+26}{(x-2)(x+4)(x+1)}$. $\boxed{\frac{2}{x-2}+\frac{3}{x+4}-\frac{3}{x+1}}$

   (k) $-\dfrac{16x^2-15x-13}{(x-2)(x+5)(2x-1)}$. $\boxed{-\frac{2}{2x-1}-\frac{6}{x+5}-\frac{1}{x-2}}$

   (l) $\dfrac{9x^2+20x-36}{x^3+x^2-12x}$. $\boxed{\frac{3}{x}+\frac{5}{x-3}+\frac{1}{x+4}}$

   (m) $\dfrac{-4x^2-12x+48}{x^3+9x^2+11x-21}$. $\boxed{\frac{1}{x-1}-\frac{2}{x+7}-\frac{3}{x+3}}$

   (n) $\dfrac{15x^2+11x-64}{2x^3+7x^2-7x-12}$. $\boxed{\frac{4}{x+1}-\frac{1}{2x-3}+\frac{4}{x+4}}$

2. Now separate the following expressions with repeated factors into partial fractions.

   (a) $\dfrac{2x^2+5x+6}{x(x+1)^2}$ $\boxed{\frac{6}{x}-\frac{4}{x+1}-\frac{3}{(x+1)^2}}$

   (b) $-\dfrac{x^2+13x-5}{2x^3-x^2}$. $\boxed{\frac{3}{x}-\frac{5}{x^2}-\frac{7}{2x-1}}$

   (c) $\dfrac{4x^3-4x^2+x+5}{x(x+1)(x-1)^2}$. $\boxed{\frac{5}{x}+\frac{1}{x+1}-\frac{2}{x-1}+\frac{3}{(x-1)^2}}$

   (d) $\dfrac{3x^3+4x^2-40x-22}{(x-3)^2(x+2)^2}$. $\boxed{\frac{2}{(x+2)^2}+\frac{3}{x-3}-\frac{1}{(x-3)^2}}$

3. Now divide out these *improper* expressions and split the remaining terms into partial fractions.

(a) $\dfrac{2x^2-2x-1}{x^2-3x+2}$. $\boxed{2+\frac{1}{x-1}+\frac{3}{x-2}}$

(b) $\dfrac{x^3+x^2-10x-2}{x^2-2x-3}$. $\boxed{x+3-\frac{2}{x+1}+\frac{1}{x-3}}$

(c) $\dfrac{2x^3+4x^2-6x-6}{x(x-1)(x+2)}$. $\boxed{2+\frac{3}{x}-\frac{2}{x-1}+\frac{1}{x+2}}$

(d) $\dfrac{2x^3+4x^2-1}{(x+1)^2}$. $\boxed{2x-\frac{2}{x+1}+\frac{1}{(x+1)^2}}$

(e) $\dfrac{3x^3-2x^2-8x-18}{(x-2)(x+1)^2}$. $\boxed{3-\frac{2}{x-2}+\frac{5}{(x+1)^2}}$

4. Find $\displaystyle\int_2^3 \frac{3x^2-4x+2}{x^2-2x+1}\,dx$. $\boxed{\frac{7+4\ln 2}{2}}$

5. Find the value of

$$\int_2^3 \frac{-2x^2+14x-6}{x(x-1)(x+2)}dx,$$

giving your answer in the form $\ln\left(\frac{a}{b}\right)$ where a and b are integers. $\boxed{\ln\left(\frac{221184}{78125}\right)}$