

D Summer New GCSE Topics

In order to cater for the new grade 9, some new topics were added to the Edexcel IGCSE syllabus. The following is a selection of questions on these topics.

Completing The Square

1. Complete the square on $x^2 + 6x - 1$. $(x + 3)^2 - 10$
2. Complete the square on $x^2 - 10x + 3$. $(x - 5)^2 - 22$
3. Complete the square on $x^2 + 5x$. $(x + \frac{5}{2})^2 - \frac{25}{4}$
4. Complete the square on $x^2 + 2ax - 1$. $(x + a)^2 - a^2 - 1$
5. Complete the square on $2x^2 + 16x + 3$. $2(x + 4)^2 - 29$
6. Complete the square on $3x^2 + 6x - 13$. $3(x + 1)^2 - 16$
7. Complete the square on $-x^2 + 8x - 3$. $-(x - 4)^2 + 13$
8. By completing the square, solve the equation $x^2 + 4x - 1 = 0$. $x = -2 \pm \sqrt{5}$
9. By completing the square, solve the equation $y^2 - y - 5 = 0$. $\frac{1 \pm \sqrt{21}}{2}$
10. By completing the square, find the vertex (i.e. the maximum or minimum) of the curve $y = x^2 - 8x + 5$. $(4, -11)$
11. By completing the square, find the vertical line of symmetry of the curve $y = x^2 - 2x + 1$. $x = 1$

Arithmetic Sequences

1. The n th term of a sequence is given by $3n + 5$. Write down the first five terms. $8, 11, 14, 17, 20$
2. The n th term of a sequence is given by $n^2 - n$. Write down the first five terms. $0, 2, 6, 12, 20$
3. Find an expression for the n th term of the following:
 - (a) 4, 7, 10, 13, 16, ... $3n + 1$
 - (b) 100, 95, 90, 85, 80, ... $-5n + 105$
 - (c) -11, -4, 3, 10, 17, ... $3n - 18$
4. Find the sum of $3 + 5 + 7 + 9 + 11 + \dots$ (200 terms). 40400
5. Find the sum of $10 + 15 + 20 + 25 + 30 + \dots$ (1000 terms). 2507500
6. Find the sum of $5 + 7 + 9 + 11 + 13 + \dots + 553$. (Hint: find how many terms there are.) 76725

Perpendicular Lines

1. Find the equation of the line perpendicular to $y = \frac{1}{3}x - 1$ which passes through $(3, -2)$. Give your answer in the form $y = mx + c$. $y = -3x + 7$
2. Find the equation of the line perpendicular to $y = 2x + 3$ which passes through $(-1, 4)$. Give your answer in the form $ax + by + d = 0$, where a , b and d are integers. $x + 2y - 7 = 0$

Surds

1. Simplify $\sqrt{8}$. $2\sqrt{2}$
2. Simplify $\sqrt{45}$. $3\sqrt{5}$
3. Simplify $\sqrt{200}$. $10\sqrt{2}$
4. Simplify $\frac{4}{\sqrt{2}}$. $2\sqrt{2}$
5. Simplify $\frac{100}{\sqrt{5}}$. $20\sqrt{5}$
6. Simplify $\frac{9}{\sqrt{27}}$. $\sqrt{3}$
7. Simplify $\frac{1}{1 + \sqrt{2}}$. $\sqrt{2} - 1$
8. Simplify $\frac{1 - \sqrt{3}}{\sqrt{3} + 1}$. $\sqrt{3} - 2$

Expanding Brackets

1. Expand and simplify $x(x - 2)(x + 5)$. $x^3 + 3x^2 - 10x$
2. Expand and simplify $(y - 4)(y - 1)(y + 2)$. $y^3 - 3y^2 - 6y + 8$
3. Expand and simplify $(x - 1)^3$. $x^3 - 3x^2 + 3x - 1$
4. Expand and simplify $(2x + 1)^3$. $8x^3 + 12x^2 + 6x + 1$
5. Expand and simplify $(1 - x^2)^3$. $1 - 3x^2 + 3x^4 - x^6$
6. Expand and simplify $(x + 1)^3 - x(2x + 3)(x - 1)$. $-x^3 + 2x^2 + 6x + 1$

Proof

1. Prove that the sum of five consecutive positive integers is always divisible by 5.
Boils down to $5n + 10 = 5(n + 2)$. So a multiple of 5.
2. Prove (for positive integer n) that $(n + 1)^2 - 2n - 1$ always represents a square number.
Boils down to n^2 . So a square.
3. Prove (for positive integer n) that $(n - 1)^2 + 4n + 1$ always represents a square number.
Boils down to $(n + 1)^2$. So a square.
4. Prove (for positive integer n) that $3n^2 - 3n + 1 + (n - 1)^3$ always represents a cube number.
Boils down to n^3 . So a cube.
5. Prove that the angle subtended in a semicircle is always 90° .
Draw radii. Two isosceles triangles. Then consider angle sum of overall triangle.

Trig Graphs & Graphical Transformations

1. Sketch the following graphs:

(a) $y = \sin x$.

Sine wave

(b) $y = \cos x$.

Cosine wave

(c) $y = \tan x$.

Tan 'wave'

(d) $y = 2 \cos x$.

Cos wave. Up to 2, down to -2

(e) $y = \sin(x + 90)$.

Cos wave

(f) $y = \cos x + 1$.

Cos wave up one unit

(g) $y = \sin(2x)$.

Sine wave squished so wavelength 180 instead of 360

(h) $y = 5 \sin 3x$.

Sine wave up to 5 down to -5, wavelength 120

(i) $y = \tan(x - 90)$.

Tan wave right 90, so asymptote on y-axis

2. By sketching the graph of $y = \sin x$ and $y = \frac{1}{2}$, find all solutions to $\sin x = \frac{1}{2}$ in the range $0 < x < 720$.

$x = 30, 150, 390, 510$