Further Pure Core – Integrating Factor

1. Use the integrating factor method to find the solution of the differential equation
$$\frac{dy}{dx} + 2y = e^{-3x}$$
for which $y = 1$ when $x = 0$. Express your answer in the form $y = f(x)$. [OCR]

2. (a) Find the general solution of the differential equation
$$\frac{dy}{dx} + xy = xe^{\frac{x^2}{2}}$$
giving your answer in the form $y = f(x)$.
(b) Find the particular solution for which $y = 1$ when $x = 0$. [OCR]

3. Find the general solution of the differential equation
$$\frac{dy}{dx} - \frac{y}{x} = x$$
giving y in terms of x in your answer. [OCR]

4. Find the solution of the differential equation
$$\frac{dy}{dx} - \frac{x^2 y}{1 + x^3} = x^2$$
for which $y = 1$ when $x = 0$, expressing your answer in the form $y = f(x)$. [OCR]

5. (a) Find the general solution of the differential equation
$$\frac{dy}{dx} + y \tan x = \cos^3 x,$$
expressing y in terms of x in your answer.
(b) Find the particular solution for which $y = 2$ when $x = \pi$. [OCR]

6. (a) Find the general solution of the differential equation
$$\frac{dy}{dx} + \frac{y}{x} = \sin 2x,$$
expressing y in terms of x in your answer.
(b) In a particular case, it is given that $y = \frac{2}{\pi}$ when $x = \frac{\pi}{4}$. Find the solution of the differential equation in this case.
(c) Write down a function to which y approximates when x is large and positive. [OCR]

7. The differential equation
$$\frac{dy}{dx} + \frac{1}{1 - x^2} y = (1 - x)^{\frac{1}{2}} \qquad \text{where } |x| < 1,$$
can be solved by the integrating factor method.
(a) Use an appropriate result given in the formula booklet to show that the integrating factor can be written as $\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$.
(b) Hence find the solution of the differential equation for which $y = 2$ when $x = 0$, giving your answer in the form $y = f(x)$. [OCR]