F Michaelmas Prime Factorisation, HCFs And LCMs

HCF = Highest Common Factor.
LCM = Lowest Common Multiple.
Patrons are reminded that it is *always* preferable to factories a number into its prime factor form. For example
$3960 \Rightarrow$ BAD,
$2^3 \times 3^2 \times 5 \times 11 \Rightarrow$ GOOD.

1. Express the following numbers as a product of primes
   (a) 60. $2^2 \times 3 \times 5$
   (b) 81. 3^4
   (c) 53. 53
   (d) 102. $2 \times 3 \times 17$
   (e) 72. $2^3 \times 3^2$
   (f) 1 485. $3^3 \times 5 \times 11$
   (g) 605. 5×11^2
   (h) 11 375. $5^3 \times 7 \times 13$
   (i) 230. $2 \times 5 \times 23$
   (j) 8 856. $2^3 \times 3^3 \times 41$
   (k) 91 500. $2^2 \times 3 \times 5^3 \times 61$
   (l) 59 290. $2 \times 5 \times 7^2 \times 11^2$
2. Find the smallest integer that 11340 needs to be multiplied by to make a perfect square.
3. Find the smallest integer that 38808 needs to be multiplied by to make a perfect cube.
4. Find the HCF and LCM of 25 and 30. HCF = 5, LCM = $2 \times 3 \times 5^2 = 150$
5. Find the HCF and LCM of 100 and 110. HCF = $2 \times 5 = 10$, LCM = $2^2 \times 5^2 \times 11 = 1100$
6. Find the HCF and LCM of 72 and 90. HCF = $2 \times 3^2 = 18$, LCM = $2^3 \times 3^2 \times 5 = 360$
7. Find the HCF and LCM of $2^3 \times 3 \times 7$ and $3^2 \times 5 \times 7$. HCF = $3 \times 7 = 21$, LCM = $2^3 \times 3^2 \times 5 \times 7 = 2520$
8. Find the HCF and LCM of $2^2 \times 5^3 \times 11^2$ and $3 \times 5^4 \times 7^2$. HCF = $5^3 = 125$, LCM = $2^2 \times 3 \times 5^4 \times 7^2 \times 11^2 = 44467500$
9. (Thanks to JM for this gem.) The LCM of two numbers is 84. If one of the numbers is 28, find all possible values of the other number. 3, 6, 12, 21, 42, 84
10. (Thanks, again, to JM for this gem.) Find the smallest integer greater than 1 with the property that division by each of 3, 4 and 5 yields a remainder of 1. □
11. (Maclaurin) How many positive integers leave a remainder of 31 when divided into 2011? □