THE COLLEGES OF OXFORD UNIVERSITY

MATHEMATICS FOR PHYSICISTS
Specimen of Written Test at Interview

Issued May 2000
Time allowed: 1 hour

For candidates applying for Physics, and Physics and Philosophy

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No calculators or tables may be used

Attempt as many questions as you can

Solve for x, giving real solutions only:

(i) $\ln(x^3) - \ln(5) = \ln(200)$; [2]

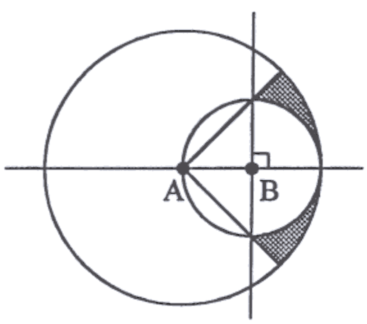
(ii) $x^4 = 0.0081$. [2]

2 The third and fifth terms of an infinite geometric series are $\frac{1}{12}$, $\frac{1}{48}$ respectively. Find:

(i) the first term of the series; [2]

(iii) the sum of the series. [3]

3



The figure shows two circles with radii $2r$, r and centres A, B respectively. Find in terms of r the area of the shaded region. [6]

4 Two identical dice are thrown, one after the other. What are the probabilities that:

(i) the total of the numbers shown is 6; [3]

(ii) the second number is greater than the first? [3]

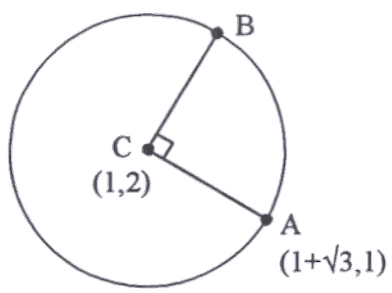
[Turn over]

5 i) Find the stationary points of the function $f(x) = x + \sin(x)$ on the interval $0 \le x < 4\pi$. [4]

ii) Identify each stationary point as a maximum, minimum or point of inflexion. [2]

6 How many solutions to the equation $\sin x \tan x = 0.001$ are there on the interval $0 \le x < 2\pi$? (You may find it helpful to sketch the graphs $y = \sin x$, $y = \tan x$ and $y = \sin x \tan x$ using one set of axes for all three sketches.) [4]

7



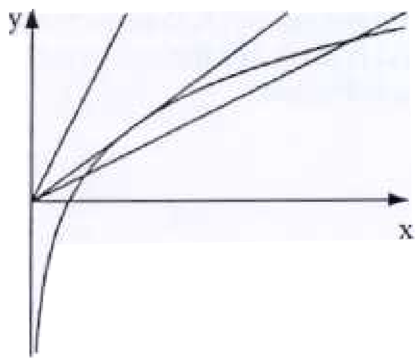
A and B are on the circumference of a circle centred at C; the coordinates of A and C are given on the diagram.
Find the coordinates of B and give the equation of the line CB in the form $y = mx + c$. [4] [2]

8 a) Differentiate with respect to x the function $y = \cos(x^2)$ [2]

b) Find $\int_{-\pi/2}^{\pi/2} \sin x \, dx$. [2]

c) Integrate by parts $\int_{-\pi/2}^{\pi/2} x \sin x \, dx$. [3]

9



The sketch shows the graphs $y = \ln x$ and $y = ax$ for three different values of the constant a. What value of a corresponds to the case in which the graphs touch at one point only? Hint: note that at this point the gradients of the two functions are equal. (Your answer should be expressed in terms of e, the base of natural logarithms.) [6]